Cosmic Microwave Background Anisotropy

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- Testing CMB angular correlation function with WMAP
- Testing direction-dependent power spectrum with Planck
- Testing early Universe models with future CMB experiments (Planck, CMBPol etc…)

Testing CMB angular correlation function with WMAP

Is the Universe statistical isotropy?

↓

Does the Universe lack large angular correlations?

Two Points Correlation Function

$$\Delta T(\theta,\phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi)$$

$Y_{lm}(n)$ are the spherical harmonics

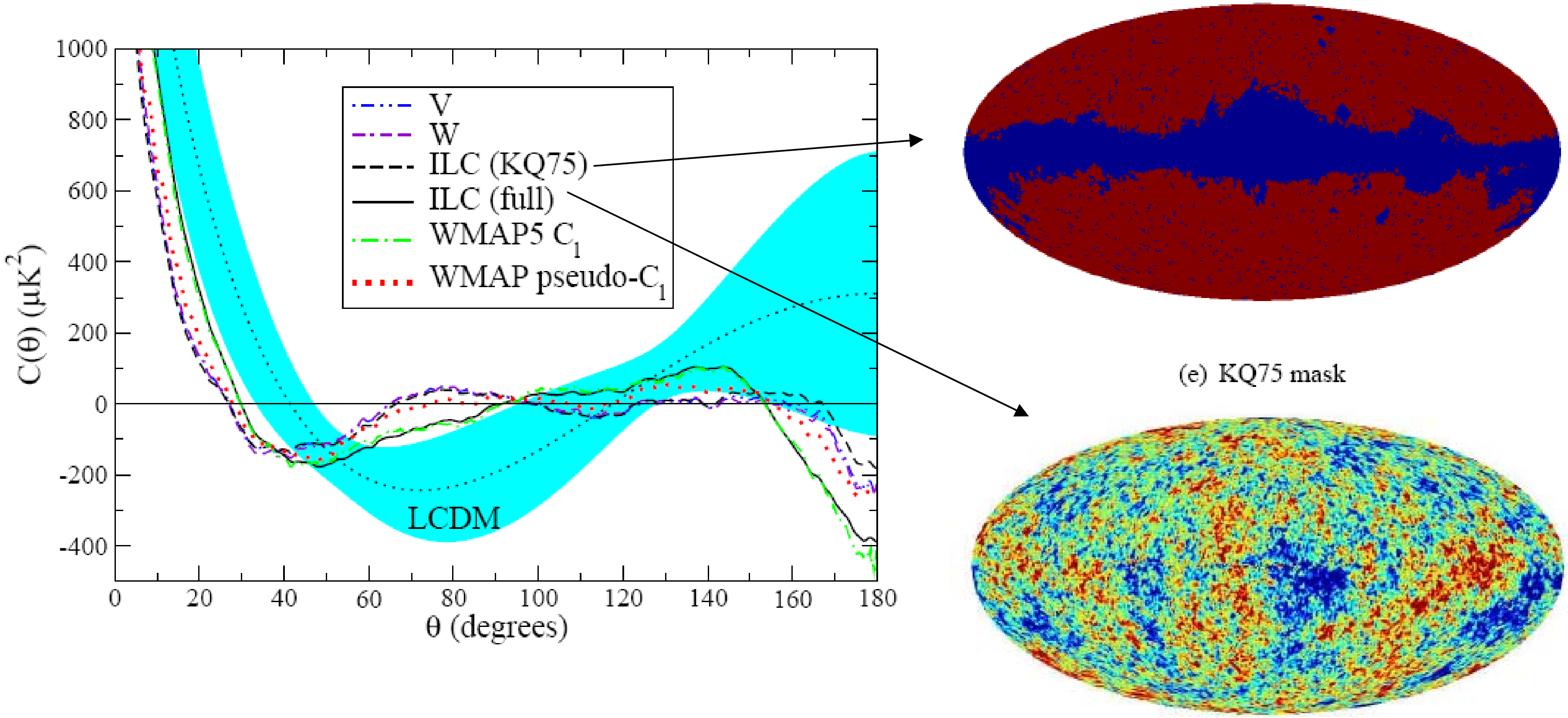
$$\begin{aligned} C(\theta) &= \langle \Delta T(\mathbf{n}) \Delta T(\mathbf{n}+\theta) \rangle_\theta \\ &= \frac{1}{4\pi} \sum_{lm} \sum_{l'm'} \langle a_{lm} a^*_{l'm'} \rangle \, Y_{lm}(\mathbf{n}) Y_{l'm'}(\mathbf{n}+\theta) \\ &= \frac{1}{4\pi} \sum_{l=2}^{l_{\max}} (2l+1) C_l P_l(\cos\theta) \end{aligned}$$

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$$

Sum to lmax=15 is enough

$$\begin{aligned} S_{1/2} &= \int_{-1}^{1/2} C^2(\theta) d\cos\theta \\ &= \int_{60^0}^{180^0} C^2(\theta) \sin\theta d\theta \end{aligned}$$

Arguments of lack of large angular correlations



(e) KQ75 mask

(a) WMAP 5-year ILC map

Copi et al. 2009

CHHS08

Data Source	$S_{1/2}$ $(\mu K)^4$	$P(S_{1/2})$ (per cent)	$6C_2/2\pi$ $(\mu K)^2$	$12C_3/2\pi$ $(\mu K)^2$	$20C_4/2\pi$ $(\mu K)^2$	$30C_5/2\pi$ $(\mu K)^2$
V3 (kp0, DQ)	1288	0.04	77	410	762	1254
W3 (kp0, DQ)	1322	0.04	68	450	771	1302
ILC3 (kp0, DQ)	1026	0.017	128	442	762	1180
ILC3 (kp0), $\mathcal{C}(>60°)=0$	0	—	84	394	875	1135
ILC3 (full, DQ)	8413	4.9	239	1051	756	1588
V5 (KQ75)	1346	0.042	60	339	745	1248
W5 (KQ75)	1330	0.038	47	379	752	1287
V5 (KQ75, DQ)	1304	0.037	77	340	746	1249
W5 (KQ75, DQ)	1284	0.034	59	379	753	1289
ILC5 (KQ75)	1146	0.025	81	320	769	1156
ILC5 (KQ75, DQ)	1152	0.025	95	320	768	1158
ILC5 (full, DQ)	8583	5.1	253	1052	730	1590
WMAP3 pseudo-C_ℓ	2093	0.18	120	602	701	1346
WMAP3 MLE C_ℓ	8334	4.2	211	1041	731	1521
Theory3 C_ℓ	52857	43	1250	1143	1051	981
WMAP5 C_ℓ	8833	4.6	213	1039	674	1527
Theory5 C_ℓ	49096	41	1207	1114	1031	968

Copi et al. 2009

Thus the full-sky results seem inconsistent with cut-sky results and they appear inconsistent in a manner that implies that *most of the large-angle correlations in reconstructed sky maps are inside the part of the sky that is contaminated by the Galaxy.*

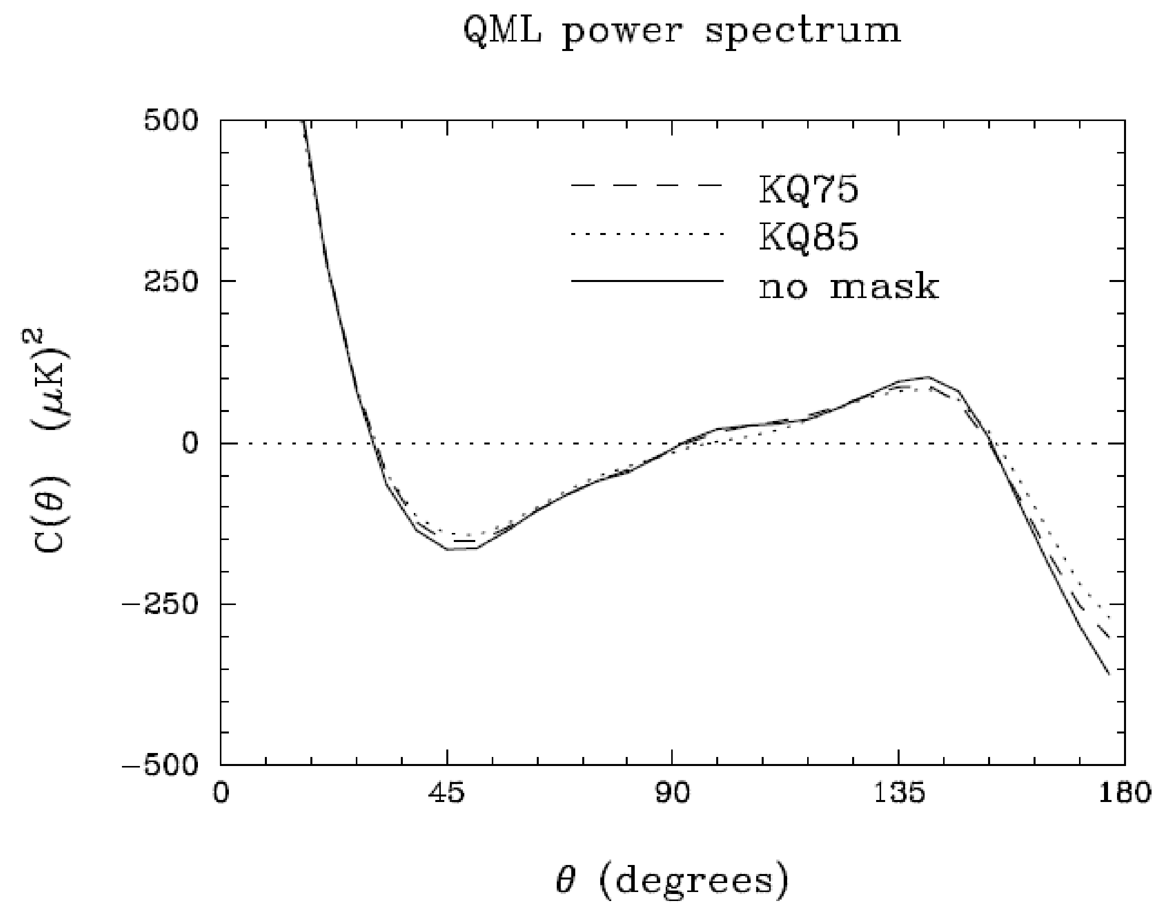
- The reason of the low probability value:

Direct Pixel Estimator of Angular Correlation Function on a cut sky

+ a posteriori choice of the $S_{1/2}$ statistic.

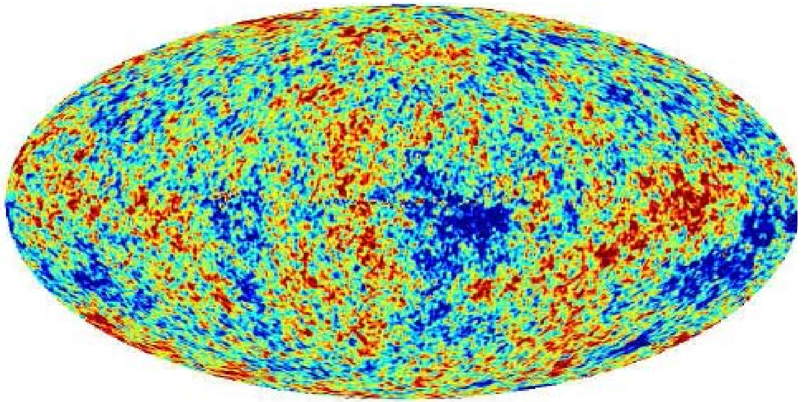
G. Efstathiou, YZM and D. Hanson, MNRAS 407(2010)2530

EMH09

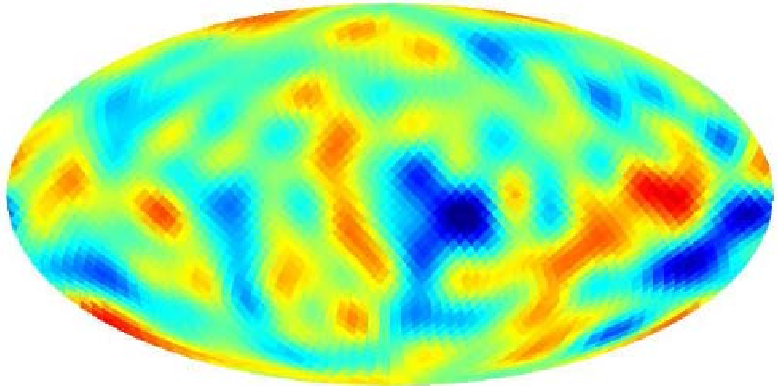


$l_{max}=32$

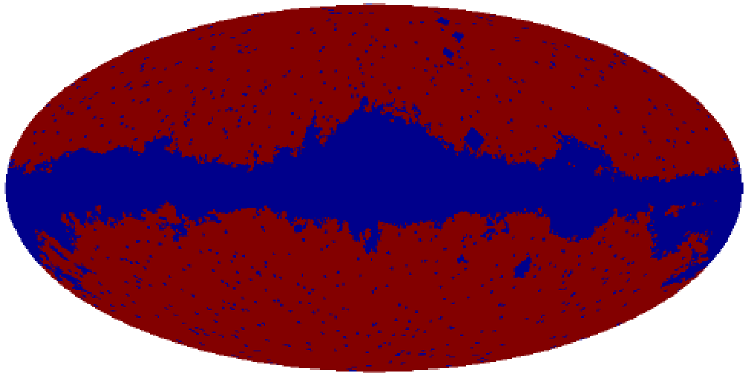
The Quadratic Maximum Likelihood estimator effectively performs the reconstruction for a_{lm} , but uses the assumption of statistical isotropy to downweight 'ambiguous' modes that are poorly constrained by the sky cut.



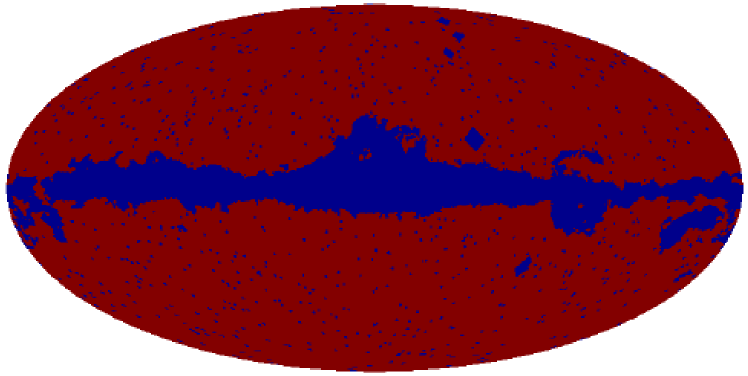
(a) WMAP 5-year ILC map



(c) whole-sky map smoothed to $l_{max} = 15$

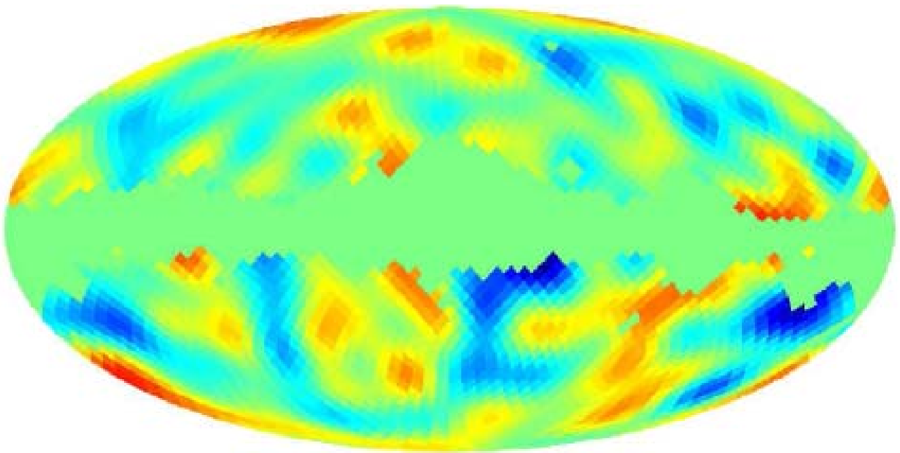


(e) KQ75 mask

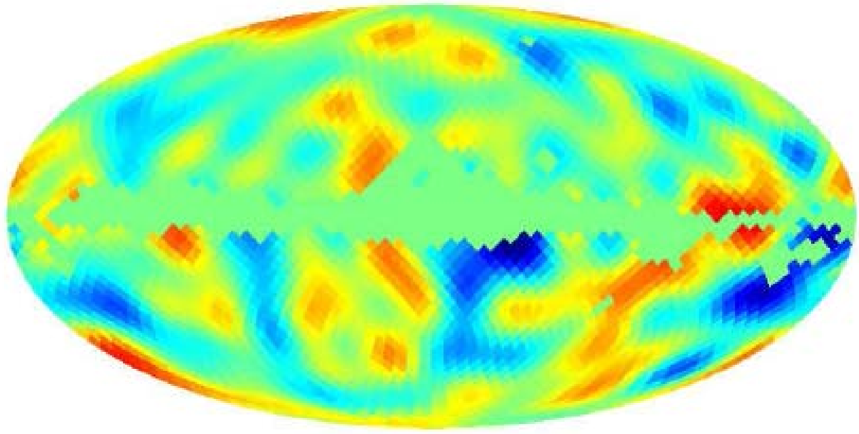


(f) KQ85 mask

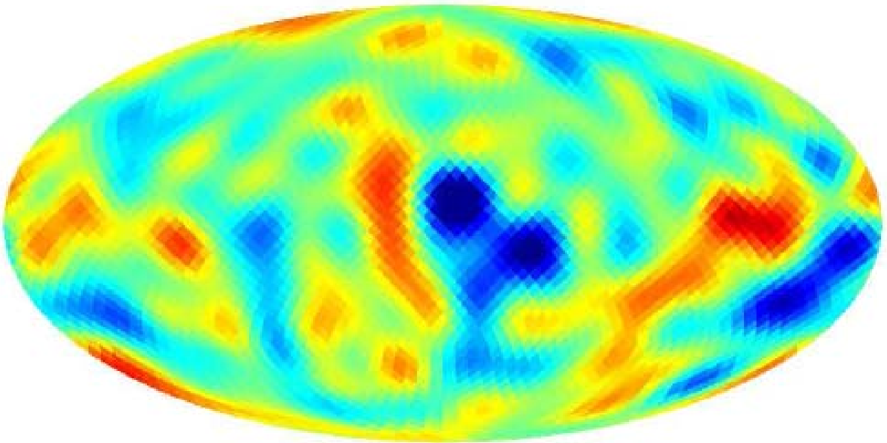
(g) Scales (Unit: mK)



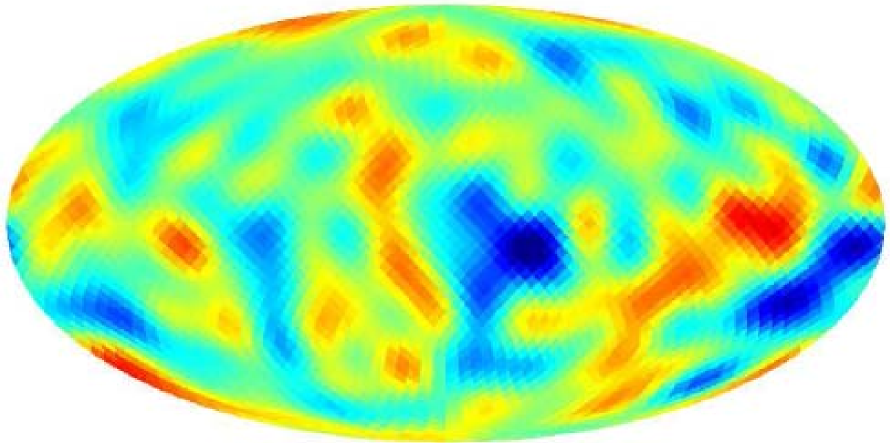
(c) Smoothed map $l_{max} = 15$ masked by KQ75



(d) Smoothed map $l_{max} = 15$ masked by KQ85

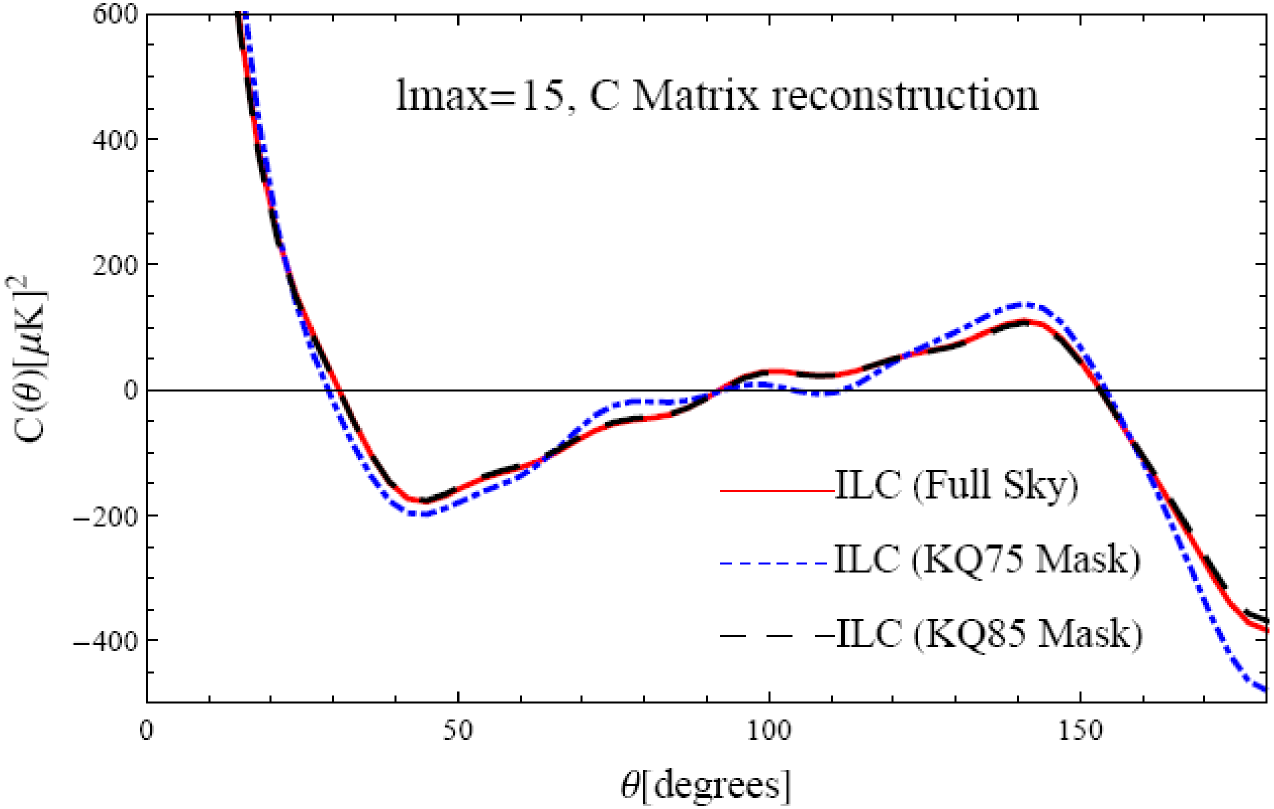


(c) Reconstruction map to $l_{max} = 15$ (KQ75 mask)



(d) Reconstruction map to $l_{max} = 15$ (KQ85 mask)

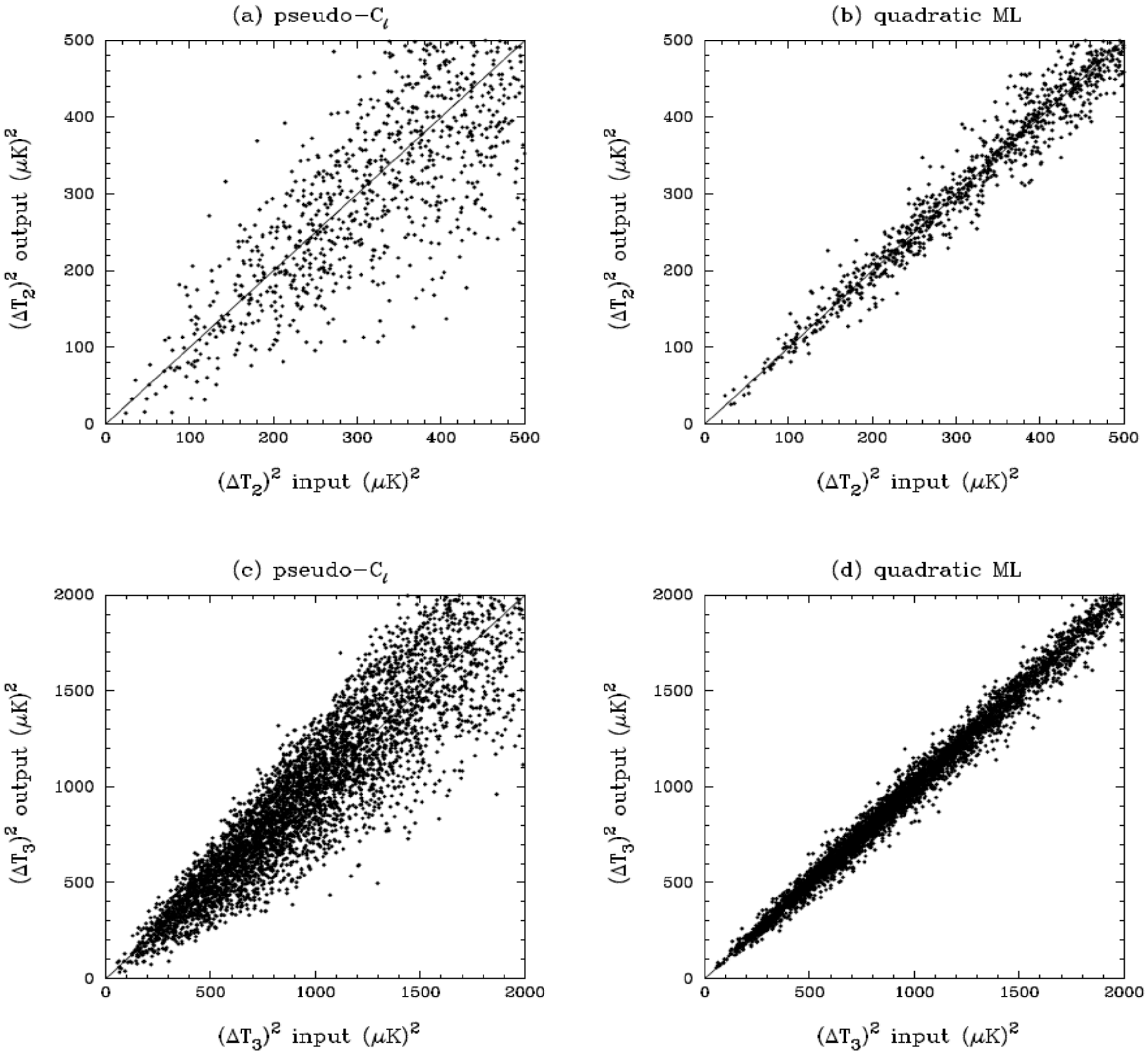
Results for $C(\theta)$



Efstathiou et al. (2009) also reconstructed the low-l multipoles across the foreground sky cut region in a manner that was numerically stable, without an assumption of statistical isotropy. Their method relied on the fact that the low multipole *WMAP* data are signal-dominated and that the cut size is modest. They showed that the small reconstruction errors introduce no bias and they did not depend on assumptions of statistical isotropy or Gaussianity. The reconstruction error only introduced a small "noise" to the angular correlation function without changing its shape.

C Bennett et al. 2010 (WMAP7)

Compare different estimators



Efstathiou 2003.

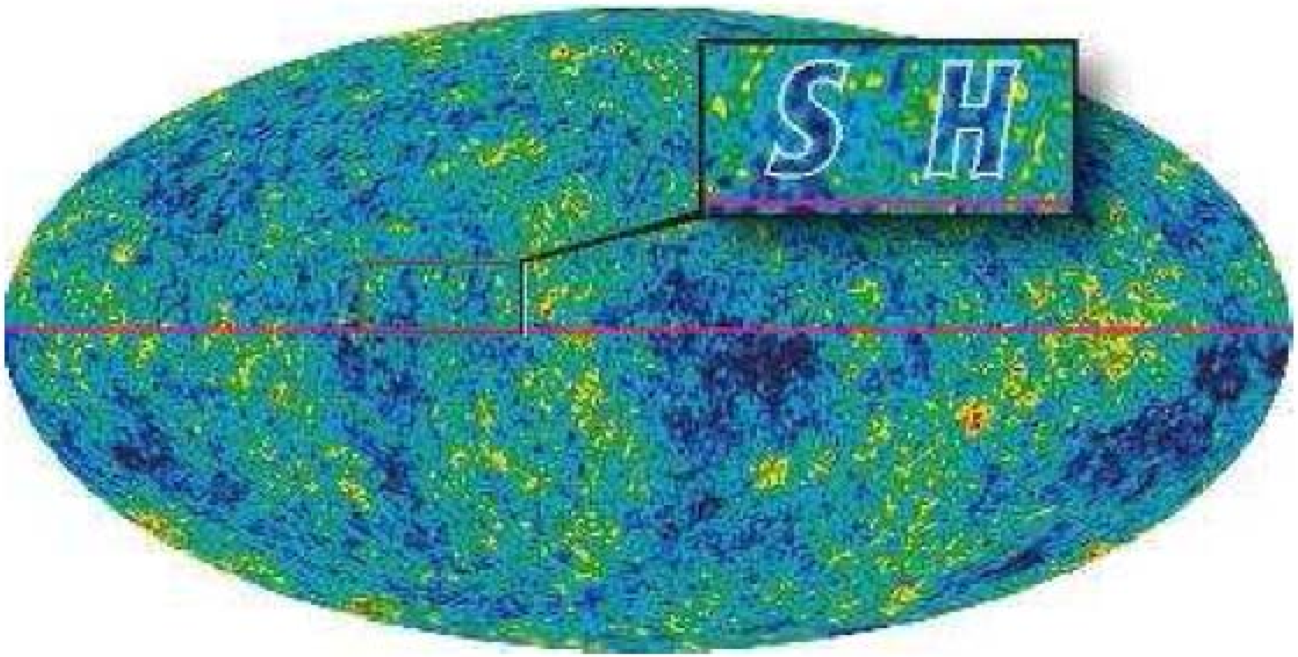
What is a posteriori statistic?

What is a posteriori statistics?

A posteriori choices can have a substantial effect on the estimated significance of features. For example, it is not unexpected to find a 2σ feature when analyzing a rich data set in a number of different ways. However, to assess whether a particular 2σ feature is interesting, one is often tempted to narrow in on it to isolate its behavior. That process involves *a posteriori* choices that amplify the apparent significance of the feature.

C Bennett et al. 2010 (WMAP7)

What is the probability of this “SH” initial occurrence?

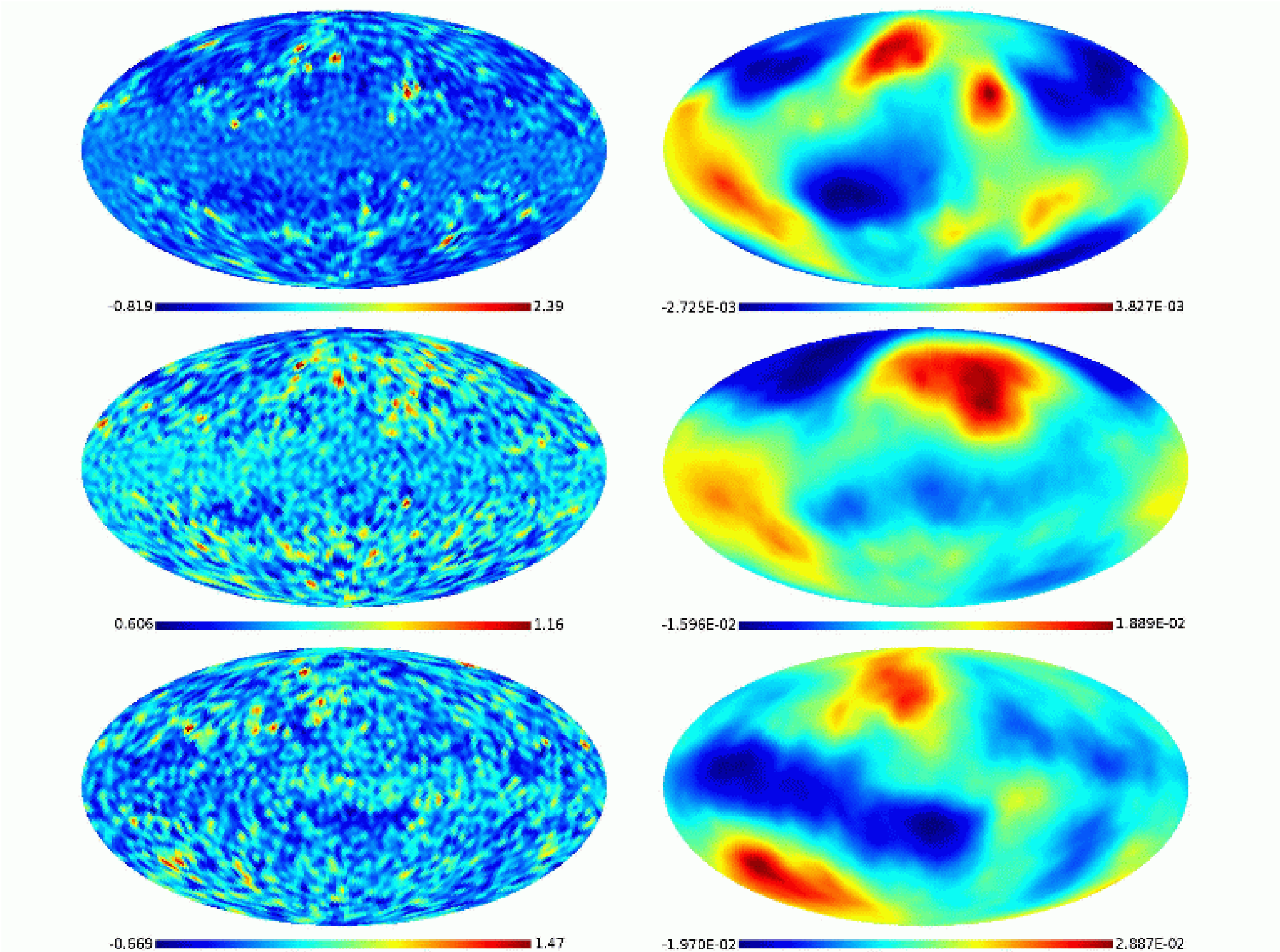


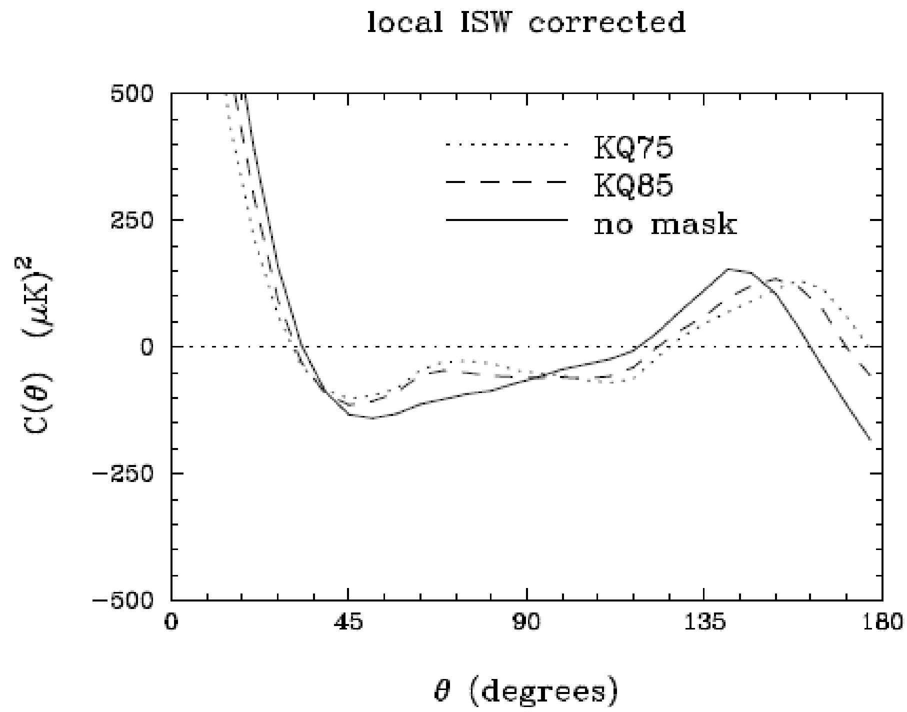
Bennett, WAMP7

Many CMB analysts ask: what is the oddities can I find in the data given the LCDM model.

However, most sensible question is: give this data, what is the probability of the LCDM model?

$$\frac{\Delta T^{\mathrm{ISW}}}{T_{\mathrm{CMB}}} = 2\int_{t_{\mathrm{LS}}}^{t_0} \frac{\dot{\Phi}(\vec{x}(t),t)}{c^2}\,\mathrm{d}t,$$





Francis and Peacock

0909.2495

$S_{1/2}$ statistic :

$$10360\ (\mu\mathrm{K})^4\ \text{(all-sky)}$$
$$6463\ (\mu\mathrm{K})^4\ \text{(KQ85 mask)}$$
$$5257\ (\mu\mathrm{K})^4\ \text{(KQ75 mask)}$$

All consistent with the concordance
LCDM model at the few percent level.

1. Argue that there is a physical alignment of local structure with potential fluctuation at LSS that conspires to remove large scale correlations outside the Galactic mask. (implausible)
2. A posteriori statistics

G. Efstathiou, YZM and D. Hanson, MNRAS 407(2010)2530

The original use of a sky cut in calculating $S_{1/2}$ was motivated by concern for residual foregrounds in the ILC map. We now recognize that this precaution was unnecessary as the ILC foreground residuals are relatively small. Values of $S_{1/2}$ are smaller on the cut sky than on the full sky, but since the full sky contains the superior sample of the universe and the cut sky estimates suffer from a loss of information, cut sky estimates must be considered sub-optimal. It now appears that the Spergel et al. (2003) and Copi et al. (2007, 2009) low p-values result from both the *a posteriori* definition of $S_{1/2}$ and a chance alignment of the Galactic plane with the CMB signal. The alignment involves Cold Spot I and the

C Bennett et al. 2010 (WMAP7)

Efstathiou et al. (2009) corrected the full-sky *WMAP* ILC map for the estimated Integrated Sachs-Wolfe (ISW) signal from redshift $z < 0.3$ as estimated by Francis & Peacock (2009). The result was a substantial increase in the $S_{1/2}$. Yet there is no large-scale cosmological significance to the orientation of the sky cut or the orientation of the local distribution of matter with respect to us, thus the result from Spergel et al. and Copi et al. must be a coincidence.

C Bennett et al. 2010 (WMAP7)

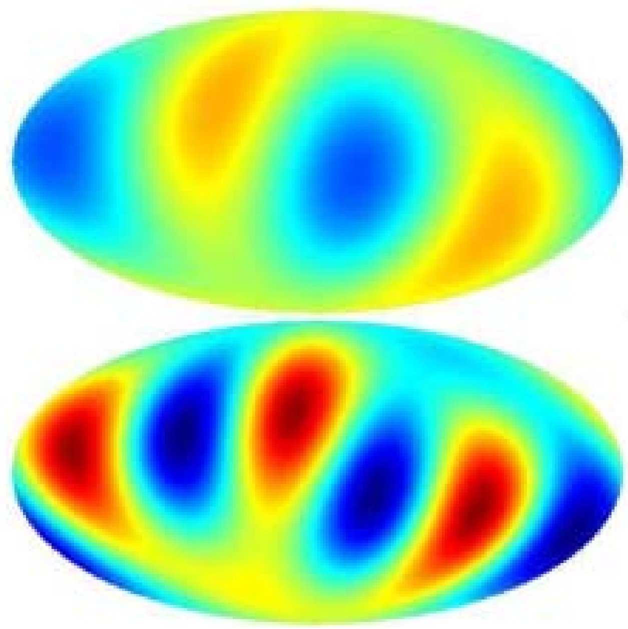
Testing direction-dependent power spectrum with Planck

Anomalies in the CMB maps (WMAP)

Alignment of Quadrupole and Octople.

M. Tegmark, A. de Oliveira-Costa and A Hamilton:

astro-ph/0302496



Some recent works:

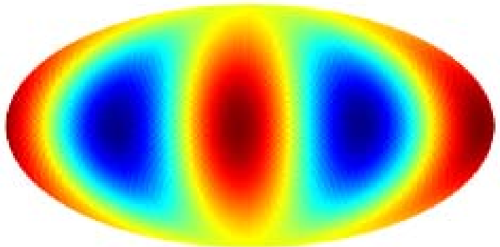
L Ackerman, S Carroll, and M. Wise:

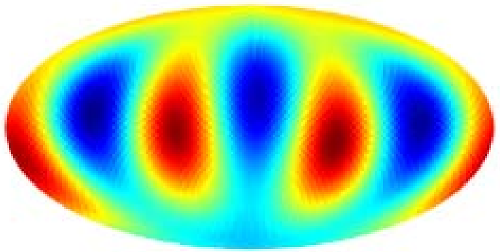
$$P'(\mathbf{k}) = P(k)\left(1 + g(k)(\hat{\mathbf{k}}\cdot\mathbf{n})^2\right)$$

C Dvorkin, H. Peiris and W. Hu:

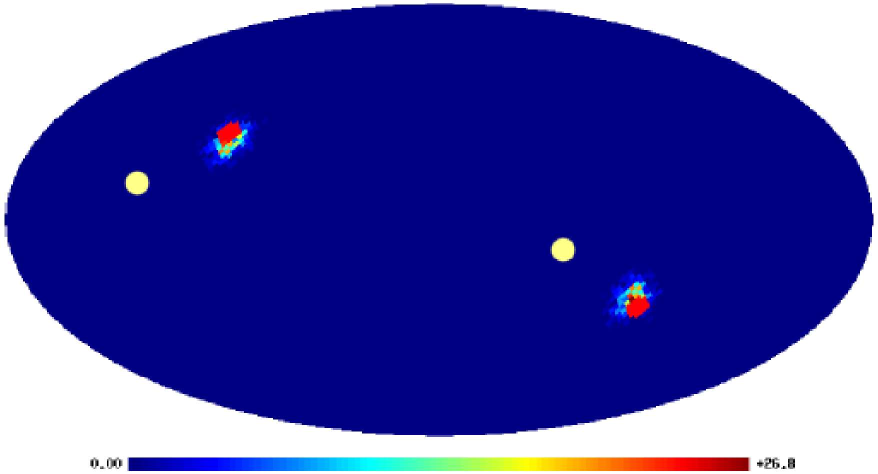
0711.2321

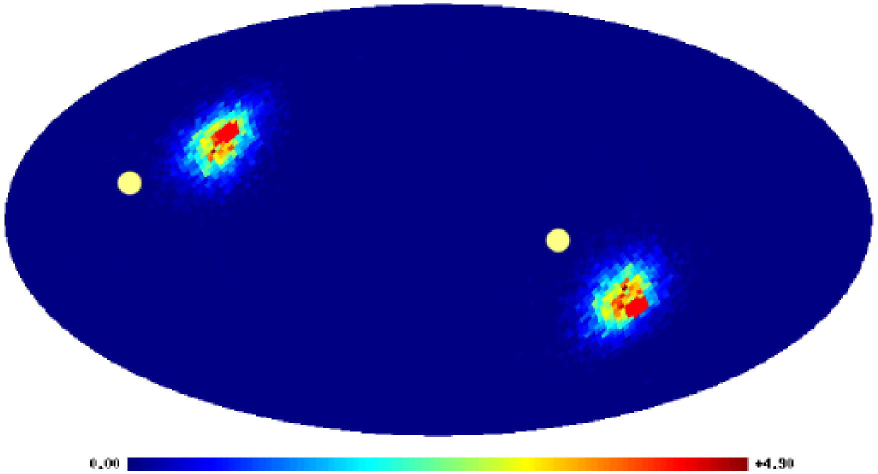
$$\frac{\Delta T}{T}(\mathbf{x}) = -\frac{1}{3}\Phi(\mathbf{x})$$





N. Groeneboom et.al: 0911.0150





Band	ℓ range	Mask	Amplitude g_*	Direction (l, b)
W1-4	$2-400$	KQ85	0.29 ± 0.031	$(94^\circ, 26^\circ) \pm 4^\circ$
V1-2	$2-400$	KQ85	0.14 ± 0.034	$(97^\circ, 27^\circ) \pm 9^\circ$
Q1-2	$2-300$	KQ85	-0.18 ± 0.040	$(99^\circ, 28^\circ) \pm 10^\circ$

NOTE. — The values for g_* indicate posterior mean and standard deviation. The ecliptic poles are located at $\pm(96^\circ, 30^\circ)$.

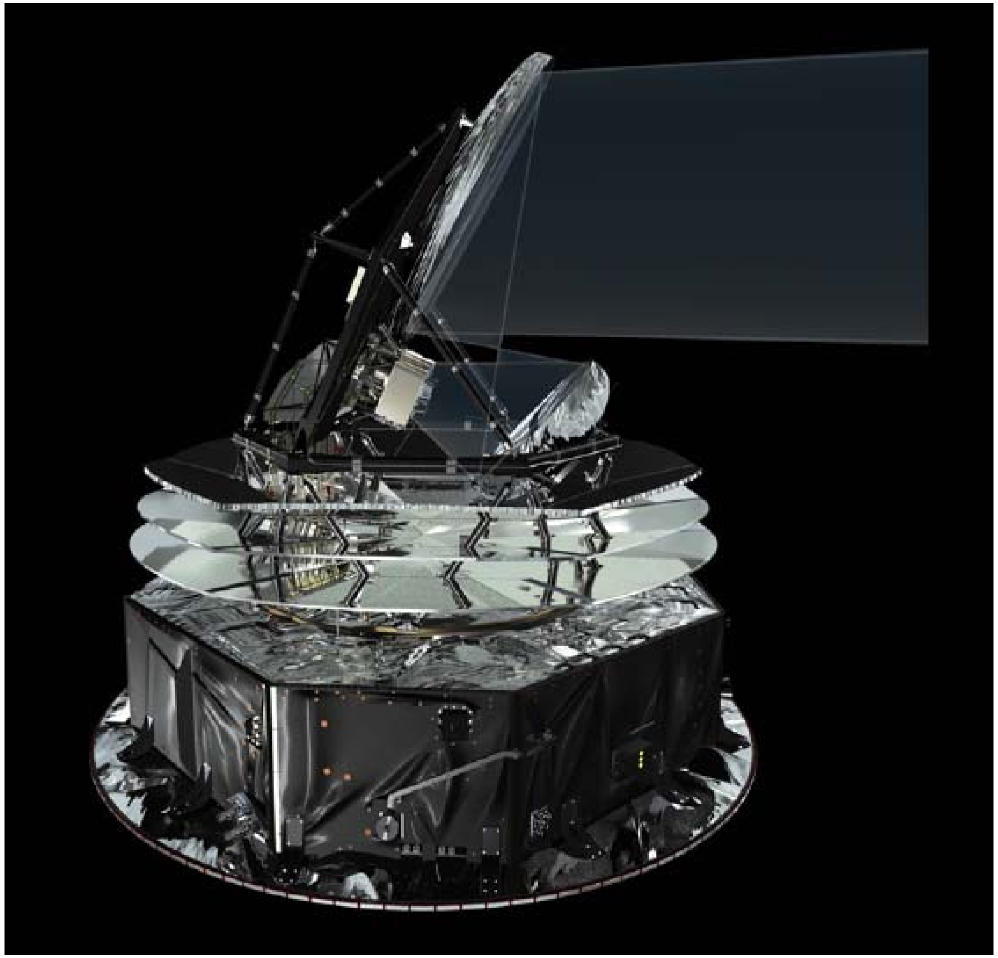
However, it may still be systematics

- Is the power spectrum really direction-dependent?
- If it is, is the anisotropy really axis-symmetric?
- How to approach the preferred axis?
- It is also very interesting to know: whether the polarization could provide stronger constraints.

Planck was launched successfully in May 14th, 2009

Planck:

1. The coolest spacecraft every built
2. Price tag is 700M euros, and mass at launch is 1.9 tonnes!
3. Time schedule and scanning strategy:



May/2009: launched

July/2009: reach the orbit of L2

August/2009: start scanning

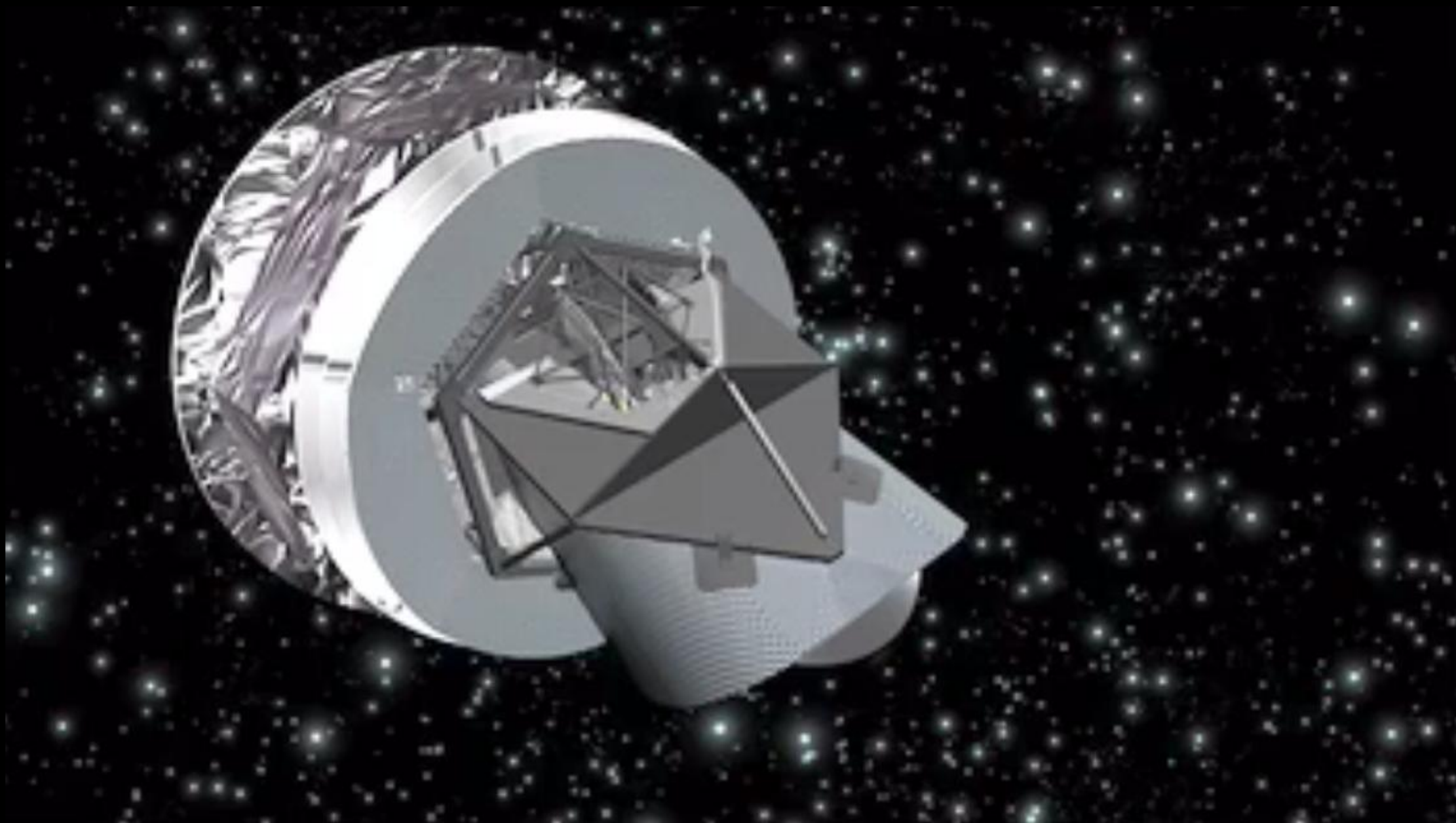
Sep/2009: first light

Feb-Mar/2010: 1st whole-sky survey

Aug-Sep/2010: 2 sky surveys

Aug-Sep/2011: 4 sky surveys

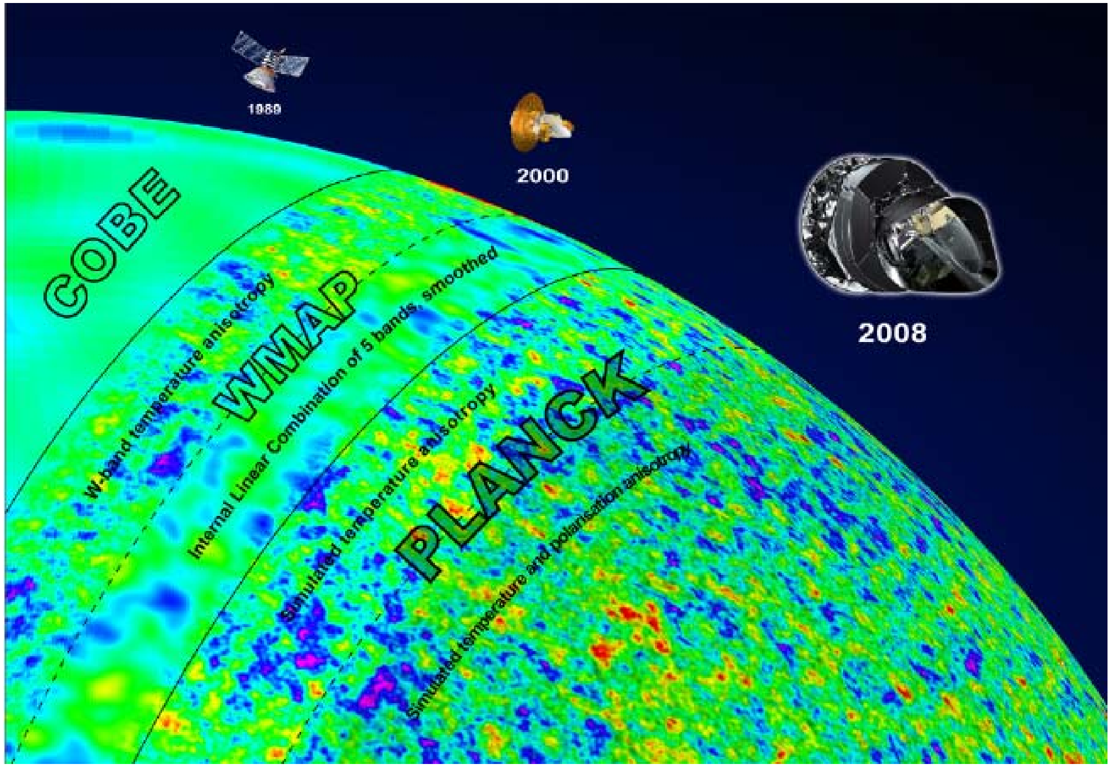
Jan/2013(AAS): Publishing results



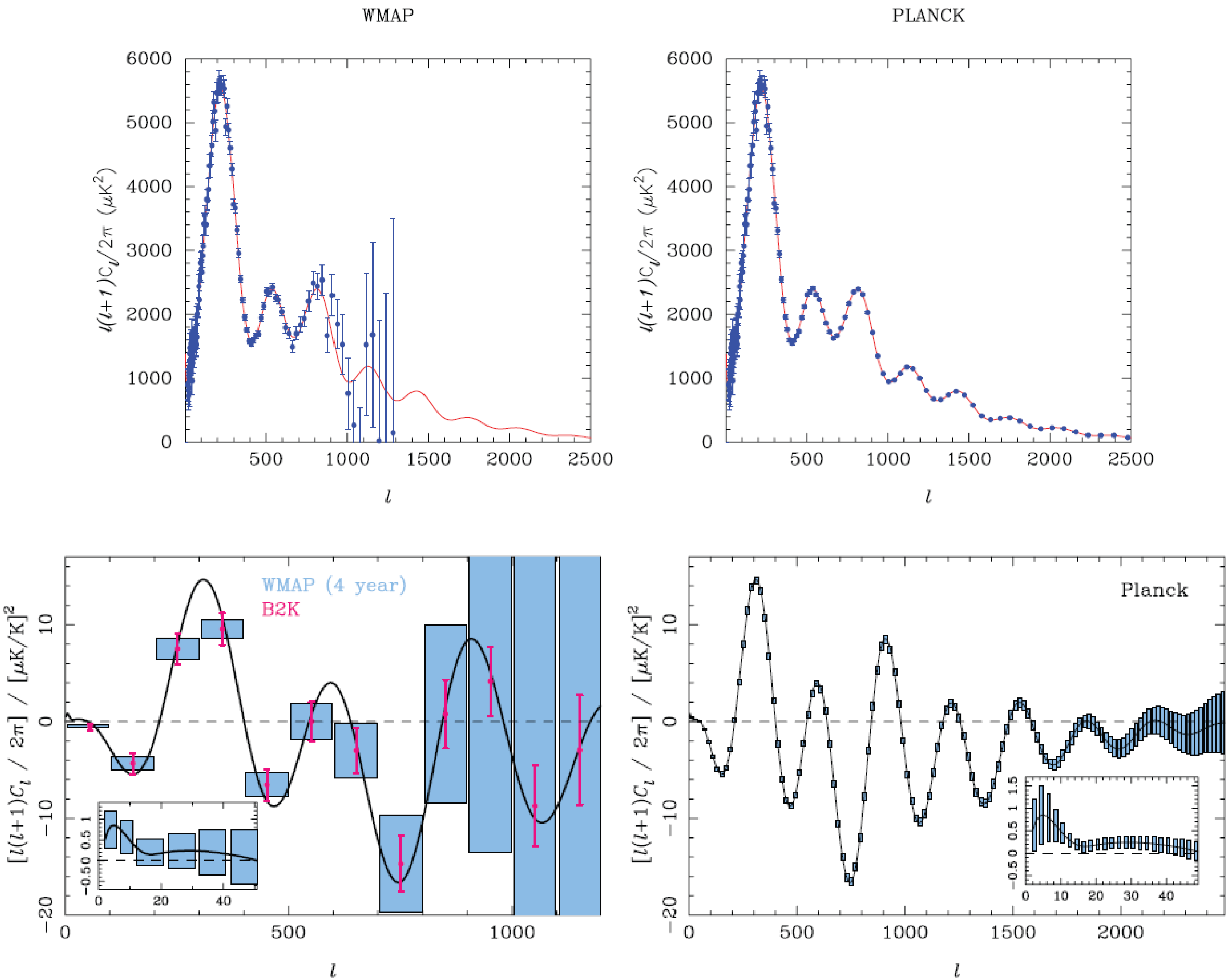
Summary of Planck Instrument Characteristics

Instrument Characteristic	LFI			HFI					
Detector Technology	HEMT arrays			Bolometer arrays					
Center Frequency [GHz]	30	44	70	100	143	217	353	545	857
Bandwidth ($\Delta\nu/\nu$)	0.2	0.2	0.2	0.33	0.33	0.33	0.33	0.33	0.33
Angular Resolution (arcmin)	33	24	14	10	7.1	5.0	5.0	5.0	5.0
$\Delta T/T$ per pixel (Stokes I)[a]	2.0	2.7	4.7	2.5	2.2	4.8	14.7	147	6700
$\Delta T/T$ per pixel (Stokes Q &U)[a]	2.8	3.9	6.7	4.0	4.2	9.8	29.8	...	...

[a] Goal (in μK/K) for 14 months integration, 1σ, for square pixels whose sides are given in the row "Angular Resolution".



Planck: scientific programme



WMAP
PLANCK
$\ell(\ell+1)C_\ell/2\pi$ (μK^2)
l
WMAP (4 year)
B2K
Planck
$[l(l+1)C_l / 2\pi] / [\mu K/K]^2$

Statistical isotropy $\longrightarrow$ a_{lm}^X are Gaussian random variable

$$a_{lm}^T = \int d\Omega Y_{lm}^*(\Omega) \Delta T(\Omega)$$

$$C_{lm,l'm'}^T = \left\langle a_{lm}^{T*} a_{l'm'}^T \right\rangle = C_l^{TT} \delta_{ll'} \delta_{mm'}$$

But, if we consider statistical anisotropic model, the covariance matrix will have off-diagonal terms

Generalizing axis-symmetric assumption:

$$P(\mathbf{k}) = P(k)[1 + \sum_{LM} g_{LM} Y_{LM}(\hat{\mathbf{k}})]$$

$$a_{lm}^X = 4\pi i^l \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Delta_l^X(k) \chi_0(\mathbf{k}) Y_{lm}^*(\hat{\mathbf{k}})$$

$$\langle \chi_0(\mathbf{k}) \chi_0^*(\mathbf{k}') \rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') \frac{2\pi^2}{k^3} P_\chi(\mathbf{k})$$

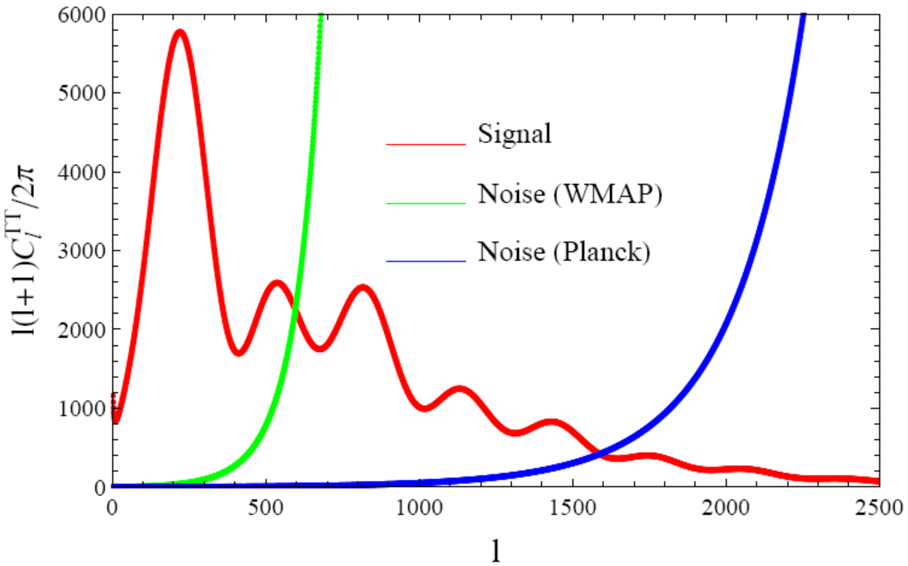
$$C_{l_1 m_1 l_2 m_2}^{XX'} = \left\langle a_{l_1 m_1}^X a_{l_2 m_2}^{X'*} \right\rangle = C_{l_1 m_1 l_2 m_2}^{XX' iso} + \delta C_{l_1 m_1 l_2 m_2}^{XX'}$$

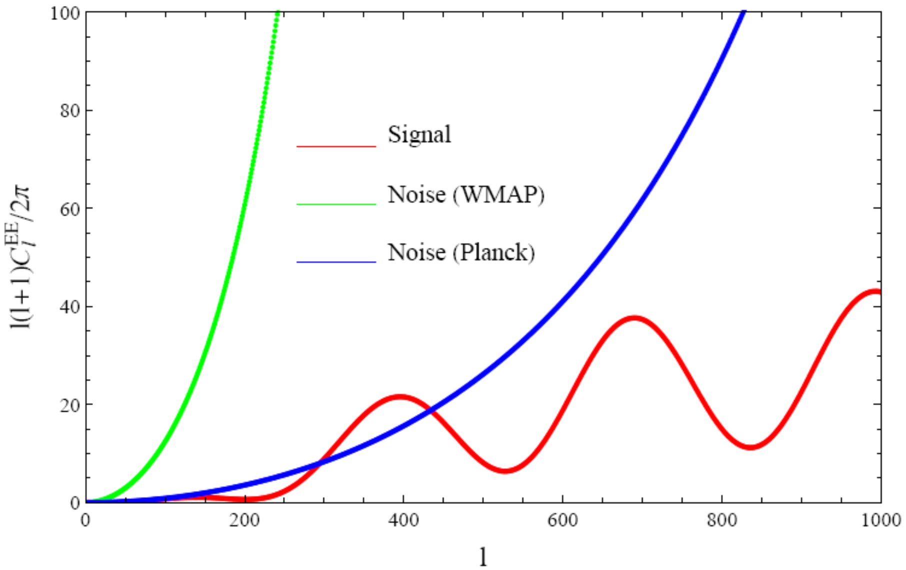
where

$$\begin{aligned} C_{l_1 m_1 l_2 m_2}^{XX' iso} &= \delta_{l_1 l_2} \delta_{m_1 m_2} C_{l_1}^{XX'} \\ &= \delta_{l_1 l_2} \delta_{m_1 m_2} (4\pi) \int d\ln k P_\chi(k) \Delta_{l_1}^X(k) \Delta_{l_1}^{X'}(k) \end{aligned}$$

$$\delta C^{XX'}_{l_1m_1l_2m_2} = i^{l_1-l_2} C^{XX'}_{l_1l_2} \sum_{M=-2}^{2} g_{2M} \int d\Omega_k Y_{2M}(\hat{\mathbf{k}}) Y^*_{l_1m_1}(\hat{\mathbf{k}}) Y_{l_2m_2}(\hat{\mathbf{k}})$$

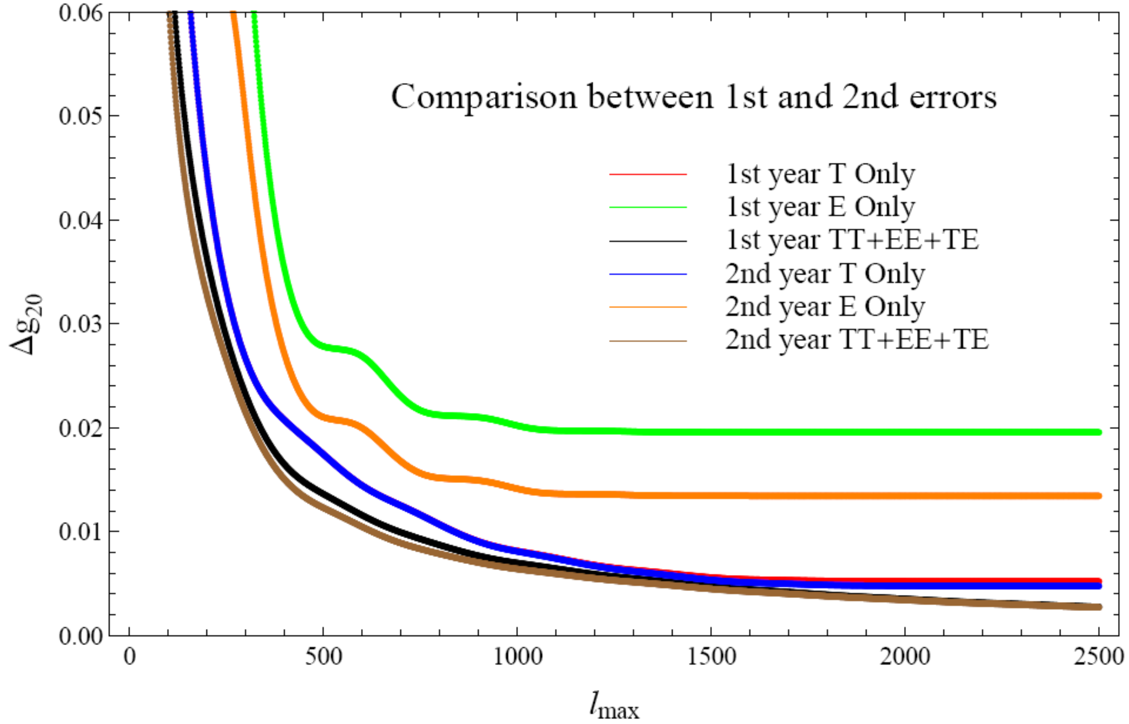
Reminds: the noise level for WMAP and Planck





Therefore, by calculating the fisher matrix, we know the error for the anisotropic parameters g_{2M}

$$F_{g_{2M}g_{2M'}} = F_{g_{20}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



Then perform the Quadratic estimator (equivalent to the maximum likelihood estimator) to the map:

$$\tilde{g}_{2M} = \frac{1}{2} \sum_{l_1 m_1 l_2 m_2} \frac{\delta C_{l_1 m_1 l_2 m_2}}{\delta g_{2M}} \bar{\Theta}^*_{l_1 m_1} \bar{\Theta}_{l_2 m_2}$$

Now we try to simulate a Planck map and do the reconstruction

Talor Expansion:

$$C = (I + \delta C [C^i]^{-1}) C^i$$

$$\downarrow$$

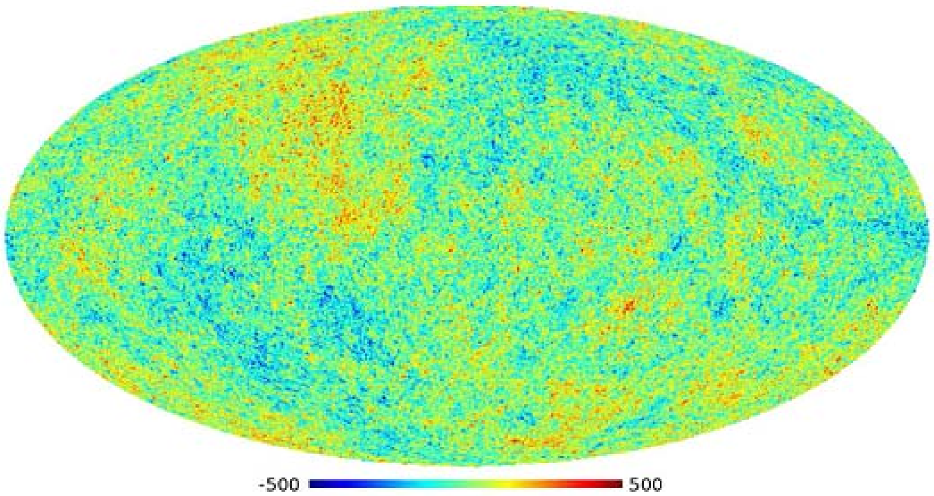
$$\Theta = \left[I + \delta C [C^i]^{-1}\right]^{1/2} \Theta^i$$

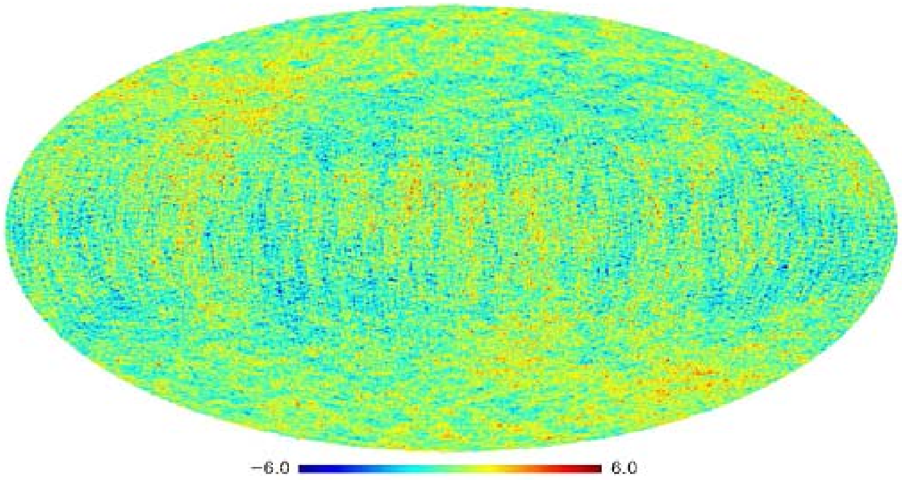
$$\downarrow$$

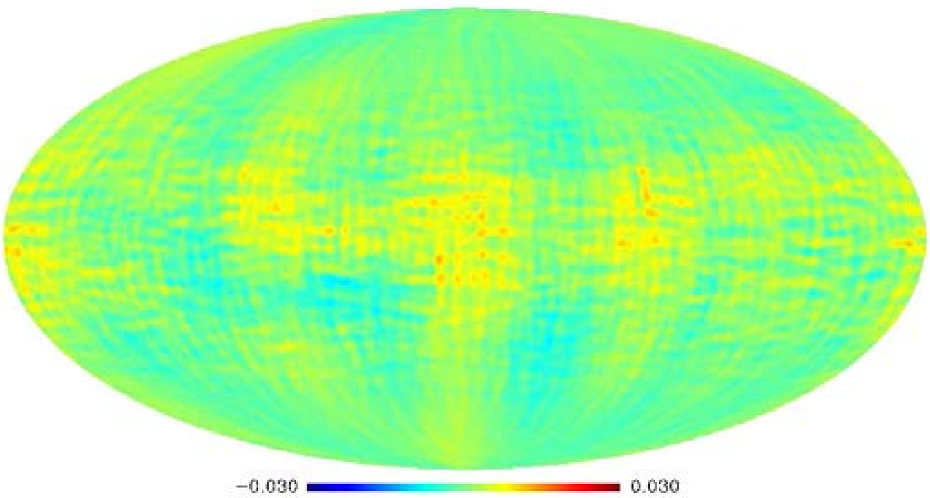
$$\Theta \approx \Theta^i + \frac{1}{2}\delta C [C^i]^{-1} \Theta^i + \ldots .$$

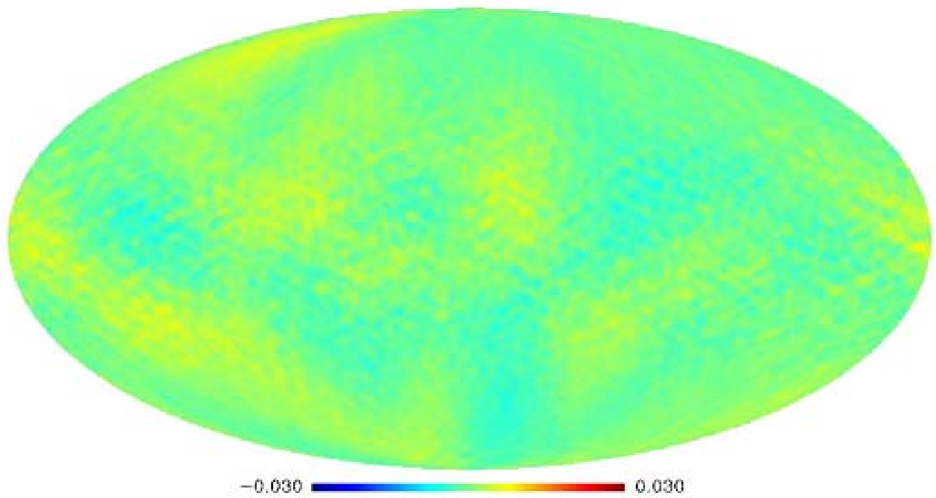
D. Hanson and A. Lewis: 0908.0963

$g_{20} = 0.1$

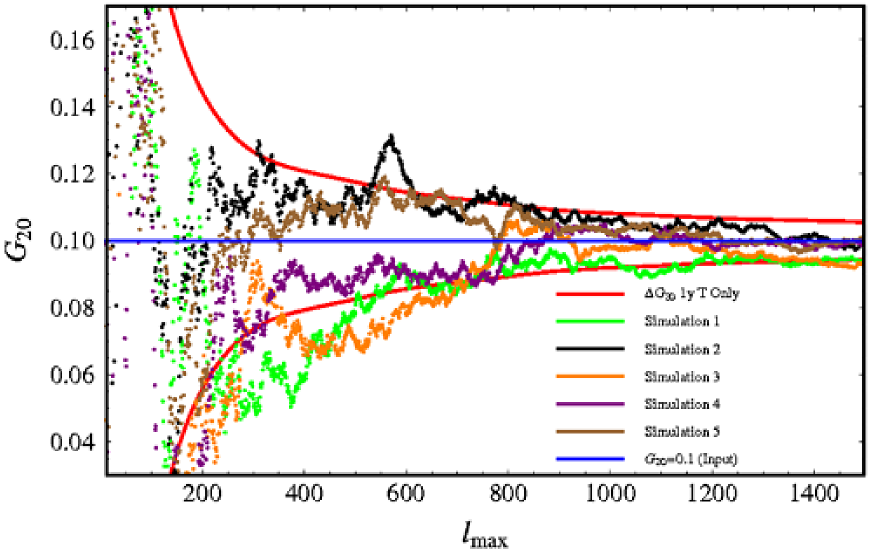


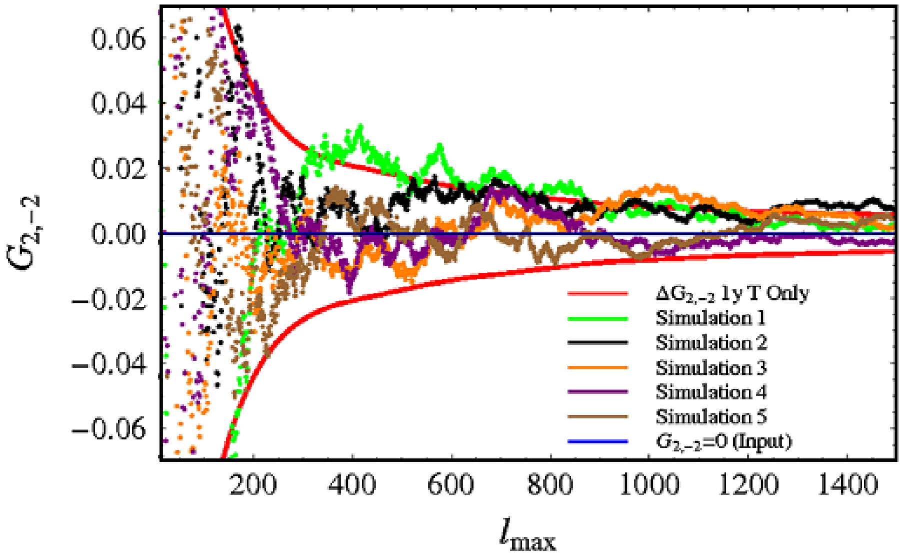



−0.030
0.030

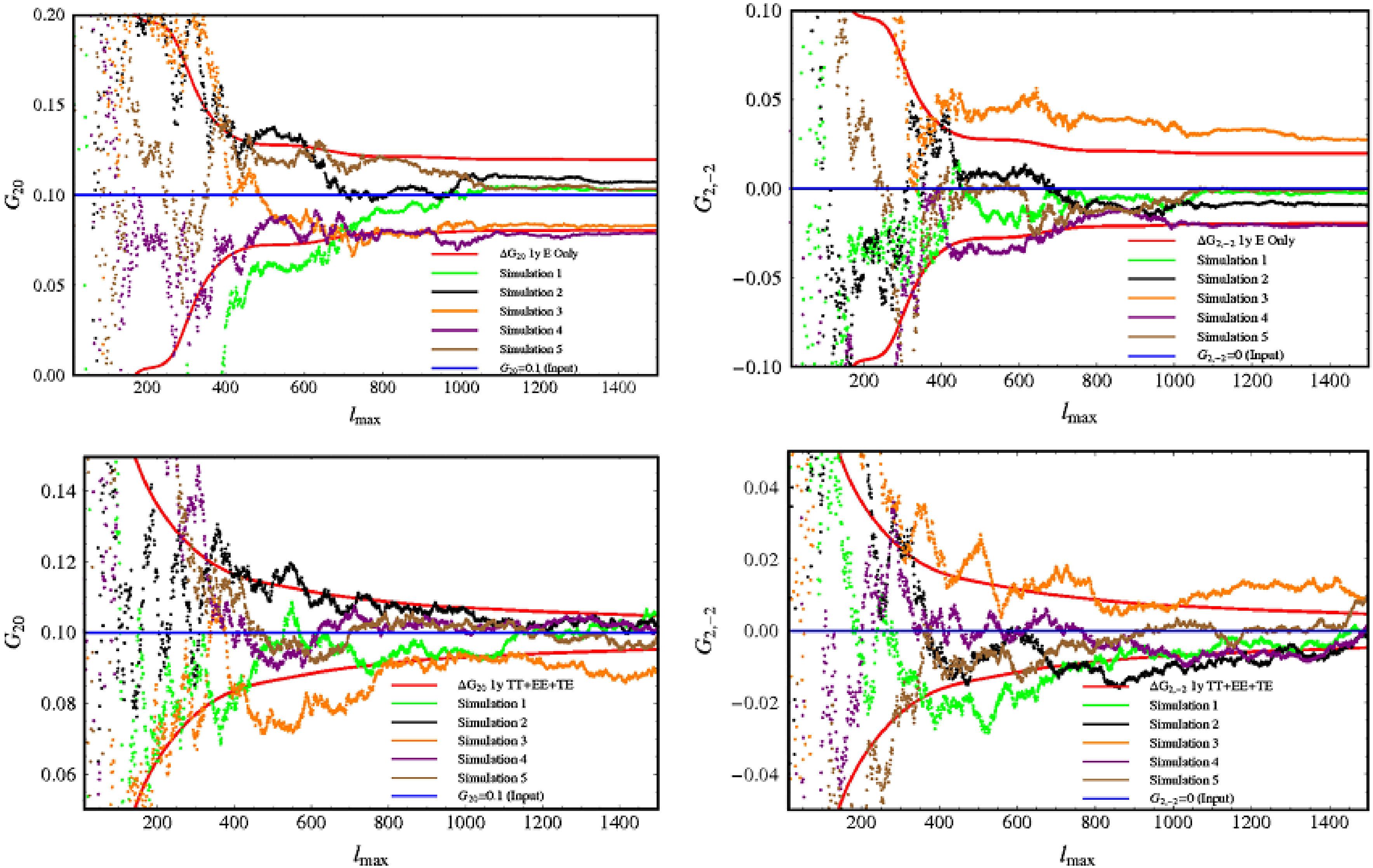

−0.030
0.030

Apply the Quadratic estimator to the map:





$$P(\mathbf{k}) = P(k)[1 + \sum_{LM} g_{LM} Y_{LM}(\hat{\mathbf{k}})]$$

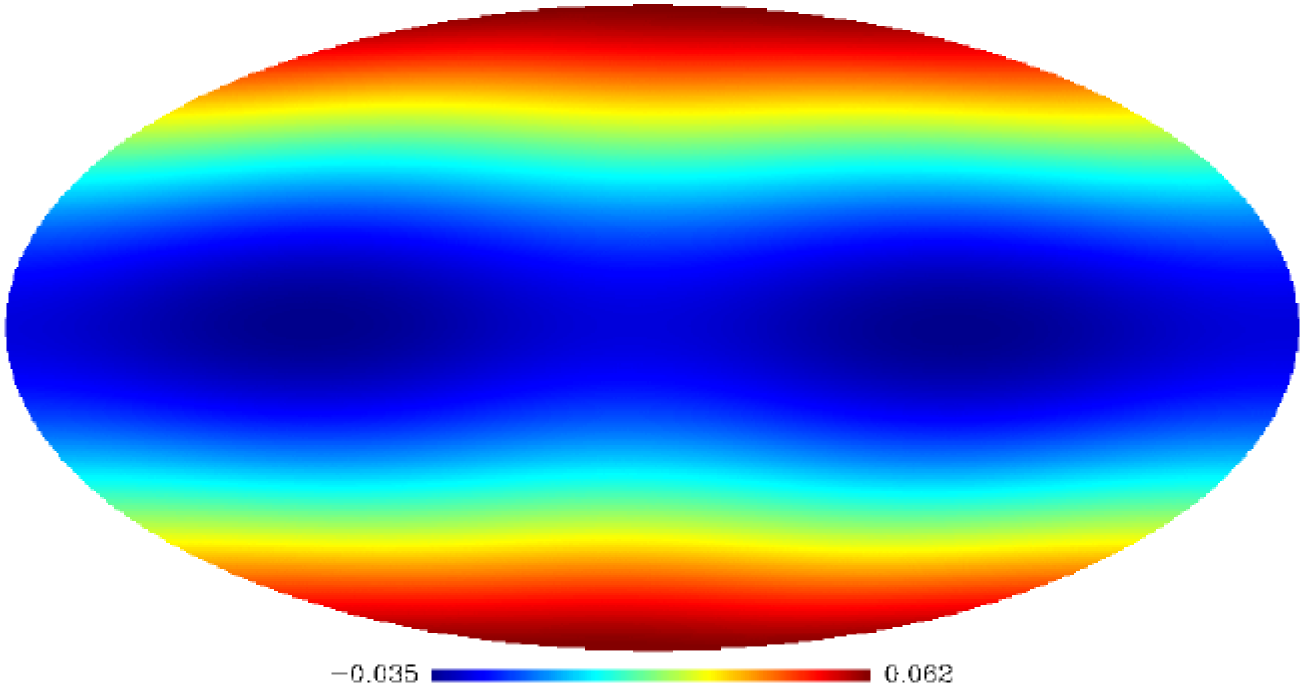


$$P(\mathbf{k}) = P(k)[1 + \sum_{LM} g_{LM} Y_{LM}(\hat{\mathbf{k}})]$$

Therefore, Planck Temperature map should be able to constrain the amplitude of any spherical multipole of a scale-invariant quadrupole asymmetry at the 0.01 level (2 sigma). Almost independent constraints can be obtained from polarization at the 0.03 level after four full-sky surveys, providing an important consistency test.

Our question is: if the anisotropic parameters are reconstructed from Planck map, and the errors are estimated, how to know that whether these sets of values indicate a preferred direction on the sky?

Method 1: reconstruct $g(\mathbf{n})$ on the whole sky and test it by eyeball.



Method 2: fitting a preferred direction model

$$g_{2M} = D^2_{0M}(\pi - \gamma, \beta, -\pi - \alpha) g_*$$

$$G^{\text{trans}}_{20} = \left[\left(\cos\frac{\beta}{2}\right)^4 + \left(\sin\frac{\beta}{2}\right)^4 - 4\left(\cos\frac{\beta}{2}\right)^2 \left(\sin\frac{\beta}{2}\right)^2 \right] g_*$$

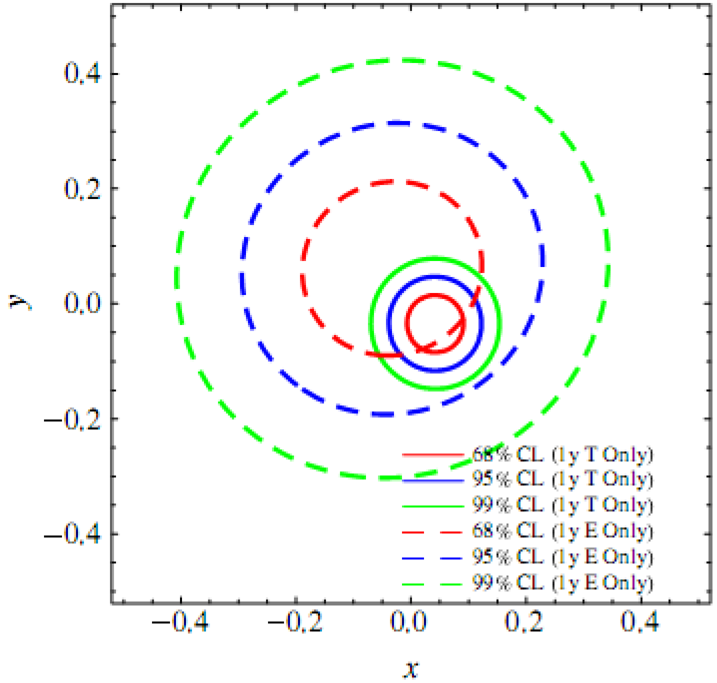
$$G^{\text{trans}}_{22} = \frac{\sqrt{6}}{4}(\cos 2\alpha)(\sin^2\beta) g_*,$$

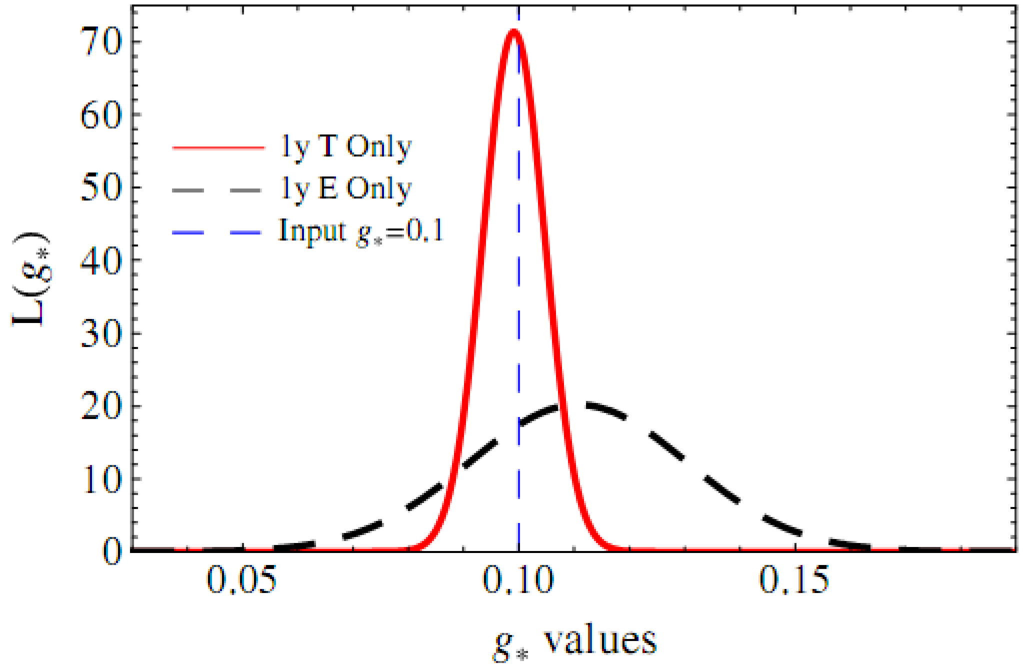
$$G^{\text{trans}}_{2,-2} = \frac{\sqrt{6}}{4}(\sin 2\alpha)(\sin^2\beta) g_*,$$

$$G^{\text{trans}}_{21} = -\frac{\sqrt{6}}{4}(\cos\alpha)(\sin 2\beta) g_*,$$

$$G^{\text{trans}}_{2,-1} = -\frac{\sqrt{6}}{4}(\sin\alpha)(\sin 2\beta) g_*.$$

$$\chi^2(\alpha, \beta, g_*) = \sum_{M=-2}^{2} \left(\frac{G^{\text{est}}_{2M} - G^{\text{trans}}_{2M}}{\sigma_{g_{2M}}} \right)^2$$





$$x = 2\sin(\beta/2) \times \cos(\alpha)$$

$$y = 2\sin(\beta/2)\sin(\alpha)$$

$$\chi^2_{min}/2 \simeq 0.519$$

Then you can transform the G_{2M}s back to the coordinate which maximize the g_{20}, if all of the other $G_{2M}(M \neq 0)$ are small comparing with the error, then there is no evidence for deviating the axis-symmetric power spectrum.

$$\tilde{g}_{2M'} = \sum_{M} D^2_{MM'}(\alpha, \beta, \gamma) g_{2M}$$

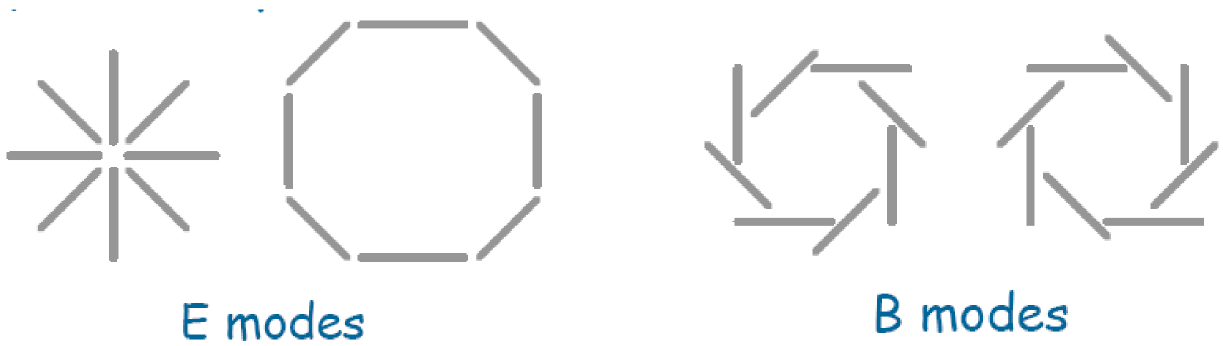
CMB B-mode polarization:

Polarization Q and U parameters:

$$(Q + iU)(\mathbf{n}) = \sum_{lm} a_{lm}^{(\pm 2)} [{}_{\pm 2}Y_{lm}(\mathbf{n})]$$

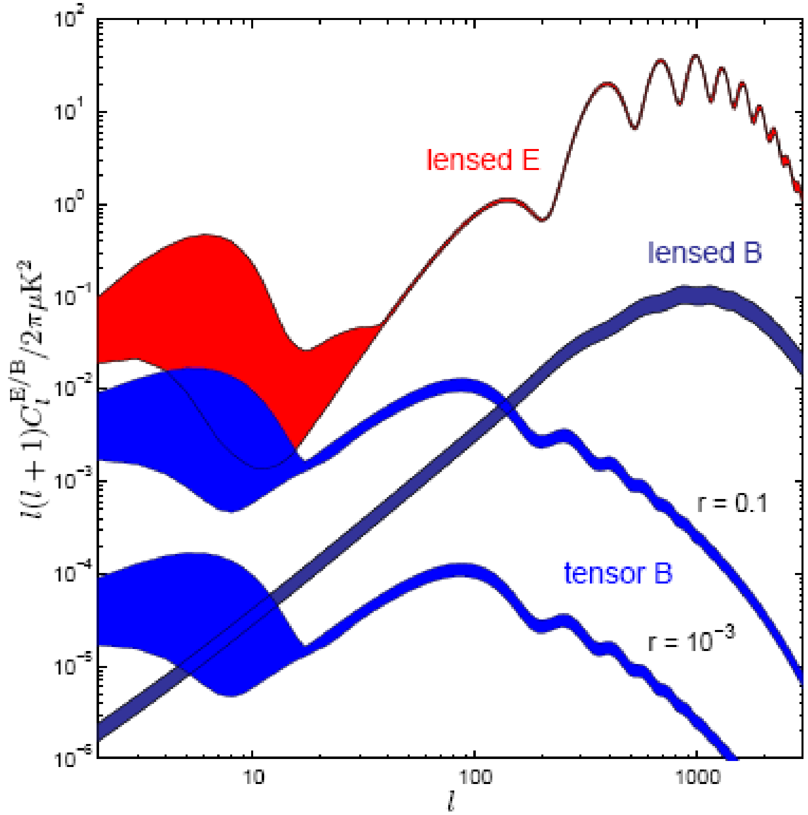
$$a_{lm}^{E} = -\frac{1}{2}(a_{lm}^{(2)} + a_{lm}^{(-2)}),\ a_{lm}^{B} = -\frac{1}{2i}(a_{lm}^{(2)} - a_{lm}^{(-2)})$$

$$E(\mathbf{n}) = \sum_{lm} a_{lm}^{E} Y_{lm}(\mathbf{n}),\ B(\mathbf{n}) = \sum_{lm} a_{lm}^{B} Y_{lm}(\mathbf{n})$$



Why B-mode polarization?

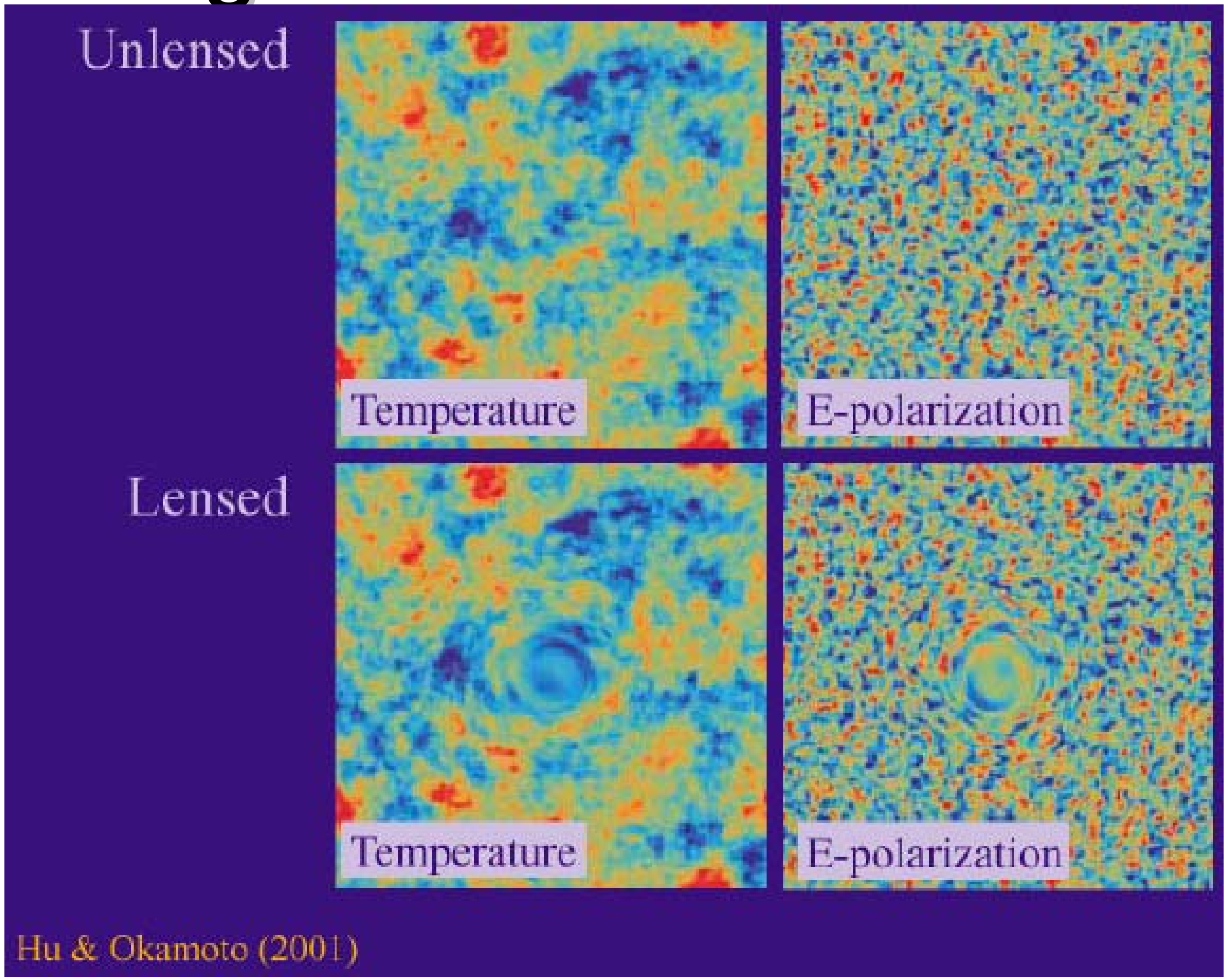
- Carrying over GW information from early Universe, not mixing with density fluctuations
- Least parameter degeneracy



Lewis and Challinor, astro-ph/0601954

Problems:

1. Dominated by noise
2. Lensing confusion



Why this is important?

Lyth bound:

$$V^{\frac{1}{4}} = 1.06 \times 10^{16}\mathrm{GeV}\left(\frac{r}{0.01}\right)^{\frac{1}{4}} \qquad \frac{\Delta\phi}{\mathrm{M_{pl}}} \gtrsim \left(\frac{r}{0.01}\right)^{\frac{1}{2}}$$

Consistency relation:

$$n_t = -\frac{r}{8}$$

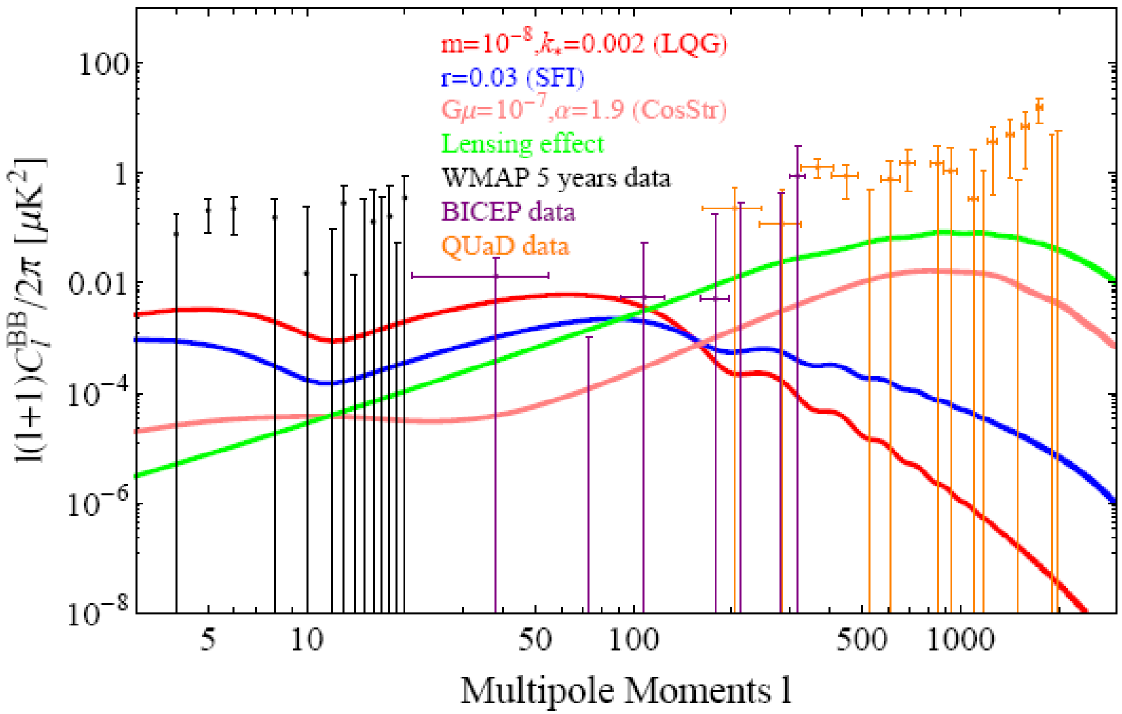
$$C_l^{BB} = \frac{\pi}{4}\int P_t(k)\Delta_l^B(k)^2 d\ln k$$

Cosmic string from Brane-Inflation

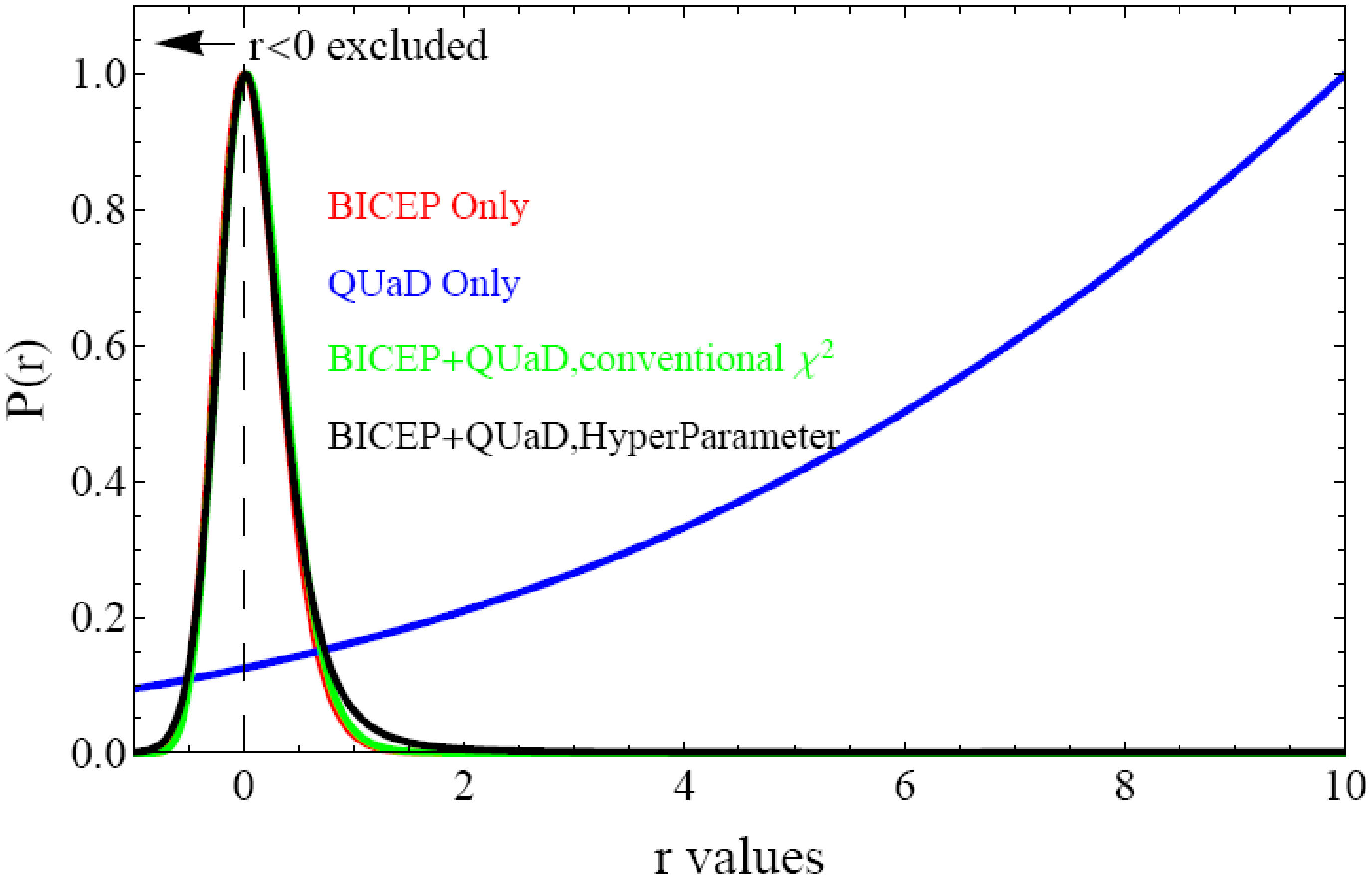
$$C_l^{BB} = C_l^{BB,0}\left(\frac{G\mu}{G\mu_0}\right)^2$$

CMBACT:

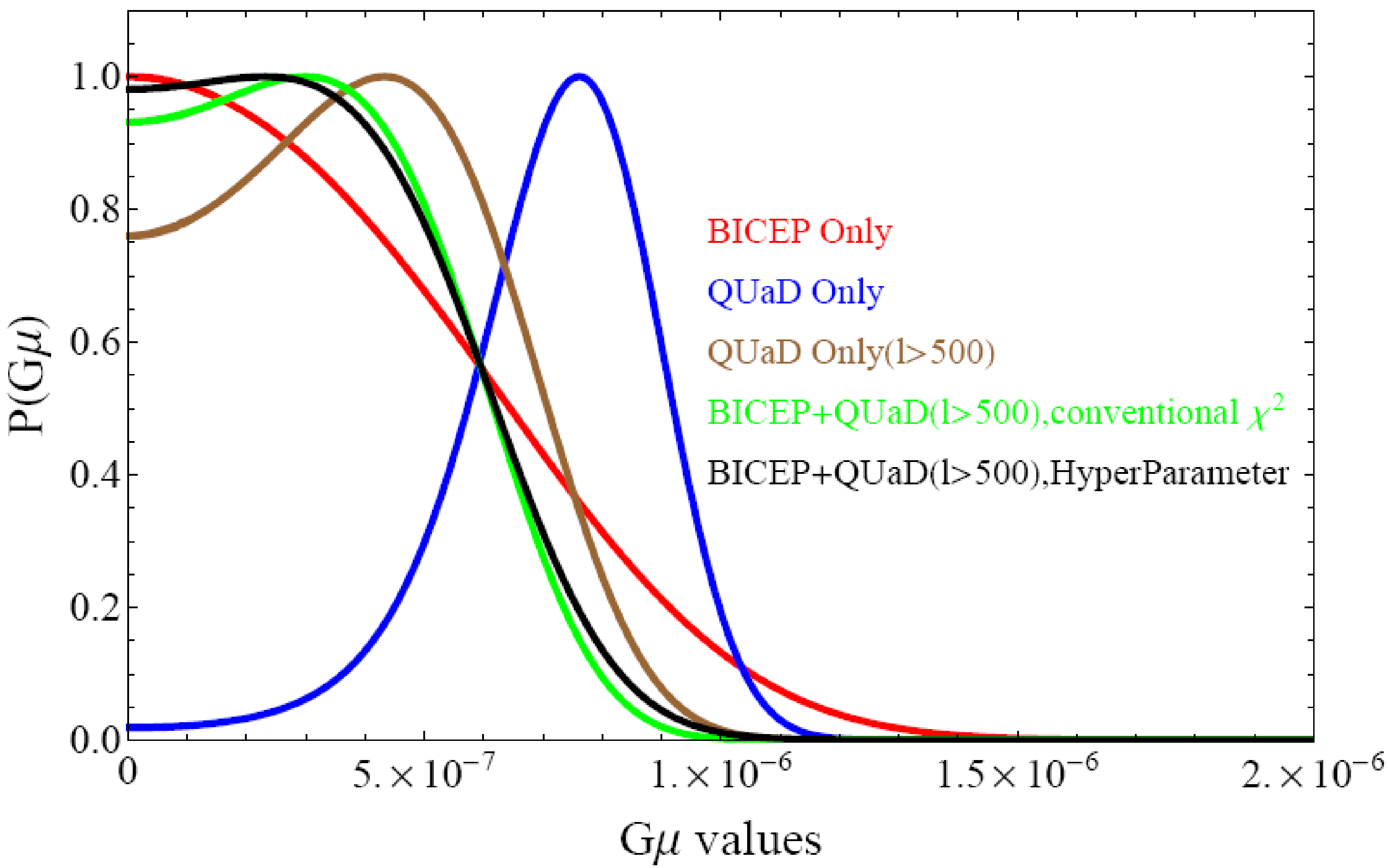
http://www.sfu.ca/levon/cmbact.html.



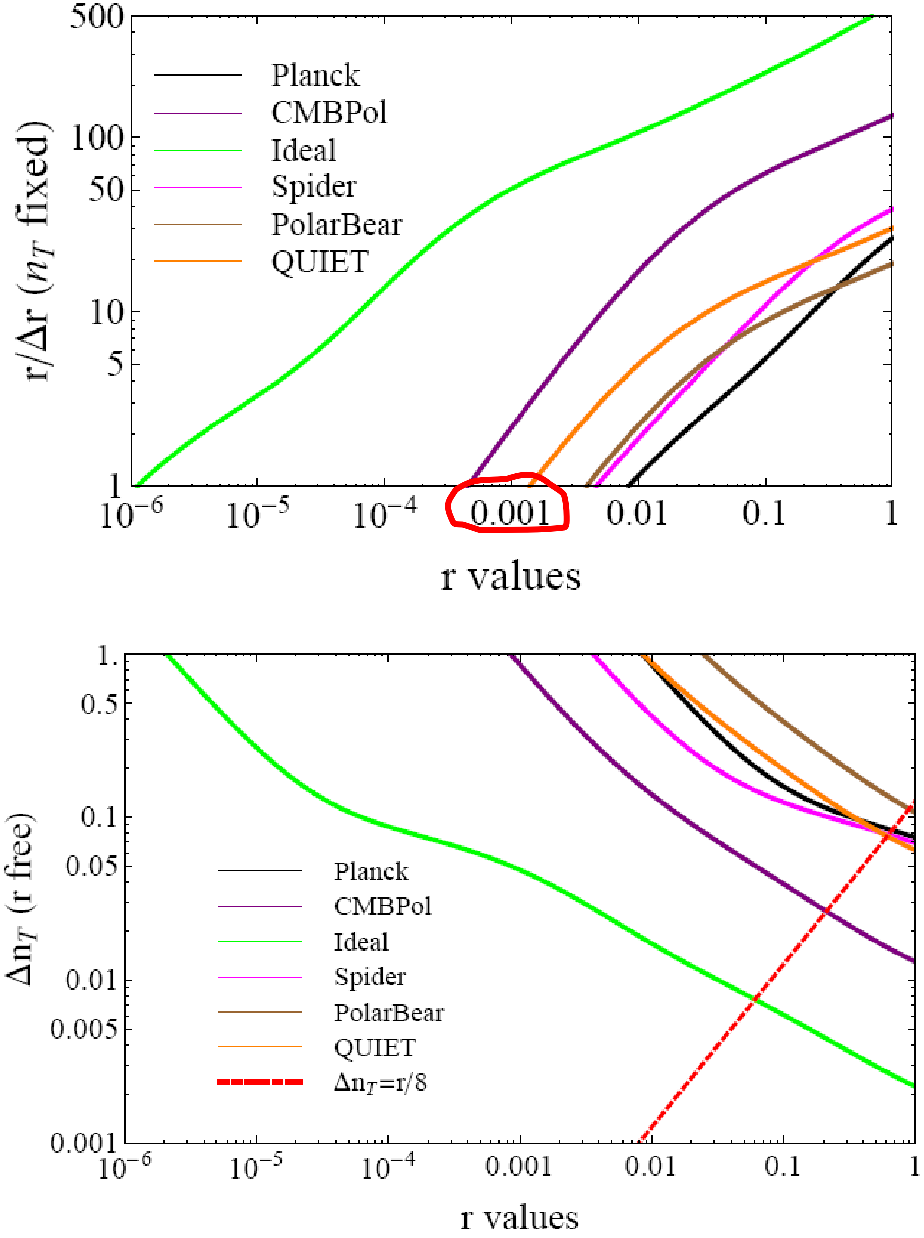
Single field slow-roll inflation:



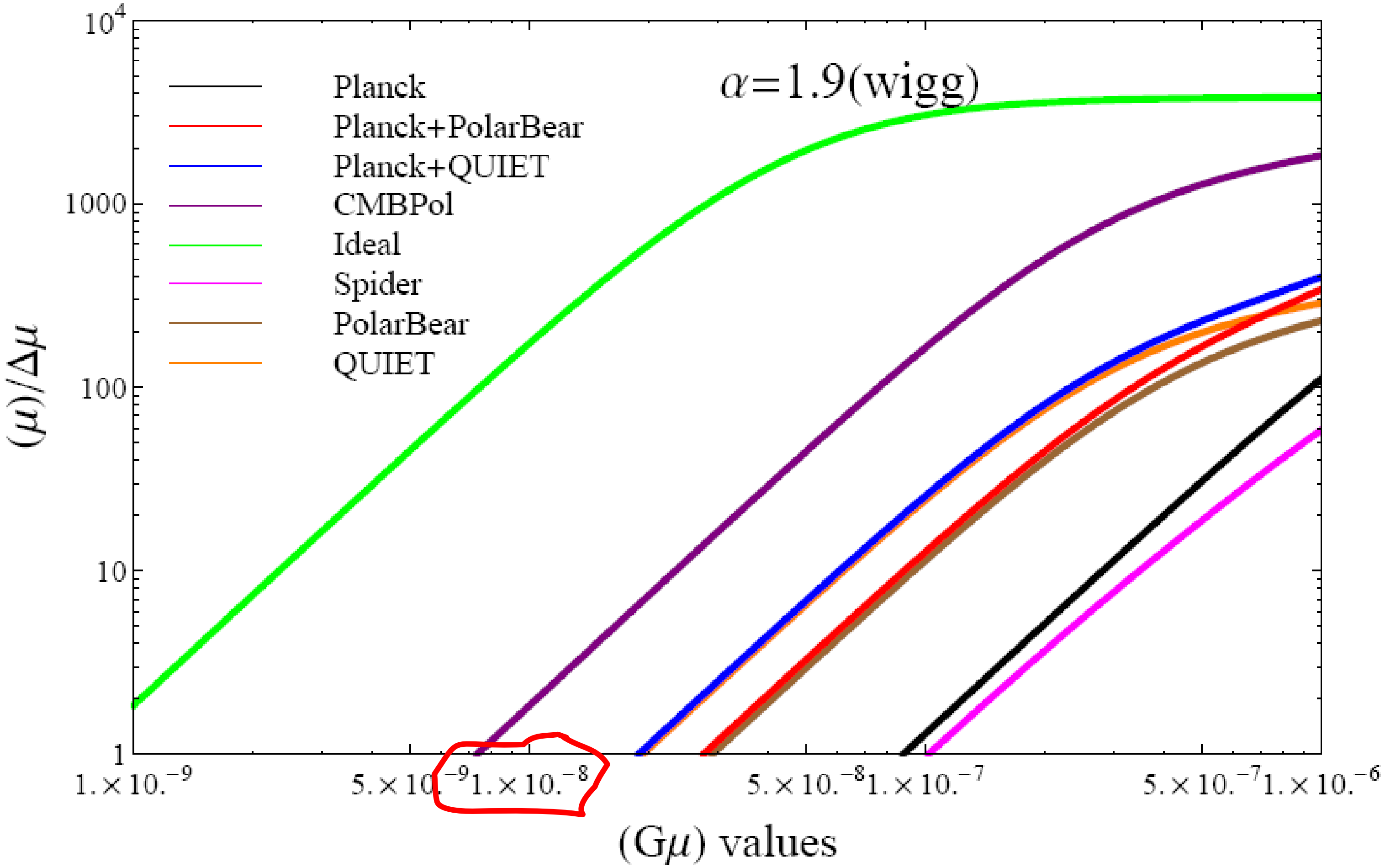
Cosmic string from D-brane Inflation:



Results: Forecast for future experiments



Results: Forecast for future experiments



Conclusion:

- There is no real lack of large angular correlation function on the CMB sky, previous used S_{1/2} statistics is a posteriori statistics.
- The WMAP V-W band data show weak evidence of direction-depedent power spectrum, and it can be strongly constrained by temperature and polarization maps of Planck.
- B-mode polarization data can provide an interesting constraints on early Universe models. The detection limit for r is ~10^(-3) and for Gmu~10^(-8).