Holography of Charged Black Holes with RF^2 Corrections

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Outline

What is Gauge/gravity duality?

- A holographic duality between a weakly-coupled theory of gravity in certain spacetime and a strongly-coupled field theory living on the boundary of that spacetime.
- A powerful new tool for investigating dynamics of strongly-coupled field theories in the dual gravity side.
- A new window on understanding real-world physics: QCD, CMT, etc.
- Two complementary approaches: bottom-up and top-down.

Two complementary approaches:

Bottom-up

- Toy-models coming from simple gravity theory;
- Basic ingredients: $g_{\mu\nu}, A_\mu, \psi$ and/or dilaton ϕ;
- Advantage(s): simplicity and universality;
- Disadvantage(s): the dual field theory is unclear.

Top-down

- Configurations originated from string/M theory;
- Exact solutions of SUGRA or Dp/Dq-branes;
- Advantage(s): good understanding on the field theory;
- Disadvantage(s): complexity.

The main results of 1010.0443[hep-th]

By R. C. Myers, S. Sachdev and A. Singh

- Charge transport near 2+1-dim strongly interacting quantum critical points;
- Background: *Schwarzschild-AdS*$_4$;
- Effective action for A_μ

$$I_{\rm vec} = \frac{1}{g_4^2}\int d^4x\sqrt{-g}\Big[-\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\Big], \qquad (1)$$

- The DC conductivity

$$\sigma_{\rm DC} = \frac{1}{g_4^2}(1+4\gamma). \qquad (2)$$

An alternative form of the corrections

$$I'_{\rm vec} = \frac{1}{\tilde{g}_4^2}\int d^4x\sqrt{-g}[-\frac{1}{4}F_{ab}F^{ab} + \alpha L^2(R_{abcd}F^{ab}F^{cd} -4R_{ab}F^{ac}F^{b}{}_{c} + RF^{ab}F_{ab})], \quad (3)$$

arising from KK reduction of 5D Gauss-Bonnet gravity.
In neutral background $R_{ab} = -3/L^2 g_{ab}$, using the definition of the Weyl tensor, the action (3) becomes

$$I'_{\rm vec} = \frac{1+8\alpha}{\tilde{g}_4^2}\int d^4x\sqrt{-g}[-\frac{1}{4}F_{ab}F^{ab} + \frac{\alpha}{1+8\alpha}L^2 C_{abcd}F^{ab}F^{cd}]. \quad (4)$$

An alternative form of the corrections Cont'd

It is equivalent to (1) with the following identifications

$$g_4^2 = \frac{\tilde{g}_4^2}{1+8\alpha}, \quad \gamma = \frac{\alpha}{1+8\alpha}. \tag{5}$$

Thus the charge transport properties are identical. In particular,

$$\sigma_{\mathrm{DC}} = \frac{1+12\alpha}{\tilde{g}_4^2}. \tag{6}$$

QUESTION: How about the case with a non-vanishing chemical potential?

The starting point

Leading order solution: *RN-AdS*$_4$. The action

$$S_0 = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[R + \frac{6}{L^2} - \frac{L^2}{g_F^2}F_{ab}F^{ab}]. \quad (7)$$

The metric

$$ds_0^2 = \frac{r^2}{L^2}[-f_0(r)dt^2 + dx^2 + dy^2] + \frac{L^2}{r^2}\frac{dr^2}{f_0(r)}, \quad (8)$$

where

$$f_0(r) = 1 - \frac{M}{r^3} + \frac{Q^2}{r^4}, \quad (9)$$

The starting point Cont'd

The gauge field

$$A_t^{(0)} = \mu_0(1 - \frac{r_0}{r}). \tag{10}$$

The horizon r_0 satisfies $f_0(r_0) = 0$, $\Rightarrow M = r_0^3 + Q^2/r_0$.
The chemical potential μ_0, charge density ρ_0, energy density ϵ_0 and entropy density s_0

$$\mu_0 = \frac{g_F Q}{L^2 r_0}, \quad \rho_0 = \frac{2Q}{\kappa^2 L^2 g_F},$$
$$\epsilon_0 = \frac{M}{\kappa^2 L^4}, \quad s_0 = \frac{2\pi r_0^2}{\kappa^2 L^2}. \tag{11}$$

The starting point Cont'd

The temperature

$$T_0 = \frac{3r_0}{4\pi L^2}(1 - \frac{Q^2}{3r_0^4}), \tag{12}$$

The extremal limit

$$T_0 = 0, \quad \Rightarrow \quad Q^2 = 3r_0^4. \tag{13}$$

One can verify that the first law of thermodynamics holds

$$d\epsilon_0 = T_0 ds_0 + \mu_0 d\rho_0. \tag{14}$$

The equations of motion

The full action including higher order corrections

$$
\begin{aligned}
S &\equiv S_0 + S_1 \\
&= \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[R + \frac{6}{L^2} - \frac{L^2}{g_F^2}F_{ab}F^{ab} \\
&\quad + \frac{\alpha L^4}{g_F^2}\big(R_{abcd}F^{ab}F^{cd} - 4R_{ab}F^{ac}F^{b}{}_{c} + RF^{ab}F_{ab}\big)\Big]. \qquad (15)
\end{aligned}
$$

α-a dimensionless constant.
The modified Maxwell equation

$$
\nabla_a[F^{ab} - \alpha L^2(R^{ab}{}_{cd}F^{cd} - 2R^{ac}F_c{}^{b} + 2R^{bc}F_c{}^{a} + RF^{ab})] = 0. \quad (16)
$$

The equations of motion Cont'd

The Einstein equation

$$
\begin{aligned}
&R_{ab} - \frac{1}{2}R g_{ab} = \frac{3}{L^2} + \frac{2L^2}{g_F^2} F_{ac}F_b{}^c - \frac{L^2}{2g_F^2} g_{ab}F^2 \\
&+\frac{\alpha L^4}{g_F^2}\Big(\frac{1}{2} g_{ab} R_{cdef}F^{cd}F^{ef} - 2R_{adef}F_b{}^d F^{ef} - 2R_{bdef}F_a{}^d F^{ef} \\
&+2\nabla^d\nabla^f F_{da}F_{bf}\Big) + \frac{\alpha L^4}{g_F^2}\Big(-2g_{ab}R_{cd}F^{ce}F^d{}_e - 2\nabla_d\nabla_a F_{bf}F^{df} \\
&-2\nabla_d\nabla_b F_{af}F^{df} + 2\Box F_{af}F_b{}^f + 2g_{ab}\nabla_c\nabla_d F^c{}_f F^{df} \\
&+4R_{ac}F_{bf}F^{cf} + 4R_{bc}F_{af}F^{cf} + 2R_{cd}F^c{}_a F^d{}_b + 2R_{cd}F^c{}_b F^d{}_a\Big) \\
&+\frac{\alpha L^4}{g_F^2}\Big(\frac{1}{2}g_{ab}RF^2 - R_{ab}F^2 + \nabla_a\nabla_b F^2 + g_{ab}\Box F^2 + 2RF_{ac}F_b{}^c\Big).
\end{aligned}
$$

The method for obtaining the perturbative solution

The ansatz for the perturbative solution

$$ds^2 = \frac{r^2}{L^2}[-f(r)dt^2 + dx^2 + dy^2] + \frac{L^2}{r^2}\frac{dr^2}{g(r)},$$
$$A_t(r) = A_t^{(0)}(r) + H(r), \quad (17)$$

where

$$f(r) = f_0(r)(1 + F(r)), \quad g(r) = f_0(r)(1 + F(r) + G(r)), \quad (18)$$

The main steps proposed in R. C. Myers, M. F. Paulos and A. Sinha, JHEP **0906**, 006 (2009) [arXiv:0903.2834 [hep-th]].

The method for obtaining the perturbative solution Cont'd

1. Considering the combination $G_t^t - f/gG_r^r$, where G_{ab} denotes the Einstein tensor, one finds a first-order linear ODE for $G(r)$, which is solvable.
2. Given $G(r)$, the modified Maxwell equation is easily solved.
3. With the above two perturbative solutions, $F(r)$ can be determined by solving the first-order linear ODE coming from the rr-component of the Einstein equation.

Step 1 gives

$$rf_0(r)\partial_r G(r) = 0, \quad \Rightarrow \quad \partial_r G(r) = 0, \;\; G(r) = \text{const.} \tag{19}$$

The explicit form of the perturbative solution

Without loss of generality

$$G(r) = 0, \quad \Rightarrow \quad f(r) = g(r) = f_0(r)(1 + F(r)). \tag{20}$$

Step 2 leads to

$$H(r) = h_0 + \frac{h_1}{r} + \frac{2\alpha\mu_0 r_0}{r} - \frac{\alpha\mu_0 r_0^4}{2r^4} - \frac{\alpha\mu_0 Q^2}{2r^4} + \frac{2\alpha\mu_0 r_0 Q^2}{5r^5}. \tag{21}$$

Step 3 gives

$$Y(r) \equiv f_0(r)F(r) = \frac{y_0}{r^3} - \frac{\alpha Q^2 r_0^3}{2r^7} - \frac{\alpha Q^4}{2r_0 r^7} + \frac{8\alpha Q^4}{5r^8}. \tag{22}$$

The explicit form of the perturbative solution Cont'd

Several constraints (following 0903.2834[hep-th])

- $r = r_0$ is still the horizon.

$$Y(r_0) = 0, \quad \Rightarrow \quad y_0 = \frac{\alpha Q^2}{2r_0} - \frac{11\alpha Q^4}{10r_0^5}. \tag{23}$$

- The charge density remains invariant. Thus the additional terms in the modified Maxwell equation do not contribute

$$\lim_{r\to\infty}[\sqrt{-g}\alpha L^2(2R_{rt}{}^{rt}F_{rt} - 2R^r_r F_{rt} + 2R^t_t F_{tr} + RF_{rt})] = 0,$$

which leads to

$$h_1 = 0. \tag{24}$$

The explicit form of the perturbative solution Cont'd

- The gauge potential $A_t(r)$ vanishes at the horizon,

$$H(r_0) = 0, \quad \Rightarrow \quad h_0 = \frac{\alpha\mu_0 Q^2}{10 r_0^4} - \frac{3}{2}\alpha\mu_0. \tag{25}$$

Thermodynamics:
The temperature

$$\begin{aligned} T &= \frac{1}{4\pi}\frac{1}{\sqrt{-g_{tt}g_{rr}}}\frac{d}{dr}g_{tt}|_{r=r_0} \\ &= \frac{1}{4\pi L^2 r_0^2}[(3M - \frac{4Q^2}{r_0^3}) + \frac{2\alpha Q^2}{r_0^3}(1 - \frac{3Q^2}{r_0^4})]. \end{aligned} \tag{26}$$

The chemical potential and the entropy density

The chemical potential

$$\mu = A_t(r \to \infty) = \mu_0 - \alpha\mu_0(\frac{3}{2} - \frac{Q^2}{10r_0^4}). \quad (27)$$

The entropy density given by Wald formula

$$s = -2\pi\frac{\partial L}{\partial R_{abcd}}\epsilon_{ab}\epsilon_{cd} = \frac{2\pi r_0^2}{\kappa^2 L^2} + \frac{2\pi\alpha Q^2}{\kappa^2 L^2 r_0^2}. \quad (28)$$

Calculating other thermodynamic quantities: the background subtraction method. Working in the grand canonical ensemble, fixed chemical potential. The reference background: pure AdS_4.

The energy density and the charge density

The energy density

$$\epsilon=\left(\frac{\partial I_{\mathrm{E}}}{\partial\beta}\right)_{\mu}-\frac{\mu}{\beta}\left(\frac{\partial I_{\mathrm{E}}}{\partial\mu}\right)_{\beta}=\frac{M}{\kappa^2L^4}-\alpha\frac{29Q^4+5Q^2r_0^4}{5\kappa^2L^4r_0^5}. \qquad (29)$$

The charge density

$$\rho=-\frac{1}{\beta}\left(\frac{\partial I_{\mathrm{E}}}{\partial\mu}\right)_{\beta}=\frac{2Q}{g_F\kappa^2L^2}+\frac{2\alpha(-29Q^5+Q^3r_0^4)}{5g_F\kappa^2L^2r_0^4(Q^2+3r_0^4)}. \qquad (30)$$

Quantities characterizing the local stability: the specific heat C_μ and the electrical permittivity ϵ_T.

The specific heat and the electrical permittivity

The specific heat

$$C_\mu = T\left(\frac{\partial s}{\partial T}\right)_\mu \frac{4\pi r_0^2(3r_0^4 - Q^2)}{\kappa^2 L^2(Q^2+3r_0^4)} + \alpha\frac{4\pi Q^2(Q^6 - 527Q^4 r_0^4 + 567Q^2 r_0^8 + 135 r_0^{12})}{5\kappa^2 L^2 r_0^2 (Q^2+3r_0^4)^3}. \quad (31)$$

The electrical permittivity

$$\epsilon_T = \left(\frac{\partial Q}{\partial \mu}\right)_T = \frac{6r_0(Q^2+r_0^4)}{g_F^2\kappa^2(Q^2+3r_0^4)} + \alpha\frac{6(-39Q^6+247Q^4r_0^4+11Q^2r_0^8+45r_0^{12})}{10g_F^2\kappa^2 r_0^3(Q^2+3r_0^4)^2}. \quad (32)$$

$T \geqslant 0 \to Q^2 \leqslant 3r_0^4$. at leading order $C_\mu \geqslant 0, \epsilon_T > 0$, locally stable. α corrections. numerical plots.

The definition of the DC conductivity

The Kubo formula

$$G^R_{xx}(\omega,\vec{k}=0)=-i\int dtd\vec{x}e^{i\omega t}\theta(t)\langle[J_x(x),J_x(0)],\tag{33}$$

J_a-CFT current dual to the bulk gauge field A_a.
The DC conductivity

$$\sigma_{\rm DC}=-\lim_{\omega\to 0}\frac{1}{\omega}{\rm Im}G^R_{xx}(\omega,\vec{k}=0),\tag{34}$$

One subtle point: since $A_t\neq 0$, the perturbation A_x can couple to the metric perturbations h_{xi}.

The strategy

Strategy: Gauge invariance imposes a relation between the two sets of perturbations which we use to integrate out the h_{xi} and obtain an action that involves only the A_x fluctuation. Introducing a new radial coordinate $u = r_0/r$, horizon $u = u_0$, the fluctuations of metric components and gauge field

$$\begin{aligned} {h_t}^x &= \int \frac{d^3k}{(2\pi)^3} t_k(u) e^{-i\omega t + iky}, \\ {h_u}^x &= \int \frac{d^3k}{(2\pi)^3} h_k(u) e^{-i\omega t + iky}, \\ A_x &= \int \frac{d^3k}{(2\pi)^3} a_k(u) e^{-i\omega t + iky}, \end{aligned} \tag{35}$$

The approach

The simplest method: considering the quadratic effective action (R. C. Myers, M. F. Paulos and A. Sinha, [arXiv:0903.2834 [hep-th]])

$$I_a^{(2)} = \frac{1}{2\kappa^2} \int \frac{d^3k}{(2\pi)^3} du (N(u) a'_k a'_{-k} + M(u) a_k a_{-k}), \tag{36}$$

where we have eliminated the contributions from $t_k(u)$ by using the corresponding Einstein equation and imposing $h_u{}^x = 0$.
The equation of motion

$$\partial_u j_k(u) = \frac{1}{\kappa^2} M(u) a_k(u), \tag{37}$$

where

The approach Cont'd

$$j_k(u) \equiv \frac{\delta I_a^{(2)}}{\delta a'_{-k}} = \frac{1}{\kappa^2} N(u) a'_k(u), \tag{38}$$

Requiring regularity at the horizon (N. Iqbal and H. Liu, Phys. Rev. D **79**, 025023 (2009) [arXiv:0809.3808 [hep-th]].)

$$j_k(u_0) = -i\omega \lim_{u \to u_0} \frac{N(u)}{\kappa^2} \sqrt{\frac{g_{uu}}{-g_{tt}}} a_k(u) + \mathcal{O}(\omega^2), \tag{39}$$

The flux factor

$$2\mathcal{F}_k = j_k(u) a_{-k}(u), \tag{40}$$

The approach Cont'd

According to (34), the conductivity is given by

$$\sigma = \lim_{u,\omega\to 0} \frac{1}{\omega}\mathrm{Im}\left[\frac{2\mathcal{F}_k}{a_k(u)a_{-k}(u)}\right]_{k=0} = \lim_{u,\omega\to 0} \mathrm{Im}\left[\frac{j_k(u)a_{-k}(u)}{\omega a_k(u)a_{-k}(u)}\right]_{k=0}, \tag{41}$$

Note that

$$\frac{d}{du}\mathrm{Im}[j_k(u)a_{-k}(u)] = \mathrm{Im}(f_1(u)a_k(u)a_{-k}(u)+f_2(u)j_k(u)j_{-k}(u)) = 0, \tag{42}$$

as the two terms are real. Thus it is conserved and can be evaluated at the horizon.

The approach Cont'd

Then the DC conductivity

$$\sigma_{\rm DC} = \frac{1}{\kappa^2} K_A^2(u_0) \frac{\mathcal{N}(u_0)}{\mathcal{N}(0)}|_{k=0}, \tag{43}$$

where

$$K_A^2(u) = -N(u)\sqrt{\frac{g_{uu}}{-g_{tt}}}, \quad \mathcal{N}(u) = a_k(u) a_{-k}(u), \tag{44}$$

Note that $\mathcal{N}(u)$ is real and independent of ω up to $\mathcal{O}(\omega^2)$. So it is regular at the horizon and is sufficient to set $\omega = 0$ in the equation of motion for a_k.

Our case

For our particular case,

$$ds^2 = -\frac{r_0^2}{L^2u^2}f(u)dt^2 + \frac{L^2du^2}{u^2f(u)} + \frac{r_0^2}{L^2u^2}(dx^2 + dy^2), \qquad (45)$$

where

$$f(u) = (1-u)[F(u) + \alpha G(u)], \quad F(u) = 1 + u + u^2 - \frac{Q^2u^3}{r_0^4},$$

$$G(u) = \frac{Q^2u^3}{10r_0^8}[5r_0^4(1 + u + u^2 + u^3) - Q^2(11 + 11u + 11u^2 + 11u^3 + 16u^4)], \qquad (46)$$

Our case Cont'd

Consider the leading order solution

$$f(u) = f_0(u) = (1-u)F(u), \tag{47}$$

according to (34), it is sufficient to set $k=0$.
The constraint for t_k

$$t_k' = -4\frac{L^4u^2}{g_F^2 r_0^2}A_t' a_k, \tag{48}$$

Therefore

$$N(u) = -\frac{r_0}{g_F^2}f_0(u), \quad M(u) = \frac{L^4\omega^2}{r_0 g_F^2 f_0(u)} - \frac{4L^4u^2}{r_0 g_F^4}A_t'^2, \tag{49}$$

Our case Cont'd

The solution for $a_k(u)$

$$a_k(u) = a_0\left(1 - \frac{4Q^2}{3(r_0^4 + Q^2)}u\right), \tag{50}$$

The DC conductivity

$$\sigma_{\mathrm{DC}} = \frac{L^2}{\kappa^2 g_F^2}\frac{(3r_0^4 - Q^2)^2}{9(r_0^4 + Q^2)^2}, \tag{51}$$

which agrees with previous result (e.g. X. H. Ge, K. Jo and S. J. Sin, [arXiv:1012.2515 [hep-th]].).

Our case Cont'd

Including corrections: the steps are more or less the same, but the equation becomes more complicated. Keeping the solution to first order of α and Q^2,

$$a_k(u) = a_0 + a_1 u + \alpha[(a_2 - 2a_1)u - \frac{1}{4}a_1 u^4 - (\frac{1}{3}a_0 + \frac{1}{4}a_1)\frac{Q^2 u^4}{r_0^4}], \quad a_1 = -\frac{4a_0 Q^2}{3(r_0^4 + Q^2)}, \tag{52}$$

a_2-integration constant. The conductivity

$$\sigma_{\rm DC} = \frac{L^2}{\kappa^2 g_F^2}[\frac{(3r_0^4 - Q^2)^2}{9(r_0^4 + Q^2)^2} + 2\alpha(a_2 + \frac{8 - 4a_2}{3}\frac{Q^2}{r_0^4})], \tag{53}$$

Our case Cont'd

In the limit of $Q = 0$,

$$\sigma_{\mathrm{DC}} = \frac{L^2}{\kappa^2 g_F^2}(1 + 2\alpha a_2), \tag{54}$$

one can reproduce the result in arXiv: 1010.0443[hep-th] by suitably choosing a_2.

The definition

The retarded Green's function

$$G^R_{xy,xy}(\omega, \vec{k}=0) = -i \int dt d\vec{x} e^{i\omega t} \theta(t) \langle [T_{xy}(x), T_{xy}(0)] \rangle, \quad (55)$$

The shear viscosity is given by

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} G^R_{xy,xy}(\omega, \vec{k}=0), \quad (56)$$

One can still apply the effective action approach.

The approach

Consider the metric perturbation

$$h_x{}^y(t,u) = \int \frac{d^3k}{(2\pi)^3}\phi(u)e^{-i\omega t}, \tag{57}$$

and expand the action to quadratic order in ϕ,

$$\begin{aligned} I_\phi^{(2)} &= \frac{1}{2\kappa^2}\int \frac{d^3k}{(2\pi)^3}du[A(u)\phi''\phi + B(u)\phi'\phi' + C(u)\phi'\phi \\ &+ D(u)\phi\phi + E(u)\phi''\phi'' + F\phi''\phi' + K_{\mathrm{GH}}], \end{aligned} \tag{58}$$

K_{GH}-contributions from the Gibbons-Hawking terms.

The approach Cont'd

After making use of the equation of motion and integrating by parts

$$\tilde{I}^{(2)}_{\phi} = \frac{1}{2\kappa^2}\int \frac{d^3k}{(2\pi)^3}du[(B-A-\frac{F'}{2})\phi'\phi' + E(u)\phi''\phi'' + (D-\frac{(C-A')'}{2})\phi\phi] + \tilde{K}_{\rm GH}. \quad (59)$$

The canonical momentum is given by

$$\Pi(u) \equiv \frac{\delta \tilde{I}^{(2)}_{\phi}}{\delta \phi'} = \frac{1}{\kappa^2}[(B-A-\frac{F'}{2}) - (E(u)\phi'')']. \quad (60)$$

The approach Cont'd

According to arXiv: 0809.3808[hep-th],

$$\eta = \lim_{u,\omega \to 0} \frac{\Pi(u)}{i\omega\phi(u)}. \tag{61}$$

In the low frequency limit $\partial_u \Pi(u) = 0$, so we can evaluate $\Pi(u)$ at the horizon.. Imposing the regularity condition,

$$\eta = \frac{1}{\kappa^2}(K_\phi^2(u_0) + K_\phi^4(u_0)), \tag{62}$$

where

$$K_\phi^{(2)}(u) = \sqrt{\frac{g_{uu}}{-g_{tt}}}(A - B + \frac{F'}{2}), \quad K_\phi^{(4)}(u) = \left[E(u)\left(\sqrt{\frac{g_{uu}}{-g_{tt}}}\right)'\right]'. \tag{63}$$

Our case

for our particular background, the nonvanishing functions in $\tilde{I}_\phi^{(2)}$,

$$A(u) = \frac{2r_0^4 f_(u)}{L^4 u^2}, \quad B(u) = \frac{3r_0^3 f(u)}{2L^4 u^2} - \frac{\alpha u^2 Q^2 f(u)}{L^4 r_0},$$
$$C(u) = -\frac{6r_0^3 f(u)}{L^4 u^3} + \frac{2r_0^3 f(u)'}{L^4 u^2} - \frac{4\alpha u Q^2 f(u)}{L^4 r_0}, \tag{64}$$

therefore

$$K_\phi^2(u) = \frac{r_0^2}{2u^2 L^2} + \frac{\alpha u^2 Q^2}{L^2 r_0^2}, \quad K_\phi^4(u) = 0, \tag{65}$$

which leads to

$$\eta = \frac{1}{\kappa^2}\left(\frac{r_0^2}{2L^2} + \frac{\alpha Q^2}{L^2 r_0^2}\right). \tag{66}$$

Thermal conductivity

The thermal conductivity determines the response of the heat flow to temperature gradients, $T^t{}_i = -\kappa_T \partial_i T$.
The expression (D. T. Son and A. O. Starinets, JHEP **0603**, 052 (2006), hep-th/0601157)

$$\kappa_T = (\frac{s}{\rho} + \frac{\mu}{T})^2 T\sigma, \tag{67}$$

One can easily obtain κ_T by substituting previous results into this expression.

η/s and $\kappa_T\mu^2/(\eta T)$

One interesting ratio η/s,

$$\frac{\eta}{s} = \frac{1}{4\pi}(1+\alpha\frac{Q^2}{r_0^4}). \qquad (68)$$

When $Q=0$, it reproduces the well-known bound $1/4\pi$. It might be violated in the presence of a chemical potential. Another ratio

$$\frac{\kappa_T\mu^2}{\eta T} = 2\pi^2 g_F^2 + \alpha\pi^2 g_F^2[(4a_2-10)+\frac{422+80a_2}{15}\frac{Q^2}{r_0^4}], \qquad (69)$$

The bound in hep-th/0601157: $8\pi^2$ can also be violated.

Summary

- We consider RF^2 corrections to $RN - AdS_4$ black holes.
- The perturbative solutions are calculated and the thermodynamic properties are discussed.
- The DC conductivity is obtained via the effective action approach, which can reproduce the result in 1010.0443[hep-th] in certain limit.
- The shear viscosity and the thermal conductivity are evaluated.
- Two interesting ratios η/s and $\kappa_T \mu^2/(\eta T)$ are obtained. The corresponding bounds can be violated.

Discussion

- Hydrodynamic quantities in extremal background, M. F. Paulos, arXiv:0910.4602[hep-th]. In particular, $\sigma \sim \omega^2$.
- The full correlation functions in the presence of RF^2 corrections.
- Holographic optics(1006.5714[hep-th]).
- $\cdots$

Thank you!