Drag force from AdS/CFT

Shankhadeep Chakrabortty

IOP (Bhubaneswar, India)

ITP, Beijing, China
December 29, 2011

Introduction

- Strong interaction ⇒QCD. Quarks and gluons (fundamental constituents in a confined phase.)
- In RHIC experiment⇒ head-on collision of highly energetic gold ions.⇒ Quark and gluons are liberated into de-confined phase. ⇒Quark Gluon Plasma (QGP). Perturbative method ($\alpha_s = .5$) and lattice methods are not well suited.
- For certain gauge theories with large coupling constant, we may compute observables via the AdS/CFT correspondence.
- Near-extremal D3-branes provide a gravitational representation of $\mathcal{N} = 4$ super-Yang- Mills theory at finite temperature, large N, and strong 't Hooft coupling, $g_{YM}^2 N \gg 1$.

Introduction

- $SU(N)$ gauge group $\Rightarrow$ Stack of N number D3 blackbrane back-reacts the geometry. The near horizon limit gives black hole in AdS_5 times a five sphere.
- $T_H \Leftrightarrow$ temperature of gauge theory.
- Gravity dual of strongly coupled QCD (gauge group $SU(3)$) is not well-understood. To progress we replace it by $\mathcal{N} = 4$ SYM (gravity dual is known).
- This is an assumption and so far an uncontrolled approximation justified only by successful results (universality of transport coefficients).

Introduction

- Example: For $\mathcal{N} = 4$ SYM, $\frac{\eta}{s} = \frac{1}{4\pi} \sim .08$ (K,S,S)

  RHIC data: $\frac{\eta}{s} \sim .1$ with experimental error.

- In QGP,it is interesting question to ask: What is the dissipative force acting on a heavy quark as an external probe while it passes through the strongly coupled thermal plasma?
- In the same spirits using AdS/CFT, S.Gubser, L. G. Yaffe et al computed the dissipative force on a heavy quark passing through hot $\mathcal{N} = 4$ super-Yang-Mills(SYM) plasma.

Introduction

- How to introduce dynamical quark charged under fundamental representation of $SU(N)$?
- We introduce $D7$ brane in $D3$ brane background such that the geometry is not back reacted. The gauge theory of $D7$ brane contains fundamental quark.(A. Karch and E. Katz.)
- The $D7$ brane is localized far away from $D3$ brane such that the fundamental quark seats on the boundary of AdS_5.
- An open string can always end on $D7$ brane.

Introduction

- A Crucial property of the gauge/gravity dictionary is that it identifies the endpoint of the string as being dual to the quark.
- The mass of the quark is proportional to length of the string.
- There is no contradiction with the fact that $\mathcal{N} = 4$ super-Yang-Mills has no fundamentally charged quarks, because the quark is an external probe of the theory, not part of the theory.

Introduction

- In this context, we summarize Gubser's prescription to calculate the drag force.
- The near horizon limit of the non-extremal D3 brane dual to $\mathcal{N} = 4$ SYM at finite temperature is given as:

  $$ds^2_{AdS_5 - Schwarzschild} = H^{-1/2}(-hdt^2 + \vec{x}^2) + H^{1/2}\frac{dr^2}{h}$$

  $$h = 1 - \frac{r_+^4}{r^4}, H = \frac{l^4}{r^4}$$
- We consider a string hanging from the boundary to horizon along the radial direction.
- A string in AdS_5-Schwarzschild background is described by Nambu-Goto action(String frame).

Introduction

- $S_{NG} = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{-detg_{\alpha\beta}}$

  $g_{\alpha\beta} = G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$, $G_{\mu\nu}$ is AdS_5-Schwarzschild metric.
- The equation of motion: $\nabla_\alpha P^\alpha{}_\mu = 0$

  where $P^\alpha{}_\mu = -\frac{1}{2\pi\alpha}G_{\mu\nu}\partial^\alpha X^\nu$.

- Consider motions of the string in only one of the spatial directions spanned by the D3- brane: the x-direction. In static gauge, $\sigma_\alpha = (t, r)$, the embedding of the worldsheet is completely specified by the function x(t, r).
- $S_{NG} = -\frac{1}{2\pi\alpha'}\int dtdr\sqrt{1 + \frac{h}{H}(\partial_r x)^2 - \frac{(\partial_t x)^2}{h}}$

Introduction

- Static quark seating $x = 0 \Rightarrow$ Dual string hanging from boundary, $X(t, r) = 0$.
- Quark moving (absence of a medium, h $= 1$) $x(t, r) = vt$; $y, z = 0$ $\Rightarrow$ String solution: $X(t, r) = vt$
- $h \neq 1$, signaling the presence of a medium, a moving quark is not the same as a static one.
- $g_{tt} \sim -h + v^2, g_{rr} = \frac{1}{h}$ (world sheet)
- t direction is time like only when $h > v^2 \Rightarrow r > r_+(1 - v^2)^{\frac{1}{4}}$
  The part of the string world sheet with $r < r_+(1 - v^2)^{\frac{1}{4}}$ is purely space like, which means that the string is locally moving faster than the speed of light. This does not make sense if we are aiming to describe the classical dynamics of a string moving in real Minkowski time.

Introduction

- Physically, it is harder and harder to move forward as one approaches the horizon. This evokes the idea that there must be some drag force from the medium.
- The string does not hang straight down from the quark, it trails out behind it.
- If the shape is assumed not to change as the quark moves forward, and if it respects the SO(2) symmetry rotating the y,z coordinates, then it must be specified by: $X(t, r) = vt + \xi(r)$.
- Location of the quark on the boundary is $X = vt$, then $\xi(r) \to 0$ as $r \to \infty$

Introduction

- The momentum conjugate to ξ is $\pi_\xi = \frac{\partial \mathcal{L}}{\partial \xi'} = -\frac{1}{2\pi\alpha} \frac{h\xi'}{H\sqrt{1+\frac{h}{H}\xi'^2 - \frac{(v)^2}{h}}}$
- Solving for ξ' we get,

  $\xi' = \pm(2\pi\alpha')\pi_\xi \frac{H}{h}\sqrt{\frac{h-v^2}{h-(2\pi\alpha')^2{\pi_\xi}^2 H}}$
- Formal solution: $\pi_\xi = 0 \Rightarrow$ Action becomes complex.
- Quark and anti-quark (not of our interest.)
- $\pi_\xi = \frac{v}{\sqrt{1-v^2}}\frac{r_+^2}{l^2}$ (string trails out in front of the quark instead of behind it $\Rightarrow$ It does not describe energy loss.)
- $\pi_\xi = -\frac{v}{\sqrt{1-v^2}}\frac{r_+^2}{l^2}$

Introduction

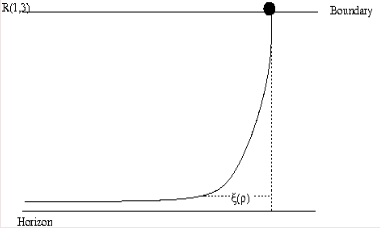

Figure: Configuration of a trailing string dangling down from boundary to horizon. Trailing profile is captured by the function$\xi(r)$

Introduction

- The change of momentum along the string (finite line segment) over a finite time interval is: $\Delta P_x = \int_{\mathcal{I}} dt \sqrt{-G} P^r{}_x = \frac{dp_x}{dt} \Delta t$
- In particular , $\frac{dp_x}{dt}$ is the drag force. The integral is so chosen that $\frac{dp_x}{dt}$ is negative, pointing opposite to the motion.
- Drag force in terms of gauge theory parameters,

$$F = -\frac{\pi \sqrt{g^2{}_{YM} N}}{2} T^2 \frac{v}{\sqrt{1-v^2}}$$

Introduction

- In this work we consider an *uniformly* distributed heavy quark cloud in this hot plasma. We then ask: how does the drag force on an external quark change with the density of the quark cloud?
- First, we construct the gravity dual of the quark cloud. In the bulk, this represents a black hole in the presence of a string cloud. These strings, assumed to be non-interacting, are aligned along the radial direction of the bulk geometry. This dual construction follows from generalizing Gubser's dual construction for single quark to large number of static quark distributed uniformly.
- We have explicitly calculated the back reaction to the metric due to the presence of string cloud.
- Without any approximation we have analyzed the effect of drag force due to the presence of quark cloud .

Plan of the talk.

- Gravity background: New black hole solution.
- Little about stability of black hole.
- Drag force.
- Summary and extension.

Black hole solution.

- In search of the dual theory we start with the gravity action in n+1 dimension ($n \geq 4$) with static string cloud source,

  $\mathcal{S} = S_{Gravity} + S_{stringcloud}$.

  $S_{Gravity} = \frac{1}{16\pi G_{n+1}} \int dx^{n+1} \sqrt{-g}(R - 2\Lambda)$.

  $S_{stringcloud} = -\frac{1}{2} \sum_i \mathcal{T}_i \int d^2\xi \sqrt{-h} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}$.
- $G^{\mu\nu}$ = space-time metric
  $h^{\alpha\beta}$ = world-sheet metric
- μ, ν represents space-time directions.
  α, β stands for world sheet coordinates.
- $\mathcal{T}_i$ is the tension of i^{th} string.

Black hole solution.

- We solve Einstein equation:
  $R_{\mu\nu} - \frac{1}{2} R G_{\mu\nu} + \Lambda G_{\mu\nu} = 8\pi G_{n+1} T_{\mu\nu}$
- $T^{\mu\nu}$ is the space time energy momentum tensor calculated from string cloud source term.
  $T^{\mu\nu} = -\sum_i \mathcal{T}_i \int d^2\xi \frac{1}{\sqrt{|G_{\mu\nu}|}} \sqrt{|h_{\alpha\beta}|} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \delta_i^{n+1}(x - X).$

Black hole solution.

- Metric ansatz:
  $ds^2 = G_{\mu\nu} dx^\mu dx^\nu = G_{tt}(r)dt^2 + G_{rr}(r)dr^2 + r^2 \delta_{i,j} dx^i dx^j$
- Gauge Choice: Static gauge, $t = \xi^0, r = \xi^1$
- Non-zero contribution for energy-momentum tensor
  $T^{tt} = -\frac{a(x)G^{tt}}{r^{n-1}}, \quad T^{rr} = -\frac{a(x)G^{rr}}{r^{n-1}}$
- $a(x) = T \sum_i \delta_i^{(n-1)}(x - X)$
  Strings with uniform tensions T are uniformly distributed over $n-1$ directions

Black hole solution.

- Looking for the gravitational solution is AdS space
  $ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2 h_{ij} dx^i dx^j,$
  h_{ij} is the metric on the $(n-1)$ dimensional boundary
- We replace $a(x)$ by an average density.
  $a = \frac{1}{V_{n-1}} \int a(x) d^{n-1}x = \frac{T}{V_{n-1}} \sum_{i=1}^{N} \int \delta_i^{(n-1)}(x - X_i) d^{n-1}x = \frac{TN}{V_{n-1}}.$
- V_{n-1} going to infinity along with the number of strings N, keeping N/V_{n-1} constant.

Black hole solution.

- The solution:

  $V(r) = K + \frac{r^2}{l^2} - \frac{2m}{r^{n-2}} - \frac{2a}{(n-1)r^{n-3}}.$

  $K = 0, 1, -1$ depending on whether the $(n-1)$ dimensional boundary is flat, spherical or hyperbolic respectively.

  $\Lambda = -n(n-1)/(2l^2).$
- The solution represents a black hole with singularity at $r = 0$ and the horizon is located at $V(r) = 0$.
- The integration constant m is related to the ADM (M) mass of the black hole $M = \frac{(n-1)V_{n-1}m}{8\pi G_{n+1}}$.

Thermodynamics

- The temperature of the black hole :
  $T = \frac{\sqrt{G^{rr}}\partial_r \sqrt{G_{tt}}}{2\pi}|_{r=r_+} = \frac{n(n-1)r_+^{n+2} - 2al^2 r_+^3}{4\pi(n-1)l^2 r_+^{n+1}}$
- The entropy is defined as $S = \int T^{-1} dM$ and the entropy density $s = \frac{r_+^{n-1}}{4G_{n+1}}$.
- s is finite even for black hole with zero mass.
- For temperature $T \geq 0$, there exists only one horizon and the radius of the horizon satisfies
  $r_+^{min} \geq \left(\frac{2al^2}{n(n-1)}\right)^{\frac{1}{n-1}}$

Thermodynamics

- It is to note that zero mass black hole has finite temperature,
  $T_0 = \frac{a}{2\pi}\left(\frac{n-1}{2al^2}\right)^{\frac{n-2}{n-1}}$.
- It is important to note, for $T = 0$
  $m^{min} = -\frac{a}{n} r_+^{min}$.
- Similar to *AdS*- Schwarzschild with negative curvature horizon. (L.Vango)

Thermodynamics

- The specific heat associated with the black hole is

$$C = \frac{\partial M}{\partial T} = \frac{V_{n-1}(n-1)r_+^{n-1}(n(n-1)r_+^n - 2al^2 r_+)}{4G_{n+1}(n(n-1)r_+^n + 2(n-2)al^2 r_+)}.$$

- As $m \geq m^{min}$ C is a continuous and positive function of r_+.

$$\Downarrow$$

Thermodynamical stability of the black hole.

- Finally, we write down the free energy of this black hole $\mathcal{F} = -\frac{(n-1)r_+^n + 2al^2(n-2)r_+}{16\pi(n-1)}$.

Gravitational stability of black hole.

- We study the stability of the black hole geometry with flat horizon($K = 0$) using the gravitational perturbation in a gauge invariant way.
- We consider perturbation on a background space time M^{2+n}

$M^{2+n} = \mathcal{N}^2 \times \mathcal{K}^n$ (Ishibashi, Kodama, Seto)

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2\delta_{ij}dx^i dx^j,$$

$$V(r) = \frac{r^2}{l^2} - \frac{2m}{r^{n-1}} - \frac{2a}{nr^{n-2}}$$

Gravitational stability of black hole.

- Each perturbed tensor realized on the maximally symmetric space $\mathcal{K}^n$ can be grouped into three classes of components, e.g, scalar, vector, and tensor such that Einstein equations of motion respect the decomposition.
- We study tensor and vector perturbation.

Gravitational stability of black hole.

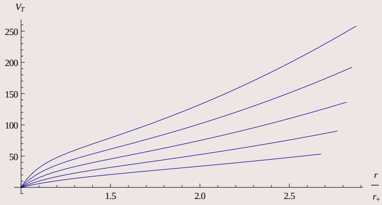

Figure: This is a plot of the effective potential in case of the tensor perturbation.The positivity of the curve beyond horizon shows the stability of the black hole we are considering.

Vector perturbation.

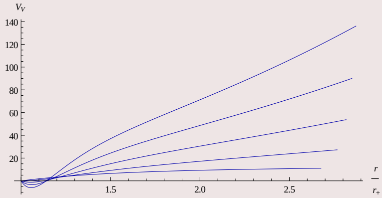

Figure: This is a plot of the effective potential V_V in case of the vector perturbation. Beyond horizon, the plot shows V_V is not always non-negative for $p > 3$.

Computation of drag force

- We now like to calculate the dissipative force experienced by the external heavy quark moving in the cloud of heavy quarks. Our aim is to study the force as a function of the cloud density.
- Here we follow Gubser's prescription. The drag force on a very massive quark with fundamental $SU(N)$ charge at finite temperature is calculated holographically by studying the motion of a infinitely long trailing string whose end point is on the boundary. This end point represents the massive quark whose mass is proportional to the length of the string.
- We will consider here the gauge theory on R^3 coordinatized by x^1, x^2, x^3. This means, for the purpose of this computation, we only consider $K = 0, n = 4$ case of the black holes discussed previously.

Computation of drag force

- Let us consider the motion of a string only in one direction, say x^1. In static gauge, $t = \xi^0, r = \xi^1$, the embedding of the world-sheet is given by the function $x^1(t, r)$.
- The induced action of the string :

$$S = -\frac{1}{2\pi\alpha'}\int dtdr\sqrt{1 + \frac{3r^4 - 2al^2r - 6ml^2}{3l^4}(\partial_r x^1)^2 - \frac{3r^4}{3r^4 - 2al^2r - 6ml^2}(\partial_t x^1)^2}.$$

- where we have scaled x^1 by l.
- The ansatz that describes the behavior of the string with attached quark moving with constant speed v along x^1 is given by:

$$x^1(r, t) = vt + \xi(r)$$

Computation of drag force

- Action simplifies to

$$S = -\frac{1}{2\pi\alpha'}\int dtdr\sqrt{1+\frac{3r^4-2al^2r-6ml^2}{3l^4}(\partial_r\xi)^2-\frac{3r^4}{3r^4-2al^2r-6ml^2}v^2}.$$

- Momentum conjugate to ξ: $\pi_\xi = \frac{\partial\mathcal{L}}{\partial\xi'}$

$$= -\frac{1}{2\pi\alpha'}\frac{(3r^4-2al^2r-6ml^2)\partial_r\xi}{3l^4\sqrt{1+\frac{3r^4-2al^2r-6ml^2}{3l^4}\xi'^2-\frac{3r^4}{3r^4-2al^2r-6ml^2}v^2}}$$

- We define: $H=\frac{l^4}{r^4}$, $h=\frac{3r^4-2al^2r-6ml^2}{3r^4}$
- Solving for ξ':

$$\xi' = -(2\pi\alpha')\pi_\xi\frac{H}{h}\sqrt{\frac{h-v^2}{h-(2\pi\alpha')^2{\pi_\xi}^2H}}$$

Computation of drag force

- $h(r_+) = 0$ and $h(r_B) = 1$. So denominator changes sign between horizon and boundary. ξ' to be real everywhere we set $h \le v^2$ and $h \le (2\pi\alpha')^2 {\pi_\xi}^2 H$
- From the equality, we set

  $$h = v^2 \Longrightarrow 3(1 - v^2) r_v^4 - 2al^2 r_v - 6ml^2 = 0$$

  $$h = (2\pi\alpha')^2 {\pi_\xi}^2 H \Longrightarrow \pi_\xi = -\frac{1}{2\pi\alpha'} \frac{v r_v^2}{l^2}$$
- To calculate the flow of momentum $\frac{dp_1}{dt}$ down the string we need the following integral:

  $$\Delta p_1 = \int_{\mathcal{I}} dt \sqrt{-g}\, p^r{}_{x^1} = \frac{dp_1}{dt} \Delta t$$

  The integral is taken over some time interval $\mathcal{I}$ of length Δt.

Computation of drag force

- $\frac{dp_1}{dt}$ is the drag force. The integral is so chosen that $\frac{dp_1}{dt}$ is negative, pointing opposite to the motion.
- Drag force $\Rightarrow \frac{dp_1}{dt} = \sqrt{-g}{P^r}_{x^1} \Rightarrow F = -\frac{1}{2\pi\alpha'}\frac{vr_v^2}{l^2}$.
- We wish to rewrite the expression of the dissipative force in terms of gauge theory parameters.
- We solve : $T = \frac{n(n-1)r_+^{n+2} - 2al^2 r_+^3}{4\pi(n-1)l^2 r_+^{n+1}}$, for r_+.

$$\Downarrow$$

$$r_+ = \frac{l^2}{6}A(T, b)$$

Computation of drag force

- $A(T,b) = ([\{2(9b+4\pi^3T^3)+6\sqrt{b(9b+8\pi^3T^3)}\}^{1/3}$
$$+2\pi T\{1+\frac{2^{2/3}\pi T}{\{(9b+4\pi^3T^3)+3\sqrt{b(9b+8\pi^3T^3)}\}^{1/3}}\}])$$
  b is the scaled quark cloud density, $b = a/l^4$.
- $\frac{l^4}{\alpha'^2} = {g_{YM}}^2 N$.
  g_{YM} = Yang-Mills(YM) gauge coupling, N = Order of the gauge group $SU(N)$.
- Modified drag force:
  $F = -\frac{A^2}{72\pi}\sqrt{{g_{YM}}^2 N}v\frac{{r_v}^2}{{r_+}^2}$.

Computation of drag force.

- Now we combine following two equations,

a.) $V(r_+) = 0 \Rightarrow \frac{r_+^2}{l^2} - \frac{2m}{r_+^{n-2}} - \frac{2a}{(n-1)r_+^{n-3}} = 0$

b.) $h(r_v) = v^2 \Rightarrow 3(1-v^2)r_v^4 - 2al^2 r_v - 6ml^2 = 0$

into a single one involving $\frac{r_v^2}{r_+^2}$

$(1-v^2)y^4 - \frac{144b}{(A(T,b))^3}(y-1) - 1 = 0$

Where $y = \frac{r_v^2}{r_+^2}$.

Computation of drag force.

- It turns out that the real positive solution of the above equation is expressible in terms of $A(T, b)$ and b itself and is denoted as $f(A, b)$.

- We write the final expression of the drag force in terms of temperature of the gauge theory and quark cloud density.

  $F = -\frac{A^2}{72\pi}\sqrt{{g_{YM}}^2 N} v f(A, b)^2$ (SC, PLB 2011)

- We study the drag force for different values of heavy quark density and for fixed T.

  For $T = 0$ we calculated the drag force, $F \sim -b^{2/3}$.

  We also calculate the free energy, $\mathcal{F} = -\frac{3ar_+}{32\pi} \sim -b^{\frac{4}{3}}$

Computation of drag force.

- For generic temperature and small b, it is possible to have a power series solution of drag force in b. However, for appreciable density, we find it more suitable to analyze F in terms of plots.

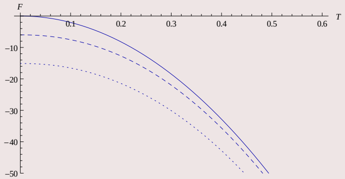

Figure: This plot shows F as a function of T for the values of quark density $b = 0$ (Solid), 0.5 (Dashed), 2 (Dotted) respectively. We see that the larger the quark density, the more is the dissipative force..

Computation of drag force..

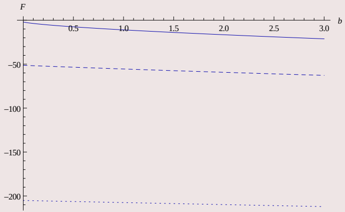

Figure: This is the plot of F as a function of b for the values of $T = .1$ (Solid), .5 (Dashed), 1 (Dotted) respectively. We see from the plot that the larger the temperature, the more is the dissipative force.

Summary

- Using the AdS/CFT duality, we have computed the dissipative force experienced by an external heavy quark with fundamental $SU(N)$ charge moving in the heavy quark cloud at finite temperature.
- We have plotted it with respect to both T and b separately while keeping one of them constant at a time. Both plots exhibit an enhancement in the drag force in the presence of evenly distributed quark cloud.

Summary

- In the context of quark gluon plasma (QGP) it is important to mention that the dynamics of a heavy quark (say charm) passing through the plasma is usually described by considering it's interaction with the medium and the resulting energy loss is calculated. In such calculations, any possible effects of other heavy quark due to the back-reaction of the plasma are neglected. In the context of $\mathcal{N} = 4$ SYM, our work can perhaps serve as an attempt to compute such back-reaction effects. Within the gauge/gravity correspondence, such effects can be modeled in terms of the deformation of the geometry due to finite density string cloud. This work shows that the back-reacted gravity background is explicitly computable.

Summary and extension.

- The geometry implicates existence of black hole parametric by it's mass m and the string cloud density a with finite horizon and singularity at $r = 0$. The black hole exhibits finite temperature behavior in the zero mass limit and emerges with negative mass at $T = 0$, thus resembles like AdS-Schwarzschild black hole with negative curvature horizon.
- We have explicitly checked the thermodynamical stability.

- Using gauge invariant perturbation method we have been able to show the stability only up to tensor and vector perturbation.
- Some yet unresolved issues: Search for the brane geometry whose near horizon geometry contains the black hole we are considering. Stability of the gravitational background under scalar perturbation.

Thank You