Left-Right Symmetric Models with Peccei-Quinn Symmetry

Pei-Hong Gu

Max-Planck-Institut für Kernphysik, Heidelberg

PHG, 1011.2380; PHG, Manfred Lindner, 1011.4905.

Institute of Theoretical Physics, Chinese Academy of Sciences, 20/12/2010

Outline

- Introduction
- Left-Right Symmetric Models
- Double and Linear Seesaw from PQ Symmetry Breaking
- Inflation and Flavor Mixing from PQ Symmetry Breaking
- Summary

Introduction

- The $SU(3)_c \times SU(2)_L \times U(1)_Y$ standard model standard model (SM) has been tested to a very high accuracy. However, there are several theoretical motivations to consider theories beyond the SM. For example, the SM left-handed fermions transform as doublets while the SM right-handed fermions transform as singlets. This unequal treatment of the left- and right-handed fermions is dictated by parity violation at low energy. It is thus natural to start with a theory with left-right symmetry (Pati, Salam, 74'; Mohapatra, Pati, 75'; Mohapatra, Senjanović, 75') at high energy, where both the left- and right-handed fermions are treated on equal footing and parity is conserved. At some high energy the left-right symmetry is spontaneously broken, which also induces a spontaneous parity violation as observed at low energy. This inspires us to consider a left-right symmetric extension of the SM.

- The strong CP problem is another big challenge to the SM. The Peccei-Quinn (Peccei, Quinn, 77') (PQ) symmetry predicts the existence of an axion (Peccei, Quinn, 77'; Weinberg, 78'; Wilczek, 78') which would solve the strong CP problem. The original axion model (Peccei, Quinn, 77'; Weinberg, 78'; Wilczek, 78') has been ruled out. Alternatively, we can consider the Kim-Shifman-Vainshtein-Zakharov (Kim, 79'; Shifman, Vainshtein, Zakharov, 80') (KSVZ) model or the Dine-Fischler-Srednicki-Zhitnitsky (Dine, Fischler, Srednicki, 81'; Zhitnitsky, 80') (DFSZ) model for the invisible axion. The axion could also be a dark matter candidate (Preskill, Wise, Wilczek, 83'; Abbott, Sikivie, 83'; Dine, 83'). The PQ symmetry could have various other interesting implications on particle physics and cosmology. For example, we can relate the PQ symmetry breaking to the neutrino mass-generation (Shin, 87').

- Recently we proposed some left-right symmetric models (PHG, 1011.2380; PHG, Lindner, 1011.4905), where the fermion singlets for double (Mohapatra, 86') and linear (Barr, 04') seesaw can naturally obtain their masses through the PQ symmetry breaking. The PQ symmetry breaking thus is tightly related to the neutrino mass-generation. The PQ symmetry breaking even can simultaneously realize the natural inflation (Freese, Frieman, Olinto, 90'; Adams *et al.*, 93') and generate the lepton and quark mixing. Our models can be embedded into the $SO(10)$ grand unification theories (GUT).

Left-Right Symmetric Models

- The left-right symmetric models (Pati, Salam, 74'; Mohapatra, Pati, 75'; Mohapatra, Senjanović, 75') are based on the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In the left-right symmetric context, the left-handed fermions are $SU(2)_L$ doublets as they are in the standard model while the right-handed fermions (the standard model right-handed fermions plus the right-handed neutrinos) are placed in $SU(2)_R$ doublets,

$$q_L(\mathbf{3},\mathbf{2},\mathbf{1},\ \tfrac{1}{3}) = \begin{bmatrix} u_L \\ d_L \end{bmatrix}, \quad q_R(\mathbf{3},\mathbf{1},\mathbf{2},\ \tfrac{1}{3}) = \begin{bmatrix} u_R \\ d_R \end{bmatrix};$$

$$l_L(\mathbf{1},\mathbf{2},\mathbf{1},-1) = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}, \quad l_R(\mathbf{1},\mathbf{2},\mathbf{1},-1) = \begin{bmatrix} \nu_R \\ e_R \end{bmatrix}.$$

In order to generate the fermion masses, we need some Higgs scalars to construct the Yukawa interactions. For example, in the most popular left-right symmetric model with two triplet and one bi-doublet Higgs scalars, the charged fermions and the neutral neutrinos can obtain their Dirac masses though the Yukawa interactions involving the Higgs bi-doublet. Since the right-handed neutrinos obtain their heavy Majorana masses through the Yukawa interactions between the right-handed lepton doublets and Higgs triplet, we can naturally realize the type-I seesaw (Minkowski, 77'; Yanagida, 79'; Gell-Mann, Ramond, Slansky, 79'; Glashow, 80'; Mohapatra, Senjanović, 80'). At the same time, the left-handed Higgs triplet, which have the Yukawa interactions with the left-handed lepton doublets, can acquire a small vacuum expectation value for the type-II seesaw (Magg, Wetterich, 80'; Schechter, Valle, 80'; Cheng, Li, 80'; Lazarides, Shafi, Wetterich, 81'; Mohapatra, Senjanović, 81'). Therefore, the small neutrino masses can be understood in a natural way. This scenario also accommodates leptogenesis (Fukugita, Yanagida, 86') — one of the most popular mechanisms to explain the cosmological baryon asymmetry.

- In the original left-right symmetric model, the Higgs sector contains two doublet and one bi-doublet Higgs scalars,

$$\chi_L(\mathbf{1},\mathbf{2},\mathbf{1},-1) = \begin{bmatrix} \chi_L^0 \\ \chi_L^- \end{bmatrix}, \quad \chi_R(\mathbf{1},\mathbf{1},\mathbf{2},-1) = \begin{bmatrix} \chi_R^0 \\ \chi_R^- \end{bmatrix},$$

$$\phi(\mathbf{1},\mathbf{2},\mathbf{2}^*,0) = \begin{bmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{bmatrix}.$$

The allowed Yukawa interactions are

$$\mathcal{L}_Y = -y_q\bar{q}_L\phi q_R - \tilde{y}_q\bar{q}_L\tilde{\phi}q_R - y_l\bar{l}_L\phi l_R - \tilde{y}_l\bar{l}_L\tilde{\phi}l_R + \text{H.c.}.$$

The discrete left-right symmetry can be parity $\mathcal{P}$ or charge-conjugation $\mathcal{C}$. The fields should transform as

$$\chi_L \stackrel{\mathcal{P}}{\longleftrightarrow} \chi_R\,, \phi \stackrel{\mathcal{P}}{\longleftrightarrow} \phi^\dagger\,, q_L \stackrel{\mathcal{P}}{\longleftrightarrow} q_R\,, l_L \stackrel{\mathcal{P}}{\longleftrightarrow} l_R\,,$$

or

$$\chi_L \stackrel{\mathcal{C}}{\longleftrightarrow} \chi_R^*\,, \phi \stackrel{\mathcal{C}}{\longleftrightarrow} \phi^T\,, q_L \stackrel{\mathcal{C}}{\leftrightarrow} q_R^c\,, l_L \stackrel{\mathcal{C}}{\longleftrightarrow} l_R^c\,.$$

Once the discrete left-right symmetry is determined, we can constrain the Yukawa couplings and other parameters in the lagrangian. For example, in the case of charge-conjugation, the Yukawa couplings should be

$$y_q = y_q^T\,, \;\; \tilde{y}_q = \tilde{y}_q^T\,, \;\; y_l = y_l^T\,, \;\; \tilde{y}_l = \tilde{y}_l^T\,.$$

Through the Yukawa interactions with the Higgs bi-doublet, the charged fermions and the neutral neutrinos can obtain their Dirac masses,

$$\tilde{m}_d = y_q\langle\phi_2^0\rangle + \tilde{y}_q\langle\phi_1^0\rangle\,, \qquad \tilde{m}_u = y_q\langle\phi_1^0\rangle + \tilde{y}_q\langle\phi_2^0\rangle\,,$$

$$\tilde{m}_e = y_l\langle\phi_2^0\rangle + \tilde{y}_l\langle\phi_1^0\rangle\,, \qquad \tilde{m}_\nu = y_l\langle\phi_1^0\rangle + \tilde{y}_l\langle\phi_2^0\rangle\,.$$

It is difficult to generate the small neutrino masses unless we fine tune the Yukawa couplings.

We can revive the original left-right symmetric model by adding some fermion singlets,

$$S_R(\mathbf{1},\mathbf{1},\mathbf{1},0) \stackrel{\mathcal{P}}{\longleftrightarrow} S_R^c(\mathbf{1},\mathbf{1},\mathbf{1},0) \ \text{ or } \ S_R(\mathbf{1},\mathbf{1},\mathbf{1},0) \stackrel{\mathcal{C}}{\longleftrightarrow} S_R(\mathbf{1},\mathbf{1},\mathbf{1},0)\,.$$

The fermion singlets with a Majorana mass term can have the Yukawa couplings with the lepton and Higgs doublets,

$$\mathcal{L} \supset -h_L \bar{l}_L \chi_L S_R - h_R \bar{l}_R^c \chi_R^* S_R - \frac{1}{2} M_S \overline{S}_R S_R^c + \text{H.c.}$$
$$\text{with} \quad h_L = h_R^* \ \text{ or } \ h_L = h_R\,.$$

The full mass terms involving the left- and right-handed neutrinos and the fermion singlets should be

$$\mathcal{L} \supset -\frac{1}{2}\left[\bar{\nu}_L \ \ \bar{\nu}_R^c \ \ \overline{S}_R^c\right] \left[\begin{array}{ccc} 0 & \tilde{m}_\nu & h_L\langle\chi_L^0\rangle \\ \tilde{m}_\nu^T & 0 & h_R\langle\chi_R^0\rangle \\ h_L^T\langle\chi_L^0\rangle & h_R^T\langle\chi_R^0\rangle & M_S \end{array}\right] \left[\begin{array}{c} \nu_L^c \\ \nu_R \\ S_R \end{array}\right] + \text{H.c.}\,.$$

When $\tilde{m}_\nu$ and $h\langle\chi_L\rangle$ are much smaller than $h\langle\chi_R\rangle$ and/or M_S, we can get the neutrino masses by the seesaw formula, i.e.

$$\mathcal{L} \supset -\frac{1}{2} m_\nu \bar{\nu}_L \nu_L^c + \text{H.c.} \quad \text{with}$$

$$m_\nu = \tilde{m}_\nu \frac{1}{h^T \langle\chi_R^0\rangle} M_S \frac{1}{h\langle\chi_R^0\rangle} \tilde{m}_\nu^T - (\tilde{m}_\nu + \tilde{m}_\nu^T) \frac{\langle\chi_L^0\rangle}{\langle\chi_R^0\rangle} .$$

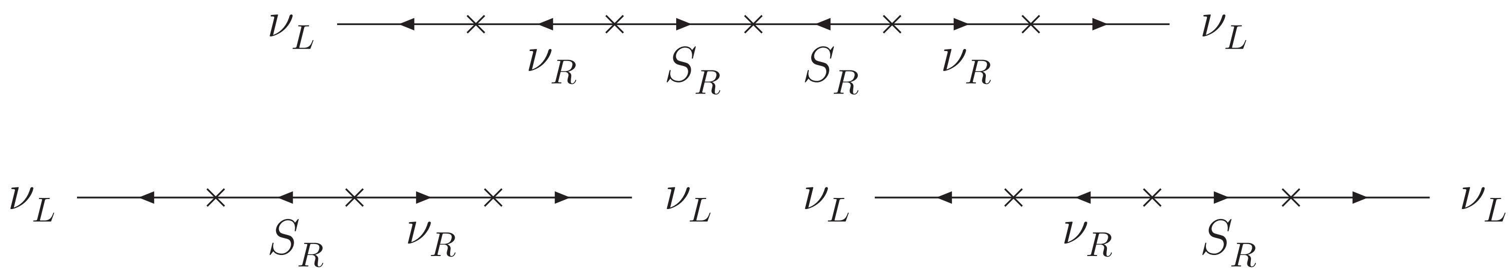

FIG. 1: Double/Inverse (top) and linear (bottom) seesaw.

In the double/inverse (Majorana, 86'; Majorana, Valle, 86') and linear (Barr, 03') seesaw scenario, we can realize the leptogenesis in two ways (PHG, Sarkar, 10'): the usual leptogenesis with double seesaw, the resonant leptogenesis with inverse seesaw.

Double and Linear Seesaw from PQ Symmetry Breaking

- The fermion singlets play an essential role to realize the double and linear seesaw in the left-right symmetric model. Their Majorana masses can be simply input by hand as they are allowed by the gauge symmetry. A more attractive possibility is to introduce certain spontaneous symmetry breaking.

  We introduce a complex scalar singlet,

  $$\sigma(\mathbf{1},\mathbf{1},\mathbf{1},0)\,.$$

  The one Higgs bi-doublet is extended to be two Higgs bi-doublet,

  $$\phi_1(\mathbf{1},\mathbf{2},\mathbf{2}^*,0)=\begin{bmatrix}\phi^0_{11} & \phi^+_{12}\\ \phi^-_{11} & \phi^0_{12}\end{bmatrix}\,,\quad \phi_2(\mathbf{1},\mathbf{2},\mathbf{2}^*,0)=\begin{bmatrix}\phi^0_{21} & \phi^+_{22}\\ \phi^-_{21} & \phi^0_{22}\end{bmatrix}\,.$$

We take the discrete left-right symmetry to be the charge-conjugation. The transformation of the complex scalar singlet should be

$$\sigma \stackrel{\mathcal{C}}{\longleftrightarrow} \sigma \, .$$

We then impose a global symmetry, under which the fields carry the quantum numbers as below,

$$\begin{aligned} 1 & \quad \text{for} \quad q_L\,, q_R^c\,, l_L\,, l_R^c\,, S_R\,; \\ 2 & \quad \text{for} \quad \phi_1\,, \phi_2\,, \sigma^*\,; \\ 0 & \quad \text{for} \quad \chi_L\,, \chi_R\,. \end{aligned}$$

Clearly, the above global symmetry is consistent with the discrete left-right symmetry.

The full scalar potential then should be

$$\begin{aligned} V &= \mu_\sigma^2|\sigma|^2 + \mu_\chi^2(|\chi_L|^2 + |\chi_R|^2) + \mu_{ij}^2\mathrm{Tr}(\phi_i^\dagger\phi_j) + \lambda_\sigma|\sigma|^4 \\ &\quad +\lambda_\chi(|\chi_L|^4 + |\chi_R|^4) + \lambda'_\chi|\chi_L|^2|\chi_R|^2 + \lambda_{ijkl}\mathrm{Tr}(\phi_i^\dagger\phi_j)\mathrm{Tr}(\phi_k^\dagger\phi_l) \\ &\quad +\lambda'_{ijkl}\mathrm{Tr}(\phi_i^\dagger\widetilde{\phi}_j)\mathrm{Tr}(\widetilde{\phi}_k^\dagger\phi_l) + \kappa_{\sigma\chi}|\sigma|^2(|\chi_L|^2 + |\chi_R|^2) + \rho_{ij}|\sigma|^2\mathrm{Tr}(\phi_i^\dagger\phi_j) \\ &\quad +\xi_{ij}(|\chi_L|^2 + |\chi_R|^2)\mathrm{Tr}(\phi_i^\dagger\phi_j) + \xi'_{ij}(\chi_L^\dagger\phi_i\phi_j^\dagger\chi_L + \chi_R^T\phi_i^T\phi_j^*\chi_R^*) \\ &\quad +[\alpha_{ij}\sigma^2\mathrm{Tr}(\widetilde{\phi}_i^\dagger\phi_j) + \beta_i\sigma\chi_L^\dagger\phi_i\chi_R + \gamma_i\sigma^*\chi_L^\dagger\widetilde{\phi}_i\chi_R + \mathrm{H.c.}] \,. \end{aligned}$$

The allowed Yukawa interactions should be

$$\mathcal{L}_Y = -y_q^i\bar{q}_L\phi_i q_R - y_l^i\bar{l}_L\phi_i l_R - h(\bar{l}_L\chi_L S_R + \bar{l}_R^c\chi_R^* S_R) - \frac{1}{2}g\sigma\overline{S}_R S_R^c + \mathrm{H.c.}\,.$$

Clearly, the lepton and quark doublets, the Higgs doublets, the Higgs bi-doublets, the fermion singlets and the scalar singlet can, respectively, belong to the $\mathbf{16}_{F_i}$, $\mathbf{16}_H$, $\mathbf{10}_{H_{1,2}}$, $\mathbf{1}_{F_i}$ and $\mathbf{1}_H$ representation in the $SO(10)$ GUTs.

- After the complex scalar singlet σ develops a vacuum expectation value to spontaneously break the global symmetry, it can be described by

$$\sigma = \frac{1}{\sqrt{2}}(f+\rho)\,\mathrm{exp}\left(i\frac{a}{f}\right).$$

In the presence of the α, β, γ-terms in the potential, like the structure of the DFSZ model, the Nambu-Goldstone boson a can couple to the quarks,

$$\begin{aligned}\mathcal{L} &\supset -\frac{1}{2f}(\partial_\mu a)(\bar{q}_L\gamma^\mu q_L - \bar{q}_R\gamma^\mu q_R)\\ &= \frac{1}{2f}(\partial_\mu a)(\bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d).\end{aligned}$$

Therefore, the global symmetry is the PQ symmetry and the Nambu-Goldstone boson is an axion.

- The charged fermions and the neutral neutrinos have the Dirac masses as below,

$$\tilde{m}_d = y_q^1 \langle \phi_{12}^0 \rangle + y_q^2 \langle \phi_{22}^0 \rangle \,, \quad \tilde{m}_u = y_q^1 \langle \phi_{11}^0 \rangle + y_q^2 \langle \phi_{21}^0 \rangle \,,$$
$$\tilde{m}_e = y_l^1 \langle \phi_{12}^0 \rangle + y_l^2 \langle \phi_{22}^0 \rangle \,, \quad \tilde{m}_\nu = y_l^1 \langle \phi_{11}^0 \rangle + y_l^2 \langle \phi_{21}^0 \rangle \,,$$

Clearly, we can get the desired mass spectrum of the charged fermions.

The fermion singlets also obtain their Majorana masses,

$$M_S = \frac{1}{\sqrt{2}} g f \,.$$

Therefore, the double and linear seesaw is ready.

Inflation and Flavor Mixing from PQ Symmetry Breaking

- We now consider a more attractive scenario where the PQ symmetry breaking can simultaneously lead to inflation (Guth, 81; Linde, 82; Albrecht, Steinhardt, 82') and flavor mixing. At the left-right level our model contains one Higgs bi-doublet for each family, two Higgs doublets, two leptoquark doublets and six complex singlets in the scalar sector while three neutral singlets and three generations of lepton and quark doublets in the fermion sector.

  The six scalar singlets are responsible for a $U(1)^6$ global symmetry breaking at the Planck scale. Because of the Yukawa interactions between the scalar and fermion singlets, the $U(1)^6$ symmetry is explicitly broken down to a $U(1)^3$ symmetry (Barbieri *et al.*, 05'; PHG, He, Sarkar, 07'). Three Nambu-Goldstone bosons will obtain heavy masses through the Coleman-Weinberg (Coleman, Weinberg, 73') potential while the other three will pick up tiny masses through the color anomaly (Adler, 69'; Bell, Jackiw, 69'; Bardeen, 69').

The heavy and light pseudo Nambu-Goldstone bosons can act as the inflaton and the axion, respectively. This inflationary scenario can also avoid the cosmological domain wall problem (Sikivie, 82').

In the absence of any off-diagonal Yukawa couplings involving the lepton and quark doublets, we can make use of the mixed fermion singlets to induce the lepton mixing by tree-level seesaw and the quark mixing by one-loop diagrams.

- Besides three generations of fermion doublets and two Higgs doublets, our model contains three Higgs bi-doublets,

$$\phi_i(\mathbf{1},\mathbf{2},\mathbf{2}^*,0) = \begin{bmatrix} \phi^0_{i1} & \phi^+_{i2} \\ \phi^-_{i1} & \phi^0_{i2} \end{bmatrix},$$

two leptoquark doublets,

$$\eta_L(\mathbf{3},\mathbf{2},\mathbf{1},\tfrac{1}{3}) = \begin{bmatrix} \eta_L^{+2/3} \\ \eta_L^{-1/3} \end{bmatrix}, \quad \eta_R(\mathbf{3},\mathbf{1},\mathbf{2},\tfrac{1}{3}) = \begin{bmatrix} \eta_R^{+2/3} \\ \eta_R^{-1/3} \end{bmatrix},$$

six complex singlets,

$$\sigma_{ij}(\mathbf{1},\mathbf{1},\mathbf{1},0) = \sigma_{ji}(\mathbf{1},\mathbf{1},\mathbf{1},0),$$

and three neutral fermion singlets,

$$S_{R_i}(\mathbf{1},\mathbf{1},\mathbf{1},0).$$

We assume a discrete left-right symmetry which is connected to charge-conjugation and the fields thus will transform as

$$\phi_i \leftrightarrow \phi_i^T\,, \quad \chi_L \leftrightarrow \chi_R^*\,, \quad \eta_L \leftrightarrow \eta_R^*\,, \quad \sigma_{ij} \leftrightarrow \sigma_{ij}\,,$$
$$q_{L_i} \leftrightarrow q_{R_i}^c\,, \quad l_{L_i} \leftrightarrow l_{R_i}^c\,, \quad S_{R_i} \leftrightarrow S_{R_i}\,.$$

In the presence of the above left-right symmetry, we can impose a family symmetry $U(1)_F = U(1)^3$, under which the left- and right-handed fermion doublets carry an equal but opposite charge for each family, i.e.

$$(-\delta_{i1}, -\delta_{i2}, -\delta_{i3}) \quad \text{for} \quad l_{L_i} \leftrightarrow l_{R_i}^c \ \text{and} \ q_{L_i} \leftrightarrow q_{R_i}^c\,.$$

We also assign the $U(1)_F$ charges for other fields,

$$\begin{array}{cl} (-\delta_{i1}, -\delta_{i2}, -\delta_{i3}) & \text{for} \quad S_{R_i}\,, \\ (\delta_{i1}+\delta_{j1}, \delta_{i2}+\delta_{j2}, \delta_{i3}+\delta_{j3}) & \text{for} \quad \sigma_{ij}\,, \\ (-2\delta_{i1}, -2\delta_{i2}, -2\delta_{i3}) & \text{for} \quad \phi_i\,, \\ (0,0,0) & \text{for} \quad \chi_{L,R}\,,\ \eta_{L,R}\,. \end{array}$$

We further introduce a global symmetry $U(1)_G$, under which the fields carry the following quantum numbers,

$$\begin{array}{lll} 2 & \text{for} & \sigma_{ij}\,, \\ 1 & \text{for} & \chi_L \leftrightarrow \chi_R^*\,, \quad \eta_L \leftrightarrow \eta_R^*\,, \quad S_{R_i}^c\,, \\ 0 & \text{for} & l_{L_i} \leftrightarrow l_{R_i}^c\,, \quad q_{L_i} \leftrightarrow q_{R_i}^c\,, \quad \phi_i\,. \end{array}$$

The allowed Yukawa interactions should be,

$$\begin{aligned} \mathcal{L}_Y = & -y_{q_i}\bar{q}_{L_i}\phi_i q_{R_i} - y_{l_i}\bar{l}_{L_i}\phi_i l_{R_i} - f_i(\bar{l}_{L_i}\chi_L S_{R_i} + \bar{l}_{R_i}^c\chi_R^* S_{R_i}) \\ & -h_i(\bar{q}_{L_i}\eta_L S_{R_i} + \bar{q}_{R_i}^c\eta_R^* S_{R_i}) - \frac{1}{2}g_{ij}\sigma_{ij}\overline{S}_{R_i}^c S_{R_j} + \text{H.c.}\,. \end{aligned}$$

For simplicity, we do not write down the full scalar potential where

$$\alpha_{ij}\sigma_{ii}\sigma_{jj}\mathrm{Tr}\tilde{\phi}_i^\dagger\phi_j + \beta_{ij}\sigma_{ij}^2\mathrm{Tr}\tilde{\phi}_i^\dagger\phi_j + \mathrm{H.c.}$$

should be absent due to the global symmetry $U(1)_G$. Instead, we only give the part relevant for generating the mixing between the left- and right-handed leptoquarks,

$$V \supset \lambda_i\sigma_{ii}\eta_L^\dagger\phi_i\eta_R + \kappa\eta_L^\dagger\chi_L\chi_R^\dagger\eta_R + \mathrm{H.c.}\,.$$

Clearly, the lepton and quark doublets, the Higgs and leptoquark doublets, the Higgs bi-doublets, the fermion singlets and the scalar singlets can, respectively, belong to the $\mathbf{16}_{F_i}$, $\mathbf{16}_H$, $\mathbf{10}_{H_i}$, $\mathbf{1}_{F_i}$ and $\mathbf{1}_{H_{ij}}$ representation in $SO(10)$ GUTs.

- Each scalar singlet σ_{ij} has an independent phase transformation to perform a $U(1)^6$ symmetry. However, the six scalar singlets have the Yukawa interactions with the three fermion singlets so that the $U(1)^6$ symmetry should be explicitly broken down to a $U(1)^3$ symmetry. After the six scalar singlets develop their vacuum expectation values, there will be six Nambu-Goldstone bosons, i.e.

$$\sigma_{ij} = \frac{1}{\sqrt{2}}(f_{ij} + \xi_{ij})e^{i\frac{\varphi_{ij}}{f_{ij}}}\,.$$

The fermion singlets then obtain their Majorana masses

$$\tilde{M}_{ij} = \frac{1}{\sqrt{2}} g_{ij} f_{ij} e^{i\frac{\varphi_{ij}}{f_{ij}}} = M_{ij} e^{i\frac{\varphi_{ij}}{f_{ij}}}\,,$$

which will result in a Coleman-Weinberg potential,

$$V = \frac{1}{32\pi^2}\mathrm{Tr}\left[(\tilde{M}\tilde{M}^\dagger\tilde{M}\tilde{M}^\dagger)\ln\left(\frac{\Lambda^2}{\tilde{M}\tilde{M}^\dagger}\right)\right]$$

with Λ being the ultraviolet cutoff.

Only three Nambu-Goldstone bosons can exist in the Coleman-Weinberg potential while the other three can be absorbed by the three fermion singlets. For example, we can take

$$S_{R_i} e^{i\frac{\varphi_{ii}}{2f_{ii}}} \Rightarrow S_{R_i} \text{ and then } \tilde{M}_{ij} = M_{ij} e^{i\frac{\varphi'_{ij}}{f'_{ij}}}$$

$$\text{with } \frac{\varphi'_{ij}}{f'_{ij}} = \frac{\varphi_{ij}}{f_{ij}} - \frac{\varphi_{ii}}{2f_{ii}} - \frac{\varphi_{jj}}{2f_{jj}}\,.$$

Clearly, we have $\varphi'_{ij} = 0$ for $i = j$. By taking a reasonable simplification on the logarithm

$$\ln\left(\frac{\Lambda^2}{\tilde{M}\tilde{M}^\dagger}\right) \simeq \text{constant} = \mathcal{O}(1)\,,$$

the Coleman-Weinberg potential can be expanded by

$$
\begin{aligned}
V \simeq \; & \frac{1}{32\pi^2}\Big\{(M_{11}^2 + M_{12}^2 + M_{13}^2)^2 + (M_{12}^2 + M_{22}^2 + M_{23}^2)^2 \\
& +(M_{13}^2 + M_{23}^2 + M_{33}^2)^2 + 2(M_{11}^2 M_{12}^2 + M_{12}^2 M_{22}^2 + M_{13}^2 M_{23}^2) \\
& +2(M_{11}^2 M_{13}^2 + M_{12}^2 M_{23}^2 + M_{13}^2 M_{33}^2) \\
& +2(M_{12}^2 M_{13}^2 + M_{22}^2 M_{23}^2 + M_{23}^2 M_{33}^2) \\
& +4M_{11}M_{22}M_{12}^2 \cos\left(\frac{2\varphi'_{12}}{f'_{12}}\right) + 4M_{11}M_{33}M_{13}^2 \cos\left(\frac{2\varphi'_{13}}{f'_{13}}\right) \\
& +4M_{22}M_{33}^2M_{23}^2 \cos\left(\frac{2\varphi'_{23}}{f'_{23}}\right) + 8M_{12}M_{13}M_{23} \\
& \times\left[M_{11}\cos\left(\frac{\varphi'_{12}}{f'_{12}} + \frac{\varphi'_{13}}{f'_{13}} - \frac{\varphi'_{23}}{f'_{23}}\right) + M_{22}\cos\left(\frac{\varphi'_{12}}{f'_{12}} - \frac{\varphi'_{13}}{f'_{13}} + \frac{\varphi'_{23}}{f'_{23}}\right)\right. \\
& \left.\left. +M_{33}\cos\left(\frac{\varphi'_{12}}{f'_{12}} - \frac{\varphi'_{13}}{f'_{13}} - \frac{\varphi'_{23}}{f'_{23}}\right)\right]\right\} .
\end{aligned}
$$

A combination of φ'_{12}, φ'_{13} and φ'_{23} can have a potential of the form as below,

$$V = \mu^4(1 \pm \cos\frac{\varphi}{f}) \text{ with } \mu = \mathcal{O}(M_{ij}) \, , \; f = \mathcal{O}(f_{ij}) \, .$$

The pseudo Nambu-Goldstone boson φ can realize the natural inflation for $\mu = \mathcal{O}(10^{15}\,\mathrm{GeV})$ and $f = \mathcal{O}(M_{\mathrm{Pl}})$. Note that the Majorana masses M_{ij} should be determined by the Yukawa couplings $g_{ij} = \mathcal{O}(10^{-4})$ for the given symmetry breaking scales $f_{ij} = \mathcal{O}(M_{\mathrm{Pl}})$.

Benefited from the inflation, our model can escape from the cosmological domain wall problem.

On the other hand, the three Nambu-Goldstone bosons absorbed in the three fermion singlets can have derivative couplings with the quarks,

$$\begin{aligned}\mathcal{L} &\supset -\frac{1}{2f_{ii}}(\partial_\mu\varphi_{ii})(\bar{q}_{L_i}\gamma^\mu q_{L_i} - \bar{q}_{R_i}\gamma^\mu q_{R_i}) \\ &= \frac{1}{2f_{ii}}(\partial_\mu\varphi_{ii})(\bar{u}_i\gamma^\mu\gamma_5 u_i + \bar{d}_i\gamma^\mu\gamma_5 d_i)\,.\end{aligned}$$

Therefore, the Nambu-Goldstone bosons φ_{ii} can obtain their tiny masses through the color anomaly. Clearly, the pseudo Nambu-Goldstone bosons φ_{ii} play the role of the invisible axion while the family symmetry $U(1)_F$ is identified with the PQ symmetry.

Note for the PQ symmetry breaking at the Planck scale, we can (Pi, 84'; Linde, 88') choose the initial value of the axion by the anthropic argument (Weinberg, 87') to give a desired dark matter relic density (Visinelli, Gondolo, 10').

- The charged leptons have a diagonal 3×3 mass matrix, i.e.

$$\mathcal{L} \supset -\tilde{m}_e \bar{e}_L e_R + \text{H.c.} \;\; \text{with} \;\; (\tilde{m}_e)_i = y_{l_i} \langle \phi^0_{i2} \rangle \,.$$

The mass terms involving the neutral leptons are given by

$$\mathcal{L} \;\supset\; -\tilde{m}_\nu \bar{\nu}_L \nu_R - f \langle \chi^0_L \rangle \bar{\nu}_L S_R - f \langle \chi^0_R \rangle \bar{\nu}^c_R S_R - \frac{1}{2} \tilde{M} \overline{S^c_R} S_R + \text{H.c.}$$

$$\text{with} \quad (\tilde{m}_\nu)_i = y_{l_i} \langle \phi^0_{i1} \rangle \,.$$

For $f \langle \chi^0_R \rangle$ and/or $\tilde{M}$ much bigger than $\tilde{m}_\nu$ and $f \langle \chi^0_L \rangle$, we can obtain the double/inverse and linear seesaw, i.e.

$$\mathcal{L} \supset -\frac{1}{2} m_\nu \bar{\nu}_L \nu^c_L + \text{H.c.} \;\; \text{with}$$

$$m_\nu = \tilde{m}_\nu \frac{1}{f^T \langle \chi^0_R \rangle} \tilde{M} \frac{1}{f \langle \chi^0_R \rangle} \tilde{m}^T_\nu - (\tilde{m}_\nu + \tilde{m}^T_\nu) \frac{\langle \chi^0_L \rangle}{\langle \chi^0_R \rangle} \,.$$

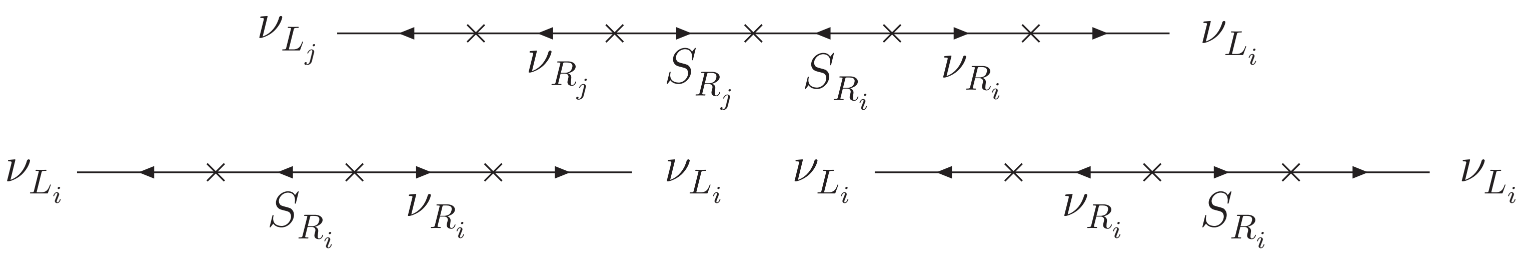

FIG. 2: Tree-level seesaw for lepton masses and mixing.

Since the charged lepton mass matrix has a diagonal structure, the neutrino mass matrix should account for the lepton mixing described by the PMNS matrix. With the six elements in the Majorana mass matrix $\tilde{M}$, we have enough flexibilities to fit the known masses and mixing (Smirnov, 93').

Note that with a left-right symmetry breaking scale $\langle\chi_R^0\rangle = \mathcal{O}(10^{13}\,\mathrm{GeV})$, the double/inverse seesaw term should be the double seesaw for $\tilde{M} = \mathcal{O}(10^{15}\,\mathrm{GeV})$. In this scenario, the inflaton should decay into the right-handed neutrinos through the off-shell fermion singlets. Subsequently, the decays of the right-handed neutrinos can realize the non-thermal or thermal leptogenesis.

- At tree level the mass matrices of the down- and up-type quarks are both diagonal,

$$\mathcal{L} \supset -\tilde{m}_d^0 \bar{d}_L d_R - \tilde{m}_u^0 \bar{u}_L u_R + \text{H.c.} \quad \text{with}$$
$$(\tilde{m}_d^0)_i = y_{q_i} \langle \phi_{i2}^0 \rangle \quad \text{and} \quad (\tilde{m}_u^0)_i = y_{q_i} \langle \phi_{i1}^0 \rangle \,.$$

At one-loop order the leptoquarks can mediate the mixing of the fermion singlets to the quark sector.

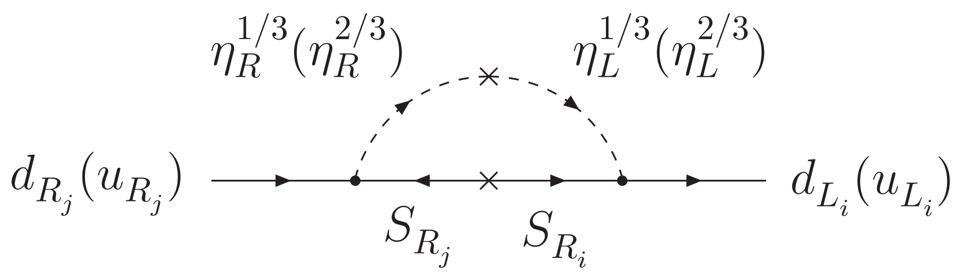

FIG. 3: One-loop diagrams for quark masses and mixing.

To calculate the loop-induced quark masses, we define the mass eigenstates of the leptoquarks,

$$\begin{cases} \eta_1^{2/3} = \eta_L^{2/3} \cos\theta_{2/3} - \eta_R^{2/3} \sin\theta_{2/3}\,, \\ \eta_2^{2/3} = \eta_L^{2/3} \sin\theta_{2/3} + \eta_R^{2/3} \cos\theta_{2/3}\,, \end{cases}$$

$$\begin{cases} \eta_1^{1/3} = \eta_L^{1/3} \cos\theta_{1/3} - \eta_R^{1/3} \sin\theta_{1/3}\,, \\ \eta_2^{1/3} = \eta_L^{1/3} \sin\theta_{1/3} + \eta_R^{1/3} \cos\theta_{1/3}\,, \end{cases}$$

with the rotation angles,

$$\theta_{2/3} \in [0,\pi] \text{ for } \delta^2_{2/3} = 0 \text{ or } \theta_{2/3} = \frac{\pi}{4} \text{ for } \delta^2_{2/3} \neq 0\,,$$
$$\theta_{1/3} \in [0,\pi] \text{ for } \delta^2_{1/3} = 0 \text{ or } \theta_{1/3} = \frac{\pi}{4} \text{ for } \delta^2_{1/3} \neq 0\,,$$

and the masses,

$$\begin{aligned} M^2_{\eta_2^{2/3}} - M^2_{\eta_1^{2/3}} &= 2\delta^2_{2/3} \sin 2\theta_{2/3}\,, \\ M^2_{\eta_2^{1/3}} - M^2_{\eta_1^{1/3}} &= 2\delta^2_{1/3} \sin 2\theta_{1/3}\,. \end{aligned}$$

Here we have defined

$$\delta^2_{2/3} = \frac{1}{\sqrt{2}}\lambda_i f_{ii}\langle\phi^0_{i1}\rangle + \kappa\langle\chi^0_L\rangle\langle\chi^0_R\rangle\,,$$
$$\delta^2_{1/3} = \frac{1}{\sqrt{2}}\lambda_i f_{ii}\langle\phi^0_{i2}\rangle\,.$$

We further rotate the fermion singlets to diagonalize their Majorana mass matrix,

$$U^*\tilde{M}U^\dagger = \text{diag}\{M_{S_1}, M_{S_2}, M_{S_3}\}\,.$$

The loop-induced quark masses can be calculated by

$$
\begin{aligned}
(\tilde{m}_d^1)_{ij} &= \frac{\sin 2\theta_{1/3}}{32\pi^2} h_i h_j U_{ik}^{\dagger} U_{kj}^T M_{S_k} \left(\frac{M^2_{\eta_2^{1/3}}}{M^2_{\eta_2^{1/3}} - M^2_{S_k}} \ln \frac{M^2_{\eta_2^{1/3}}}{M^2_{S_k}} \right. \\
&\quad \left. - \frac{M^2_{\eta_1^{1/3}}}{M^2_{\eta_1^{1/3}} - M^2_{S_k}} \ln \frac{M^2_{\eta_1^{1/3}}}{M^2_{S_k}} \right) \simeq \frac{\sin^2 2\theta_{1/3}}{16\pi^2} \frac{h_i h_j \tilde{M}^*_{ij} \delta^2_{1/3}}{M^2_{\eta_1^{1/3}}} , \\
(\tilde{m}_u^1)_{ij} &= \frac{\sin 2\theta_{2/3}}{32\pi^2} h_i h_j U_{ik}^{\dagger} U_{kj}^T M_{S_k} \left(\frac{M^2_{\eta_2^{2/3}}}{M^2_{\eta_2^{2/3}} - M^2_{S_k}} \ln \frac{M^2_{\eta_2^{2/3}}}{M^2_{S_k}} \right. \\
&\quad \left. - \frac{M^2_{\eta_1^{2/3}}}{M^2_{\eta_1^{2/3}} - M^2_{S_k}} \ln \frac{M^2_{\eta_1^{2/3}}}{M^2_{S_k}} \right) \simeq \frac{\sin^2 2\theta_{2/3}}{16\pi^2} \frac{h_i h_j \tilde{M}^*_{ij} \delta^2_{2/3}}{M^2_{\eta_1^{2/3}}} .
\end{aligned}
$$

Therefore we have

$$(\tilde{m}_d^1)_{ij} = \left(\frac{\sin 2\theta_{1/3}}{1}\right)^2 \left(\frac{h_i h_j}{1}\right) \left(\frac{\tilde{M}^*_{ij}}{10^{15}\,\mathrm{GeV}}\right) \left(\frac{\delta^2_{1/3}}{10^{19}\mathrm{GeV}\cdot 100\,\mathrm{GeV}}\right) \times \left(\frac{10^{16}\,\mathrm{GeV}}{M_{\eta_1^{1/3}}}\right)^2 63.3\,\mathrm{GeV}\,,$$

$$(\tilde{m}_u^1)_{ij} = \left(\frac{\sin 2\theta_{2/3}}{1}\right)^2 \left(\frac{h_i h_j}{1}\right) \left(\frac{\tilde{M}^*_{ij}}{10^{15}\,\mathrm{GeV}}\right) \left(\frac{\delta^2_{2/3}}{10^{19}\,\mathrm{GeV}\cdot 100\,\mathrm{GeV}}\right) \times \left(\frac{10^{16}\,\mathrm{GeV}}{M_{\eta_1^{2/3}}}\right)^2 63.3\,\mathrm{GeV}\,.$$

We now check that the tree-level and loop-order quark mass matrices definitely can induce the desired quark masses and mixing for a proper parameter choice.

Since the discrete left-right symmetry is charge-conjugation, the quark mass matrices can be diagonalized by

$$\tilde{m}_d = \tilde{m}_d^0 + \tilde{m}_d^1 = V_d^\dagger \mathrm{diag}\{m_d, m_s, m_b\} V_d \,,$$

$$\tilde{m}_u = \tilde{m}_u^0 + \tilde{m}_u^1 = V_u^\dagger \mathrm{diag}\{m_u, m_c, m_t\} V_u \,.$$

The CKM matrix can be defined by

$$V_{\mathrm{CKM}} = V_u V_d^\dagger = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} .$$

For demonstration, let's consider a limiting case where

$$V_d = V_{\mathrm{CKM}}^{\dagger}\,, \quad V_u = 1\,.$$

Now the off-diagonal quark masses can be given by

$$(\tilde{m}_d^1)_{12} = \; m_d V_{ud} V_{cd}^* + m_s V_{us} V_{cs}^* + m_b V_{ub} V_{cb}^* \; = (\tilde{m}_d^1)_{21}^*\,,$$

$$(\tilde{m}_d^1)_{13} = \; m_d V_{ud} V_{td}^* + m_s V_{us} V_{ts}^* + m_b V_{ub} V_{tb}^* \; = (\tilde{m}_d^1)_{31}^*\,,$$

$$(\tilde{m}_d^1)_{23} = \; m_d V_{cd} V_{td}^* + m_s V_{cs} V_{ts}^* + m_b V_{cb} V_{tb}^* \; = (\tilde{m}_d^1)_{32}^*\,.$$

We then read

$$|(\tilde{m}_d^1)_{12}| = |(\tilde{m}_d^1)_{21}| = 9.508 - 17.06\,\mathrm{MeV}\,,$$
$$|(\tilde{m}_d^1)_{13}| = |(\tilde{m}_d^1)_{31}| = 9.966 - 11.87\,\mathrm{MeV}\,,$$
$$|(\tilde{m}_d^1)_{23}| = |(\tilde{m}_d^1)_{32}| = 114.8 - 129.4\,\mathrm{MeV}\,,$$

by taking the CKM matrix (PDG 08') and the down-type quark masses (Fusaoka, Koide, 98'; Xing, Zhang, Zhou, 08') at $\mu = m_Z$,

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix},$$

$$\begin{cases} m_d = 2.90^{+1.24}_{-1.19}\,\mathrm{MeV}\,, \\ m_s = 55^{+16}_{-15}\,\mathrm{MeV}\,, \\ m_b = 2.89 \pm 0.09\,\mathrm{GeV}\,. \end{cases}$$

Summary

- The PQ symmetry can be embedded into the left-right symmetric models and then into the $SO(10)$ GUTs.

- The PQ symmetry breaking can naturally lead to the double and linear seesaw for generating the small neutrino masses.

- The PQ symmetry breaking can simultaneously realize the natural inflation and the lepton and quark mixing.

Thanks!