Physical consequences of the QED theta angle

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ITP, Beijing, December 2010

Outline

1. Quantum phases from the QED theta angle
2. Path integral
3. Functional Schrodinger equation and Aharonov-Bohm effect
4. Non-Abelian generalizations
5. Remarks

arxiv: 1012.2906

QED theta angle

$$\frac{\theta}{4}\, F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{\theta}{8} F_{\mu\nu}F_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = -\theta\, E \cdot B\ .$$

Does this term have physical consequences?

No topology, total derivative.

$$K^{\mu} = 1/4\ \epsilon^{\mu\alpha\beta\gamma} F_{\alpha\beta} A_{\gamma}$$

$$\partial_{\mu} K^{\mu} = 1/4\, F\tilde{F} = -E \cdot B$$

$$\int d^4x\, E \cdot B = \int d^3x\, K^0(A_f) - \int d^3x\, K^0(A_i)$$

QED theta angle

No effect on equations of motion.

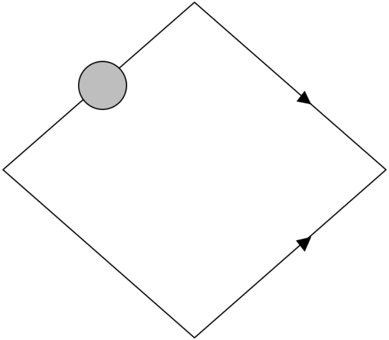
No effect on Feynman diagrams (perturbation theory)

Perhaps effects are exponentially small? $\exp(-1/\alpha)$?

But what about strong fields? Or subtle effects like quantum phases?

IF theta term has physical consequences, then θ is an unmeasured parameter of the standard model!

Gedanken experiment

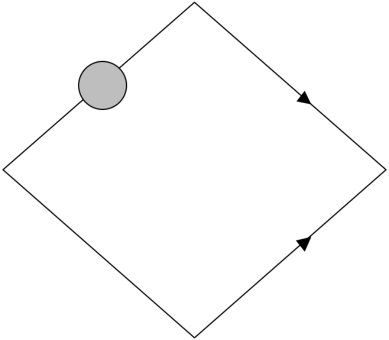


Two identical wave packets of light are sent along upper and lower paths of the same length. The upper packet is exposed to a background electromagnetic field depicted by the shaded circle. This background field is chosen so that $E \cdot B$ is non-zero in the interaction region. Interference between the recombined packets depends on the parameter θ.

Relative phase:

$$-\theta \int d^4x \, E \cdot B \; \equiv \; \theta \, \Phi \, ,$$

Gedanken experiment

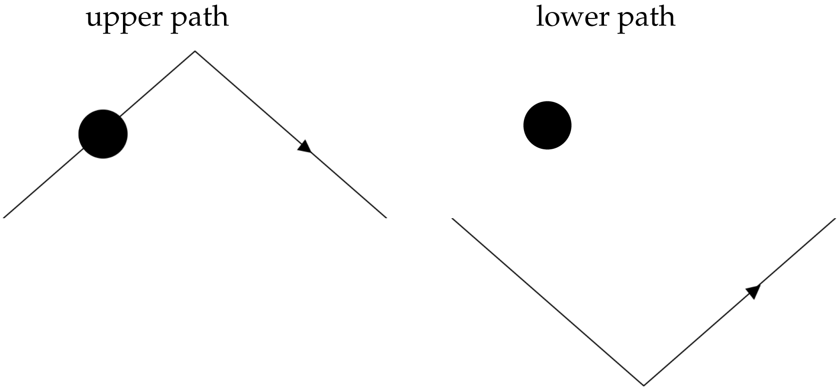


Example: wavepackets produced by beam splitter; B field normal to plane of diagram. In shaded region, have E field normal to plane of diagram. Outside of shaded region $E \cdot B = 0$.

Relative phase:

$$-\theta \int d^4x \, E \cdot B \; \equiv \; \theta \, \Phi \, ,$$

Gedanken experiment



$$\mathcal{A}(i \to f) = \mathcal{A}_{\text{upper}}(i \to f) + \mathcal{A}_{\text{lower}}(i \to f)$$

Path integral

$$\mathcal{A}(i \to f) = \mathcal{A}_{\text{upper}}(i \to f) + \mathcal{A}_{\text{lower}}(i \to f) \quad .$$

Each of $\mathcal{A}_{\text{upper/lower}}$ can be expressed as a path integral:

$$\mathcal{A} = \int dA_i \, dA_f \int_{A_i}^{A_f} DA \exp\left(-i \int d^4x \, \frac{1}{4} F^2 + \theta E \cdot B\right) \Psi^*[A_i] \, \Psi[A_f]$$

1. i, f denote initial and final times

2. wavefunction factors Ψ are the overlap between eigenstates of the field operator $\hat{A}_\mu(x)$ and physical state $|\Psi\rangle$: $\Psi[A] = \langle A|\Psi\rangle$

3. Paths differ in value of Φ.

Path integral: semi-classical approximation

Expand about solution to classical equation of motion (Maxwell's equations):

$$A(x) = \bar{A}(x) + \delta A(x) \ ,$$

where $\bar{A} = A_{\rm bkgnd} + \bar{A}_{\rm packet}$

1. Along upper path, obtain extra phase:

$$\mathcal{A}_{\rm upper} \approx \mathcal{A}_{\rm lower}\, e^{i\theta\Phi}$$

2. Action still quadratic, even with theta term – can do fluctuation integral exactly.

$$S[A] = S[\bar{A}] + \delta S[A]/\delta A|_{\bar{A}}\, \delta A + \delta^2 S[A]/\delta A^2|_{\bar{A}}\, (\delta A)^2$$

Path integral: beyond semi-classical approximation

Phase factor is exact: $\mathcal{A}_{\text{upper}} = \mathcal{A}_{\text{lower}}\, e^{i\theta\Phi}$

$$S[A] = S[\bar{A}] + \delta S[A]/\delta A|_{\bar{A}}\, \delta A + \delta^2 S[A]/\delta A^2|_{\bar{A}}\, (\delta A)^2$$

1. First term contains phase factor.

2. Second term vanishes on solution to equations of motion. Also, independent of θ. * Same for upper and lower paths.

3. Third term (fluctuation kernel) independent of θ. * Same for upper and lower paths.

* Theta term a total derivative; write in terms of boundary integrals $\int d^3x\, K^0(A)$, so no effect on $\delta/\delta A(x)$.

Gauge symmetry and redundant description

$$\int d^4x \, E \cdot B = \int d^3x \, K^0(A_f) - \int d^3x \, K^0(A_i)$$

1. If $\int d^4x \, E \cdot B$ is different for two trajectories (upper, lower), then the boundary conditions of the trajectories must be different. Assume the difference is in A_f.

2. Therefore, we are interfering two configurations with different A_f's, although they have same E and B fields (up to reflection).

Upper and lower final states differ by a reflection and a gauge transformation.

Gauge symmetry and redundant description

Classical EM: $A^U = A + iU^\dagger \partial U$ and A are the same state. (Two descriptions of the same thing.)

It's possible that the corresponding quantum states differ by a phase:

$$|A^U\rangle = e^{i\delta} |A\rangle \; ,$$

where $\hat{A}(x)|A\rangle = A(x)|A\rangle$; eigenstates of the field operator. Could even have $\delta = \delta(A, U)$!

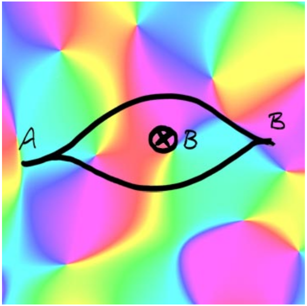
We assume $|A\rangle = |A^U\rangle$ are the same physical state; this choice of basis is required to define what is meant by θ.

Aharonov-Bohm effect

Vector potential causes shift in momentum operator

$$-i\partial_i \quad \rightarrow \quad -i\partial_i + A_i \ .$$

Induces A-dependent phase into wavefunction. But no force or modification of classical motion.



$$\exp\left(i\int dx \cdot A\right)\psi = \exp\left(i\int dt\,\frac{dx}{dt}\cdot A\right)\psi$$

Functional Schrodinger equation

Similar result in gauge field theory due to theta term.

Recall $\mathcal{L} = -F^2/4 = 1/2\left(E^2 - B^2\right)$. In $A_0 = 0$ gauge $E = -\dot{A}$, so the conjugate momentum $\partial\mathcal{L}/\partial\dot{A} = \dot{A} = -E$. With no theta term the functional Schrodinger equation is

$$\frac{1}{2}\left((-i\delta/\delta A(x))^2 + B(x)^2\right)\Psi[A] = i\frac{\partial}{\partial t}\Psi[A].$$

Theta term $-\theta\, E \cdot B = \theta\, \dot{A}B$ shifts the conjugate momentum by $\theta B(x)$. Momentum operator $-i\delta/\delta A(x)$ in the Schrodinger equation becomes

$$-i\frac{\delta}{\delta A(x)} + \theta B(x)\ .$$

Functional Schrodinger equation

This causes the wave functional $\Psi[A]$ to acquire a phase (analogous to the Aharonov-Bohm phase) associated with motion in the configuration space:

$$\Psi_\theta[A] \; = \; \exp\left(i\theta \int d^3x \, \frac{A \cdot B}{2}\right) \Psi[A] \, . \tag{1}$$

If $\Psi[A]$ is a soln to Schrodinger Eqn for $\theta = 0$, then $\Psi_\theta[A]$ is corresponding solution in presence of theta term. $\delta/\delta A(x)$ of the integral in (1) yields $-\theta B(x)$, which cancels the shift in momentum operator.

Integrand $\frac{1}{2} \, A \cdot B$ is simply K^0, where

$$K^\mu = 1/4 \, \epsilon^{\mu\alpha\beta\gamma} F_{\alpha\beta} A_\gamma$$

$$\partial_\mu K^\mu = 1/4 \, F\tilde{F} = -E \cdot B$$

Functional Schrodinger equation

Fix the phase relative to some reference configuration A_*:

$$i\theta\left(\int d^3x\, K^0(A) - \int d^3x\, K^0(A_*)\right) = -i\theta \int d^4x\, E\cdot B$$

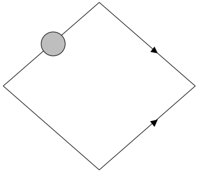
Where boundary conditions on spacetime integral are $A(t_i, x) = A_*(x)$ and $A(t_f, x) = A(x)$.

Define this quantity to be the phase

$$i\theta\, \Phi(A|A_*)$$

Note that $\Phi[A_1|A_3] = \Phi[A_1|A_2] + \Phi[A_2|A_3]$.

Functional Schrodinger equation



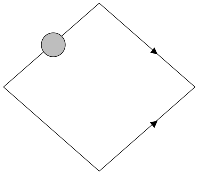
Now revisit gedanken experiment. Want to show that

$$\Phi[A_{f,\mathrm{upper}}|A_{f,\mathrm{lower}}] = \Phi[A_{f,\mathrm{upper}}|A_{i,\mathrm{upper}}]$$

$$\Phi[A_{f,\mathrm{upper}}|A_{f,\mathrm{lower}}] = \Phi[A_{f,\mathrm{upper}}|A_{i,\mathrm{upper}}] + \Phi[A_{i,\mathrm{upper}}|A_{i,\mathrm{lower}}]$$

$$+ \Phi[A_{i,\mathrm{lower}}|A_{f,\mathrm{lower}}]$$

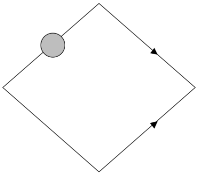
Functional Schrodinger equation



1. $\Phi[A_{i,\text{upper}}|A_{i,\text{lower}}]$ is zero by assumption – the two initial states are produced with no relative phase, e.g., by a perfect beam splitter. (Also zero by interpolation, because $E \cdot B$ is always zero; wave packet initial states are far from the background field region.)

2. $\Phi[A_{i,\text{lower}}|A_{f,\text{lower}}]$ is zero because the interpolation between the initial and final configurations on the lower trajectory have $E \cdot B = 0$ at all times.

Realistic configurations



Interference if:

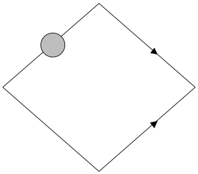
$$\Phi[A_{i,\text{lower}}|A_{f,\text{lower}}] \neq \Phi[A_{i,\text{upper}}|A_{f,\text{upper}}]$$

For simplicity, we have assumed $E \cdot B$ non-zero on upper path, zero on lower path. This may not be a realistic assumption.

Interference depends only on whether Φ is different along the upper and lower paths.

$$\int_{t_i}^{t_f} d^3x\, K^0(A) = \int_{t_i}^{t_f} d^3x\, \frac{1}{2} A \cdot B$$

Realistic configurations



$$\int_{t_i}^{t_f} d^3x \, K^0(A) = \int_{t_i}^{t_f} d^3x \, \frac{1}{2} A \cdot B$$

Let $A = A_b + A_p$ (background, packet), then

$$A \cdot B = A \cdot (\nabla \times A) = A_p \cdot (\nabla \times A_p) \; + \; A_b \cdot (\nabla \times A_b) \; + \; (\text{mixed terms})$$

Phase difference due to mixed terms:

$$A_p \cdot B_b + A_b \cdot B_p$$

One might expect that a generic source of background field placed asymmetrically wrt axis of symmetry could produce interference. (Might want time dependence.)

Non-Abelian generalization

Can repeat functional Schrodinger calculation for Non-Abelian gauge theory. Obtain phase factor

$$i\theta\,\Phi = i\,\frac{\theta}{4}\int d^4x\,\mathrm{tr}\,F\tilde{F}$$

$$K^{\mu} = \epsilon^{\mu\alpha\beta\gamma}\,\mathrm{tr}\left(F_{\alpha\beta}A_{\gamma} - \frac{2}{3}A_{\alpha}A_{\beta}A_{\gamma}\right)$$

Note: for generic gauge configurations $A_i(x)$ and $A_f(x)$ (i.e., not necessarily vacuum configurations, nor related by a gauge transformation) the topological charge is not quantized, but rather takes on continuous values.

Remarks

1. Theta term can have a quantum mechanical effect on local physics despite the fact that it is a total divergence and has no effect on the classical equations of motion (cf. the Aharonov-Bohm effect).

2. Although the spacetime integral of $E \cdot B$ over a region is fixed by the values of the potential A on the boundary, the specific arrangement of the density $E \cdot B$ within the region can lead to observable consequences: relative phases for different photon states.

3. Not so different from the case of the electric charge Q: the total Q on a spacelike slice is fixed, but the distribution of charge density has local consequences.

Remarks: baryogenesis?

1. These effects violate CP symmetry, so it is possible they may have some relevance to the baryon asymmetry of the universe.

2. The SU(2) theta angle has no physical consequences, because it can be canceled by appropriate chiral rotation of the left handed fermions.

3. But electroweak baryon number violating processes (i.e., mediated by sphaleron-like configurations) typically involve strong electromagnetic fields, so might be affected by the CP violating QED theta angle.

Remarks: GUTs?

In grand unified theories such as SU(5) or SO(10), the theta angles for each of the standard model gauge forces (i.e., SU(3), SU(2), U(1)) are related by group theoretical factors.

Therefore, low energy measurements of these angles have interesting implications for very high energy physics.

A new constant of Nature?

We thought we understood QED perfectly – no new mysteries.

Are there new quantum phases?

Who will be the first to measure this parameter of the Standard Model?