

Traversable wormholes and the exoticity in the HL gravity

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Outline

- 1 Introduction
- 2 A glimpse of Hořava-Lifshitz gravity
- 3 Flaring-out condition and exotic matter
- 4 Constructing a traversable wormhole
- 5 Discussion

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Introduction

- Recently, the **Hořava-Lifshitz (HL) gravity** has been proposed as an **ultraviolet (UV) completion of general relativity**, motivated by the Lifshitz theory in the condensed matter physics. [Hořava '09]
- The HL gravity is established as a **power-counting renormalizable theory** at the cost of **the violation of the Lorentz symmetry**.
- However, in the IR limit, the Lorentz symmetry is somehow recovered.
- A mechanism for recovering the Lorentz symmetry or the renormalization group flow is yet unsolved issue, but the HL gravity has been intensively studied in the area of cosmology and black hole physics.
- For example, it has been shown that the higher curvature terms in HL cosmology can make **negative contributions to energy density** so that the early accelerated expanding universe can be obtained without any inflaton-like fields. [Brandenberger '09, SK '10 & '11]

Introduction cont'd

- On the other hand, a **spacetime wormhole** is a widely known object providing a conceivable method for rapid interstellar travel. [Morris-Thorne '88]
- One important property for traversable wormholes is obviously that there should be **no horizon**, since it would prevent two-way travel, which is actually a critical problem of Schwarzschild wormholes.
- Considering matter sources, we can get rid of the horizon out of the geometry, though the matter which is needed to make wormholes traversable **violates (some of) the desired energy conditions**. This kind of matter is called **the exotic matter**.
- Motivated by the fact that the higher curvature terms in HL cosmology can make negative contributions to energy density, we are trying to see if they can also play the role of exotic matters.

Some key features of the traversable wormhole solutions [Morris-Thorne '88]

- to remain always open
- very small tidal forces
- two-way travel (no horizons)
- rapid transit times (as seen by both travelers and external observers)
- lack of intense, singularity-produced radiation fluxes
- the requirement that some sort of matter or field with radial tension thread the wormhole

⇒ Desired properties of traversable wormholes

- The metric should be both spherically symmetric and static (to simplify the calculations).
- The solution must everywhere obey the Einstein field equations.
- The solution must have a throat that connects two asymptotically flat regions of space-time.
- There should be no horizon.
- ... (some criteria for practical usability)

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Hořava-Lifshitz gravity

- ADM decomposition: $ds^2 = -N^2 c^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
- An anisotropic scaling between time and space: $t \rightarrow b^z t$ and $x^i \rightarrow b x^i$

The deformed HL gravity

$$I_{\text{HL}} = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2\zeta^4} \left(C_{ij} - \frac{\mu\zeta^2}{2} R_{ij} \right) \left(C^{ij} - \frac{\mu\zeta^2}{2} R^{ij} \right) + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{4\lambda - 1}{4} R^2 + (\omega - \Lambda_W) R + 3\Lambda_W^2 \right) \right],$$

- $K_{ij} \equiv \frac{1}{2N} [\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i]$ is the extrinsic curvature of $t = \text{constant}$ hypersurface, and the dot denotes the derivative with respect to time t .
- Here, g_{ij} , R , and ∇_i are the metric, the intrinsic curvature, and the covariant derivative in the three-dimensional hypersurface, respectively.
- C_{ij} is the Cotton-York tensor defined by $C^{ij} = \varepsilon^{ik\ell} \nabla_k \left(R_\ell^j - \frac{1}{4} \delta_\ell^j R \right)$, κ^2 is a coupling related to the Newton constant G_N , and λ is an additional dimensionless coupling constant.

Hořava-Lifshitz gravity cont'd

The deformed HL gravity

$$I_{\text{HL}} = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2\zeta^4} \left(C_{ij} - \frac{\mu\zeta^2}{2} R_{ij} \right) \left(C^{ij} - \frac{\mu\zeta^2}{2} R^{ij} \right) + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{4\lambda - 1}{4} R^2 + (\omega - \Lambda_W) R + 3\Lambda_W^2 \right) \right],$$

- ω represents the IR modification which is essential to have asymptotically flat solutions, and the coupling constants μ , Λ_W , and ζ come from the three-dimensional Euclidean topologically massive gravity action,

$$W = \mu \int d^3x \sqrt{g} (R - 2\Lambda_W) + \frac{1}{\zeta^2} \int \chi(\Gamma),$$

by the “detailed balance” condition (DBC), $\mathcal{L}_V = \sqrt{g} N E^{ij} \mathcal{G}_{ijkl} E^{kl}$, where $\chi(\Gamma)$ represents the gravitational Chern-Simons term, $E^{ij} = g^{-1/2} \delta W / \delta g_{ij}$, and \mathcal{G}_{ijkl} is the inverse of the generalized De Witt metric $G^{ijkl} = \frac{1}{2} (g^{ik} g^{jl} + g^{il} g^{jk}) - \lambda g^{ij} g^{kl}$.

- Then, the scaling of couplings can be obtained as $\kappa^2 \rightarrow \kappa^2$, $\mu \rightarrow b^{-1} \mu$, $\Lambda_W \rightarrow b^{-2} \Lambda_W$, $\zeta \rightarrow \zeta$ and $\omega \rightarrow b^{-2} \omega$.

Einstein limit

Identifying the fundamental constants

$$c = \frac{\kappa^2}{4} \sqrt{\frac{\mu^2(\omega - \Lambda_W)}{3\lambda - 1}}, \quad G_N = \frac{\kappa^2 c^2}{32\pi}, \quad \Lambda = -\frac{3\Lambda_W^2}{2(\omega - \Lambda_W)}$$

The Einstein-Hilbert action in the IR limit ($\lambda = 1$)

$$\begin{aligned} I_{\text{EH}} &= \frac{c^3}{16\pi G_N} \int d^4x \sqrt{-\mathcal{G}} [\mathcal{R} - 2\Lambda] \\ &= \frac{c^2}{16\pi G_N} \int dt d^3x \sqrt{g} N \left[K_{ij} K^{ij} - K^2 + c^2 (R - 2\Lambda) \right], \end{aligned}$$

- \mathcal{G} and \mathcal{R} are the metric and curvature scalar of four-dimensional spacetime.
- We will assume $\lambda = 1$ to keep the Einstein limit and $\Lambda_W = 0$ to consider an asymptotically flat spacetime.
- The HL action can be split by $I_{\text{HL}} = I_{\text{EH}} + I_{\text{HC}}$ for the purpose of considering I_{HC} as contributions to matter-energy in what follows and eventually investigating the possibility to get a traversable wormhole without any external exotic sources.

A homogeneous cosmology with the RW metric w/o DBC [SK '10]

Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right]$$

Equations of motion

$$\begin{aligned} 3(3\lambda - 1)H^2 &= \frac{\kappa^2}{2}\rho + 6 \left[\frac{\alpha}{6} + \frac{k\beta}{a^2} + \frac{2k^2(3\gamma_1 + \gamma_2)}{a^4} + \frac{4k^3(9\sigma_1 + 3\sigma_2 + \sigma_3)}{a^6} \right], \\ &= \frac{\kappa^2}{2} [\rho + \rho_{\text{vac}} + \rho_k + \rho_{\text{dr}} + \rho_{\text{ds}}], \end{aligned}$$

$$\begin{aligned} (3\lambda - 1) \left(\dot{H} + \frac{3}{2}H^2 \right) &= -\frac{\kappa^2}{4}p - 3 \left[-\frac{\alpha}{6} - \frac{k\beta}{3a^2} + \frac{2k^2(3\gamma_1 + \gamma_2)}{3a^4} + \frac{4k^3(9\sigma_1 + 3\sigma_2 + \sigma_3)}{a^6} \right], \\ &= -\frac{\kappa^2}{4} [p + p_{\text{vac}} + p_k + p_{\text{dr}} + p_{\text{ds}}], \end{aligned}$$

where $H = \dot{a}/a$ is the Hubble parameter and we will assume $\lambda > 1/3$ to keep the Einstein limit.

Energy contributions from the potential terms

Additional energy

$$\rho_{\text{vac}} = -\rho_{\text{vac}} = -\frac{3c^2}{8\pi G_N} \frac{\alpha}{6},$$

$$\rho_k = -\frac{1}{3}\rho_k = -\frac{c^2}{8\pi G_N} \frac{k\beta}{a^2},$$

$$\rho_{\text{dr}} = \frac{1}{3}\rho_{\text{dr}} = \frac{c^2}{8\pi G_N} \frac{2k^2(3\gamma_1 + \gamma_2)}{a^4},$$

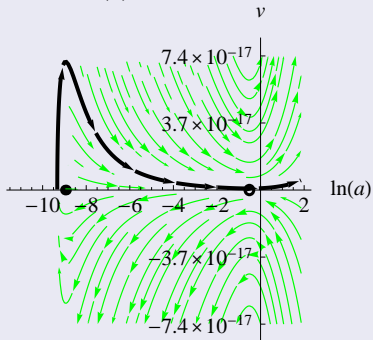
$$\rho_{\text{ds}} = \rho_{\text{ds}} = \frac{3c^2}{8\pi G_N} \frac{4k^3(9\sigma_1 + 3\sigma_2 + \sigma_3)}{a^6}.$$

- It is similar to the general relativity that ρ_{vac} and ρ_k come from the vacuum energy (cosmological constant) and the spatial curvature contributions, respectively. In particular, in the HL theory, ρ_{dr} is called the *dark radiation*.
- Moreover, ρ_{ds} is a *dark scalar*, since it is characterized by $\rho_{\text{ds}} = p_{\text{ds}} \sim a^{-6}$, which is the characteristic of a free scalar.
- It is interesting to note that for the spatial flat geometry, the equations of motion are reduced to those of general relativity up to a factor $(3\lambda - 1)/2$; in other words, **the given HL cosmology is prominent for the nonvanishing spatial curvature.**
- The total *actual* energy can be defined by $\rho_{\text{tot}} = \rho + \rho_{\text{vac}} + \rho_{\text{dr}} + \rho_{\text{ds}}$.

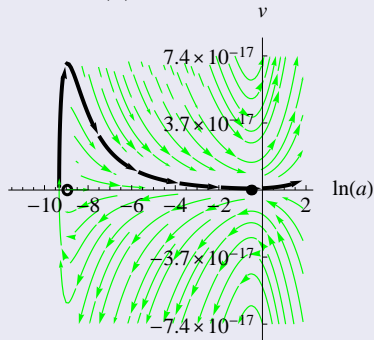
Phase portraits w/o DBC (in log scale)

Phase portraits in a log scale

(a) $k = +1$



(b) $k = -1$



Stress-energy tensors w/ DBC

The metric ansatz: $ds^2 = -e^{2\Phi(r)} c^2 dt^2 + \frac{dr^2}{1-f(r)/r} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$

The equations of motion

$$\begin{aligned} \frac{4}{\kappa^2 r^2} e^{-2\Phi} f' &= T^{tt}, & -\frac{4c^2}{\kappa^2 r^2} (1-f/r) \left[\frac{f}{r} - 2(r-f)\Phi' \right] &= T^{rr}, \\ -\frac{4c^2}{\kappa^2 r^4} \left[\frac{1}{2} r(f/r)' - r(r-f)(\Phi'^2 + \Phi'') - \left(r-f - \frac{1}{2} r^2 (f/r)' \right) \Phi' \right] &= T^{\theta\theta} = T^{\phi\phi} \sin^2 \theta, \end{aligned}$$

where $T^{\mu\nu} = T_{\text{HC}}^{\mu\nu} + T_m^{\mu\nu} \equiv -2(\delta I_{\text{HC}}/\delta \mathcal{G}_{\mu\nu} + \delta I_m/\delta \mathcal{G}_{\mu\nu})$ is the **total stress-energy tensor** and the prime denotes the derivative with respect to r .

The higher curvature contributions to stress-energy

$$\begin{aligned} T_{\text{HC}}^{tt} &= -\frac{\kappa^2 \mu^2}{16c^2 r^2} e^{-2\Phi} \left(\frac{f^2}{r^3} \right)', & T_{\text{HC}}^{rr} &= -\frac{\kappa^2 \mu^2}{16r^7} (r-f) \left[f^2 + 4rf(r-f)\Phi' \right], \\ T_{\text{HC}}^{\theta\theta} = T_{\text{HC}}^{\phi\phi} \sin^2 \theta &= -\frac{\kappa^2 \mu^2}{16r^5} \left[f \left(\frac{f}{r^2} \right)' + \frac{2f}{r} (r-f)(\Phi'^2 + \Phi'' - \Phi') - \left(\frac{f}{r} \right)' (-2r + 3f)\Phi' \right] \end{aligned}$$

Note that the higher curvature terms do not contain the parameter ζ , because the above spherically symmetric metric ansatz makes the Cotton-York tensor vanish. So, in some sense, the static, spherically symmetric metric reduces our model to effectively HL gravity of $z = 2$.

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Flaring-out condition

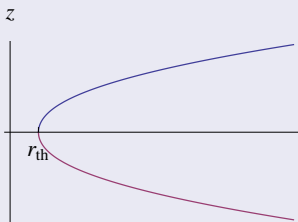
$$\text{Embedding: } ds^2 = \frac{dr^2}{1-f/r} + r^2 d\phi^2 = dz^2 + dr^2 + r^2 d\phi^2 \Rightarrow dz/dr = \pm \sqrt{f/(r-f)}$$

The flaring-out condition near the throat

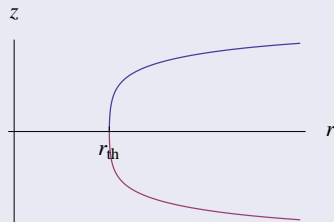
$$\frac{d^2 r}{dz^2} = -\frac{r^2(f/r)'}{2f^2} > 0 \quad \text{near } r = r_{\text{th}}$$

Note that (f/r) should be a decreasing function at least near the throat and eventually vanish as $r \rightarrow \infty$ due to the asymptotic flatness.

The shape of wormhole for $f(r) = -r^3\omega + \tilde{f}(r)$



$$(a) \tilde{f}(r) = \sqrt{r^3(r^3\omega^2 + B)}$$



$$(b) \tilde{f}(r) = \sqrt{r^3[r^3\omega^2 + B(r/r_{\text{th}})^2 e^{-\sqrt{\omega}(r-r_{\text{th}})}]}$$

Exotic matter

Definition of the exoticity function [MT '88]

$$\zeta \equiv \frac{\tau - \rho}{|\rho|}, \text{ where } \tau = -p_r$$

If $\zeta > 0$, then the energy density is less than the tension so that **a certain energy condition should be violated**. In this manner, a matter provided by $\zeta > 0$ is called **an exotic matter**.

For HC contribution, $T_{\text{HC}}^{\mu}{}_{\nu} = \text{diag}(-\rho_{\text{HC}}, -\tau_{\text{HC}}, \rho_{\text{HC}}, \rho_{\text{HC}})$

$$\rho_{\text{HC}} = -\frac{\kappa^2 \mu^2}{16r^2} \left(\frac{f^2}{r^3} \right)', \quad \tau_{\text{HC}} = \frac{\kappa^2 \mu^2}{16r^6} [f^2 + 4rf(r-f)\Phi'],$$

$$\rho_{\text{HC}} = \frac{r}{2} [(\rho_{\text{HC}} - \tau_{\text{HC}})\Phi' - \tau'_{\text{HC}}] - \tau_{\text{HC}},$$

The exoticity function for HC

$$\zeta_{\text{HC}} = \frac{\tau_{\text{HC}} - \rho_{\text{HC}}}{|\rho_{\text{HC}}|} = -\frac{4f^2}{|3f - 2rf'|} \frac{d^2 r}{dz^2} + \frac{4r(r-f)}{|3f - 2rf'|} \Phi'.$$

Traversability and the exotic source

Asymptotically flat Kehagias-Sfetsos wormhole solution with $T_m^{\mu\nu} = 0$ [KS '09]

$$f = -r^3\omega + \sqrt{r^3(r^3\omega^2 + B)}, \quad \Phi = \frac{1}{2} \ln(1 - f/r)$$

Note that the exoticity function of the higher curvature terms vanishes in this case, $\zeta_{\text{HC}} = 0$.

The external source, $T_m^{\mu\nu} = \text{diag}(-\rho, -\tau, p, p)$, considering $f(r) = -r^3\omega + r\sqrt{r^3(r^3\omega^2 + b(r))}$

$$\rho = \frac{\kappa^2 \mu^2 b'}{16r^2}, \quad p = \frac{r}{2} \left[(\rho - \tau)\Phi' - \tau' \right] - \tau,$$

$$\tau = \frac{\kappa^2 \mu^2}{16r^3} \left[-4r^3\omega^2 - b + 4r\omega\sqrt{r(r^3\omega^2 + b)} - 4\sqrt{r(r^3\omega^2 + b)} \left(1 + r^2\omega - \sqrt{r(r^3\omega^2 + b)} \right) \Phi' \right]$$

Note that the function $b(r)$ should be monotonically increasing if the energy density is assumed to be positive.

The exoticity function

$$\zeta = \frac{\tau - \rho}{|\rho|} = \frac{4}{r^4} \left(f + r^3\omega \right) \left[\frac{f^2}{|b'|} \frac{d^2 r}{dz^2} - \frac{r(r-f)}{|b'|} \Phi' \right]$$

Note that the flaring-out condition tells us that the external source is exotic near the throat.

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Traversable wormholes: Case I

The line element with a new radial coordinate $dl = \pm dr / \sqrt{1 - f/r}$

$$ds^2 = -e^{2\Phi} c^2 dt^2 + dl^2 + r^2(\ell) (d\theta^2 + \sin^2 \theta d\phi^2)$$

All traversable wormholes should have a finite $\Phi(r)$ for all $r \geq r_{\text{th}}$. Note that the asymptotic flatness is guaranteed only for the case of $b(r) \ll r\omega$ for large r .

A traversable wormhole (exp. case)

$$b(r) = B(r/r_{\text{th}})^2 e^{-\sqrt{\omega}(r-r_{\text{th}})}, \quad \Phi(r) = 0$$

$$\Rightarrow \rho = \frac{\kappa^2 \mu^2 B(2 - \sqrt{\omega}r)}{16r_{\text{th}}^2 r} e^{-\sqrt{\omega}(r-r_{\text{th}})}$$

Note that the energy density of the exotic matter decays exponentially to zero as desired, but it becomes (small) negative for $r > 2/\sqrt{\omega}$, though it may be positive near the throat. This kind of wormhole require an exotic matter spreading out all the spaces, which seems more or less unphysical.

Traversable wormholes: Case II

A traversable wormhole (WH+KS)

$$b(r) = \begin{cases} B \left[1 + \sin \frac{\pi}{16} \left(\frac{r}{r_{\text{th}}} - 1 \right) \right], & r \leq r_0 \equiv 9r_{\text{th}}, \\ 2B, & r > r_0, \end{cases}$$

$$\Phi(r) = \begin{cases} \frac{1}{2} \ln \left(1 - \frac{f(r_0)}{r_0} \right) + \left(1 - \frac{r}{r_0} \right) \frac{r_0 f'(r_0) - f(r_0)}{2(r_0 - f(r_0))}, & r \leq r_0, \\ \frac{1}{2} \ln \left(1 - \frac{f(r)}{r} \right), & r > r_0. \end{cases}$$

$$\Rightarrow \rho = \begin{cases} \frac{\kappa^2 \mu^2 \pi B}{256 r_{\text{th}} r^2} \cos \frac{\pi}{16} \left(\frac{r}{r_{\text{th}}} - 1 \right), & \text{for } r \leq r_0, \\ 0, & \text{for } r > r_0. \end{cases}$$

The energy density is positive and confined in a sphere of the radius r_0 , and the KS solution outside the sphere tells that $\rho = \tau = 0$.

Traversable wormholes: Case III

A traversable wormhole (WH+Minkowski)

$$b(r) = \begin{cases} B \left[1 - \sin \frac{\pi}{16} \left(\frac{r}{r_{\text{th}}} - 1 \right) \right], & r \leq r_0 \equiv 9r_{\text{th}}, \\ 0 & r > r_0, \end{cases} \quad \Phi(r) = 0.$$

$$\Rightarrow \rho = \begin{cases} -\frac{\kappa^2 \mu^2 \pi B}{256 r_{\text{th}} r^2} \cos \frac{\pi}{16} \left(\frac{r}{r_{\text{th}}} - 1 \right), & \text{for } r \leq r_0, \\ 0, & \text{for } r > r_0. \end{cases}$$

The confined exotic energy is negative, and the outside region is the Minkowski vacuum with $\rho = \tau = \rho_{\text{HC}} = \tau_{\text{HC}} = 0$.

Traversable wormholes & Exotic matter

Energy density & Exoticity function

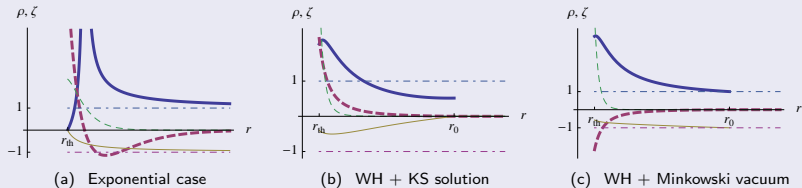


Figure: The exoticity functions (solid lines) and the energy densities (dashed lines) of the exotic source (thick lines) and the higher curvature contribution (thin lines) are plotted.

- Since $\zeta_{\text{HC}} = -1 + \tau_{\text{HC}}/\rho_{\text{HC}}$, the vanishing exoticity function in (b) tells us that $\rho_{\text{HC}}^r/\rho_{\text{HC}} = -1$, and $\zeta_{\text{HC}} \rightarrow -1$ in (a) and (c) reflects $\rho_{\text{HC}}^r/\rho_{\text{HC}} \rightarrow 0$.
- Similarly, for the exotic source, the equation of state becomes $p^r/\rho \rightarrow 0$ for (a) and (c), and $p^r/\rho \rightarrow -1.5$ for (b).
- Note that the equation of state for (b) implies that the exotic matter is a phantom energy in this case.

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Discussion

Summary

- We have tried to build an asymptotically flat wormhole in the deformed HL gravity.
- First of all, the KS wormhole can be constructed without any exotic source, but it turns out to be nontraversable since it has a horizon right at the throat.
- Requiring the absence of horizon, then, the exotic source is crucial to make a traversable wormhole at least in the deformed HL theory, since it cannot be replaced by the higher curvature terms.
- If DBC is relaxed, however, the higher curvature contributions might play the role of the exotic source, because it has been seen that the early acceleration of the Universe can be obtained without any inflaton fields in HL cosmology. [SK '11]
- The exoticity function for $\rho_{\text{HC}} < 0$ can be written by $\zeta_{\text{HC}} = 1 + p_{\text{HC}}^r / \rho_{\text{HC}}$, and the equation-of-state parameter for the dark scalar in HL cosmology turns out to be $\omega_{\text{ds}} = p_{\text{ds}} / \rho_{\text{ds}} = 1$. It naturally yields the positive $\zeta_{\text{HC}} \simeq 2 > 0$.
- What it means is that the higher curvature contribution can play the role of the exotic source when the DBC is relaxed, without resorting to the external exotic matter. Although this argument was not based on the explicit wormhole geometry but just the cosmological side, it gives some insight into the possibility to create the traversable wormhole at the early stage of the Universe in the HL gravity.

Discussion cont'd

Summary cont'd

- We have constructed three sorts of traversable wormholes: one is the wormhole with an exponentially decaying exotic source in the radial direction, another is the wormhole with an exotic source confined in the middle of the KS vacuum, and the other is the wormhole with an exotic source confined in the middle of the Minkowski vacuum.
- Interestingly, in the second case, the exotic source and the higher curvature contributions, respectively, behave like the phantom energy and the dark energy near the boundary of the confined region, while in the first and third cases, both the exotic source and the higher curvature contributions behave like the (dark) matter in the boundary.

It is interesting to note that a spherically symmetric matter distribution and the corresponding higher curvature contribution satisfy the four-dimensional covariant form of energy conservation in the deformed HL gravity, $\nabla_{\mu}^{(4)} T_m^{\mu\nu} = \nabla_{\mu}^{(4)} T_{\text{HC}}^{\mu\nu} = 0$, though it is obvious that the total energy satisfies the conservation of energy in the covariant form in four dimensions, $\nabla_{\mu}^{(4)} T^{\mu\nu} = 0$, because the left-hand sides of the equations of motion are the same as the four-dimensional Einstein tensor, $\mathcal{R}^{\mu\nu} - (1/2)\mathcal{G}^{\mu\nu}\mathcal{R}$, up to the factor $4/\kappa^2$. This behavior is not supposed to be seen in HL theory if DBC is not considered at all. However, we should not say that it is due to DBC.