Quasiparticle states in heavy nuclei

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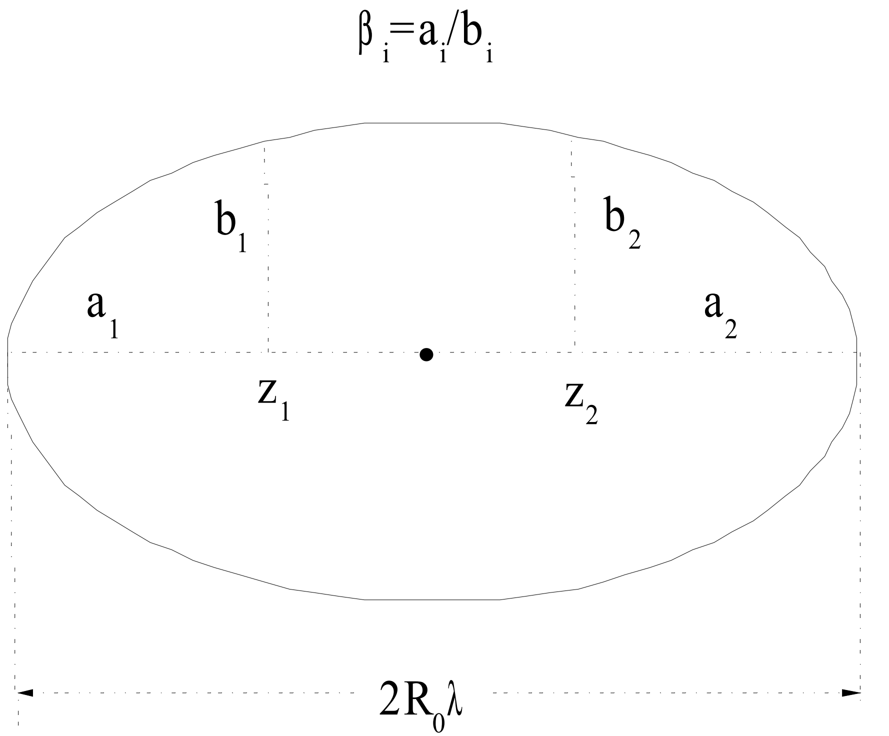
-Study of structure of heaviest nuclei to understand the mechanism of their formation in fusion reactions

-Identification of heavy nuclei by α-decay needs the analysis of isomer states

-Population of isomer states in reactions

-Search of shells and subshells closure



$\beta_1 = \beta_2$, even multipolarities; $\beta_1 \neq \beta_2$, odd and even multipolarities

R_0 is the radius of spherical nucleus

Parametrisation of nuclear shape with TCSM

$$H = T + V(\rho, z) + V_{LS} + V_{L^2}$$

$$V(\rho, z) = \begin{cases} \frac{1}{2} m\omega_z^2 (z - z_1)^2 + \frac{1}{2} m\omega_\rho^2 \rho^2, z < z_1 \\ \frac{1}{2} m\omega_\rho^2 \rho^2, z_1 < z < z_2 \\ \frac{1}{2} m\omega_z^2 (z - z_2)^2 + \frac{1}{2} m\omega_\rho^2 \rho^2, z > z_2 \end{cases}$$

$$V_{LS} = -\frac{2\hbar\kappa_i}{m\omega_{0\mathrm{i}}} (\nabla V \times \vec{p}) \vec{s}$$

$$V_{L^2} = -\hbar\omega_{0\mathrm{i}} \kappa_i \mu_i \hat{l}^2 + \hbar\kappa_i \mu_i \omega_{0\mathrm{i}} N_1 (N_1 + 3)/2\, \delta_{if}$$

$$\omega_{0\mathrm{i}} = 41\, MeV / A_i^{1/3}, A_i = a_i b_i^2 / 1.22^3, \omega_\rho / \omega_z = a_i / b_i, z_2 - z_1 = 2\mathrm{R}_0 \lambda - a_1 - a_2$$

Comparison with other calculations

^{248}Fm gs.:

λ=1.18, β=1.28 -> β_2=0.25, β_4=0.027

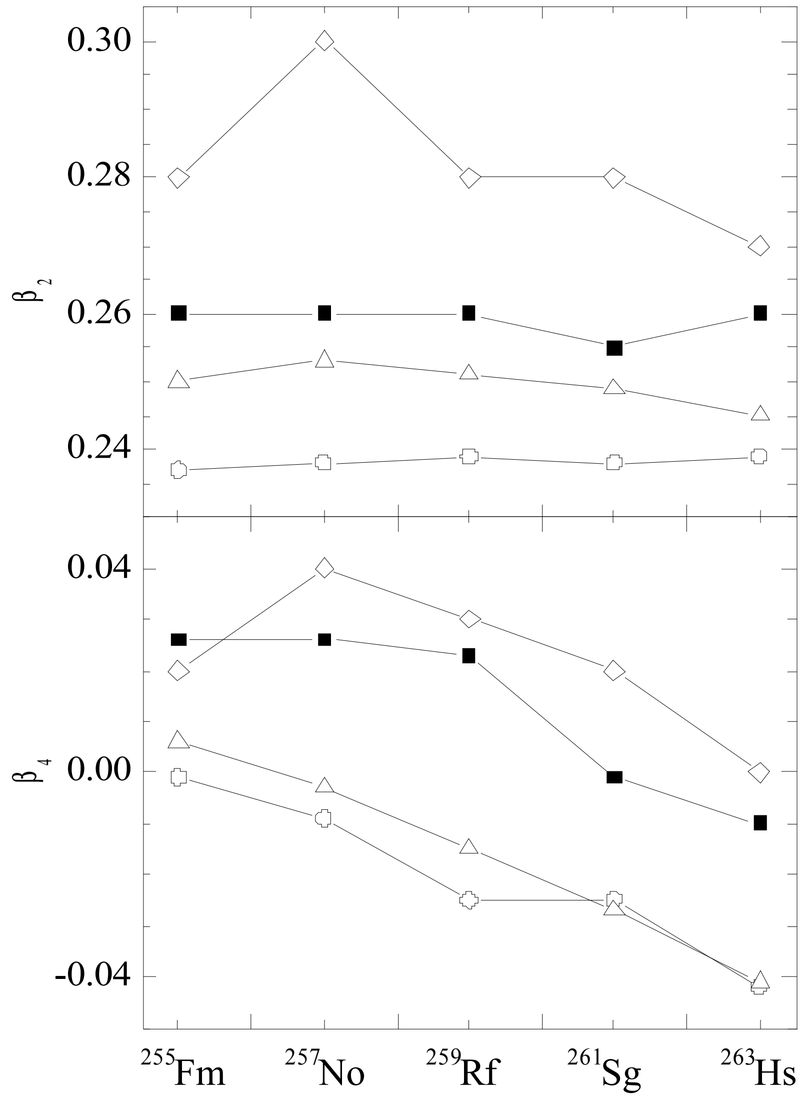
P.Möller et al. β_2=0.235, β_4=0.049

For ^{247,248,249}Fm, the microscopic corrections are

-3.85, -3.88, and -4.3 MeV.

P.Möller et al.: -3.52, -3.57, and -3.97 MeV

The values of Δ differ within 0.1 MeV.



▪ TCSM

△ A. Sobiczewski et al.

◦ P. Möller et al.

◇ S. Goriely et al.

N=155

Potential energy

$$U(Z, A, \lambda, \beta) = U_{LDM}(Z, A, \lambda, \beta) + \delta U_{mic}(Z, A, \lambda, \beta)$$

Binding energy

$$B(Z, A) = U(Z, A, \lambda_{gs}, \beta_{gs}) - a_v\left(1 - 1.78\left(\frac{N-Z}{A}\right)^2\right) A + \ldots$$

$$a_v = 15.83\ MeV$$

Q_α energy

$$Q_\alpha(Z, A) = B(Z, A) + 28.296 - B(Z-2, A-4)$$

Alpha decay half-lives T_α (A. Sobiczewski et al.)

$$\log_{10} T_\alpha(Z, A) = 1.5372\, Z\, Q_\alpha^{-1/2} - 0.1607\, Z - 36.573$$

Strength parameters of paring interaction

$$G_{\substack{n\\p}} = \left(19.2 \mp 7.4 \frac{N-Z}{A}\right) A^{-1}\, MeV$$

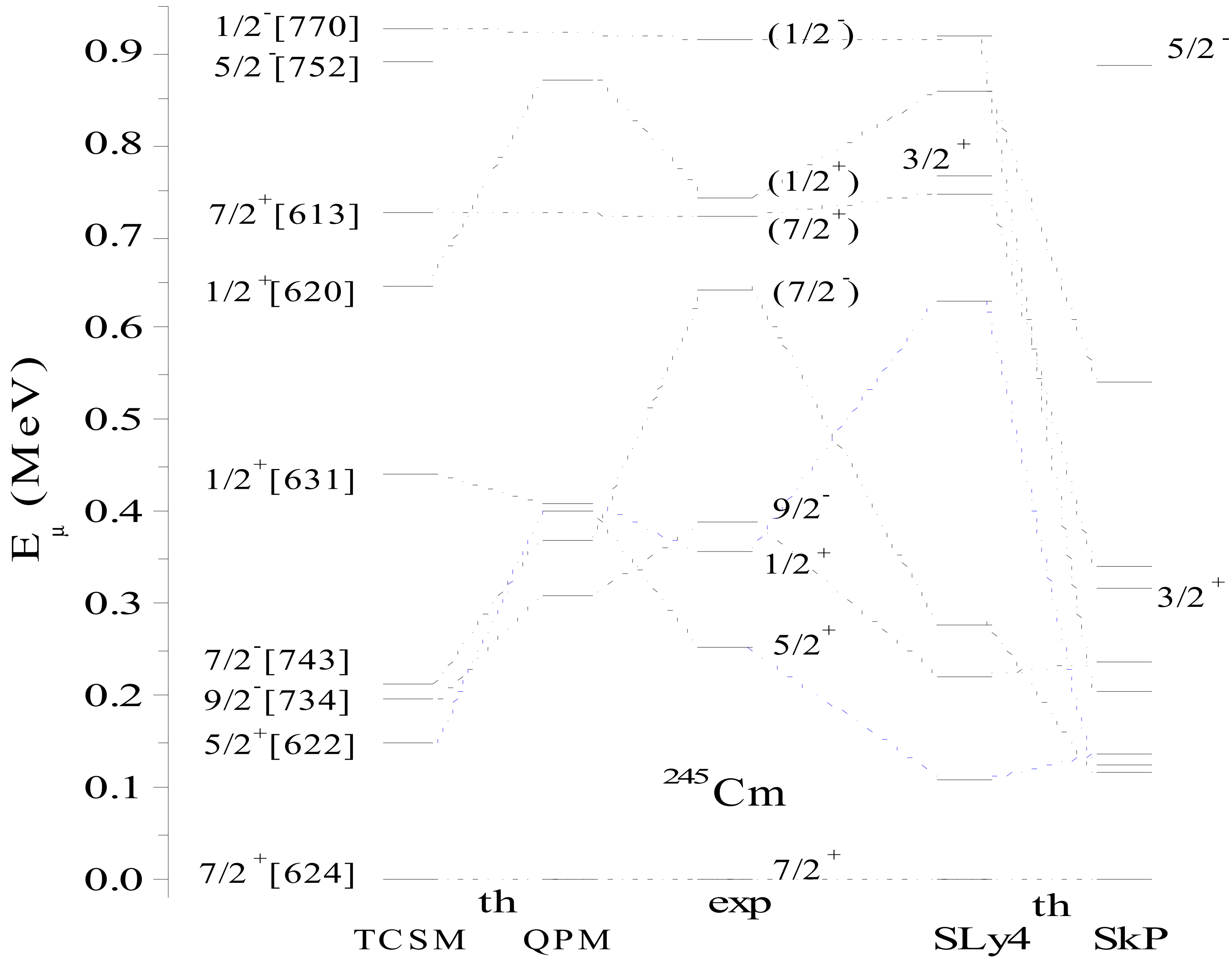
$$A \approx 250 \rightarrow G_n \approx 0.075\, MeV\ ,\ G_p \approx 0.085\, MeV$$

One-quasiparticle excitations

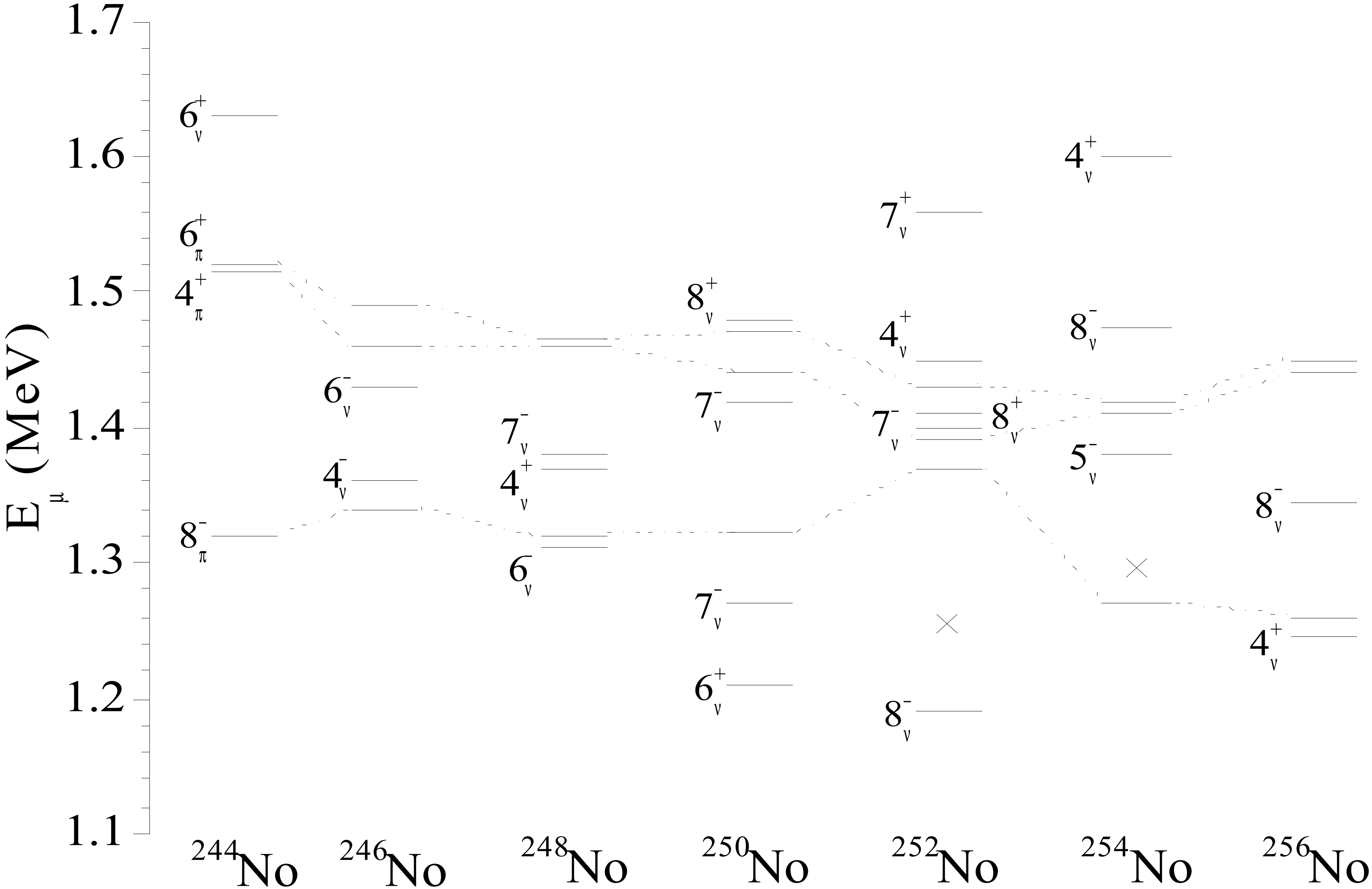
$$E_\mu = \sqrt{(e_\mu - e_F)^2 + \Delta^2} - \sqrt{(e_\mu{}' - e_F)^2 + \Delta^2}$$

Two-quasiparticle excitations

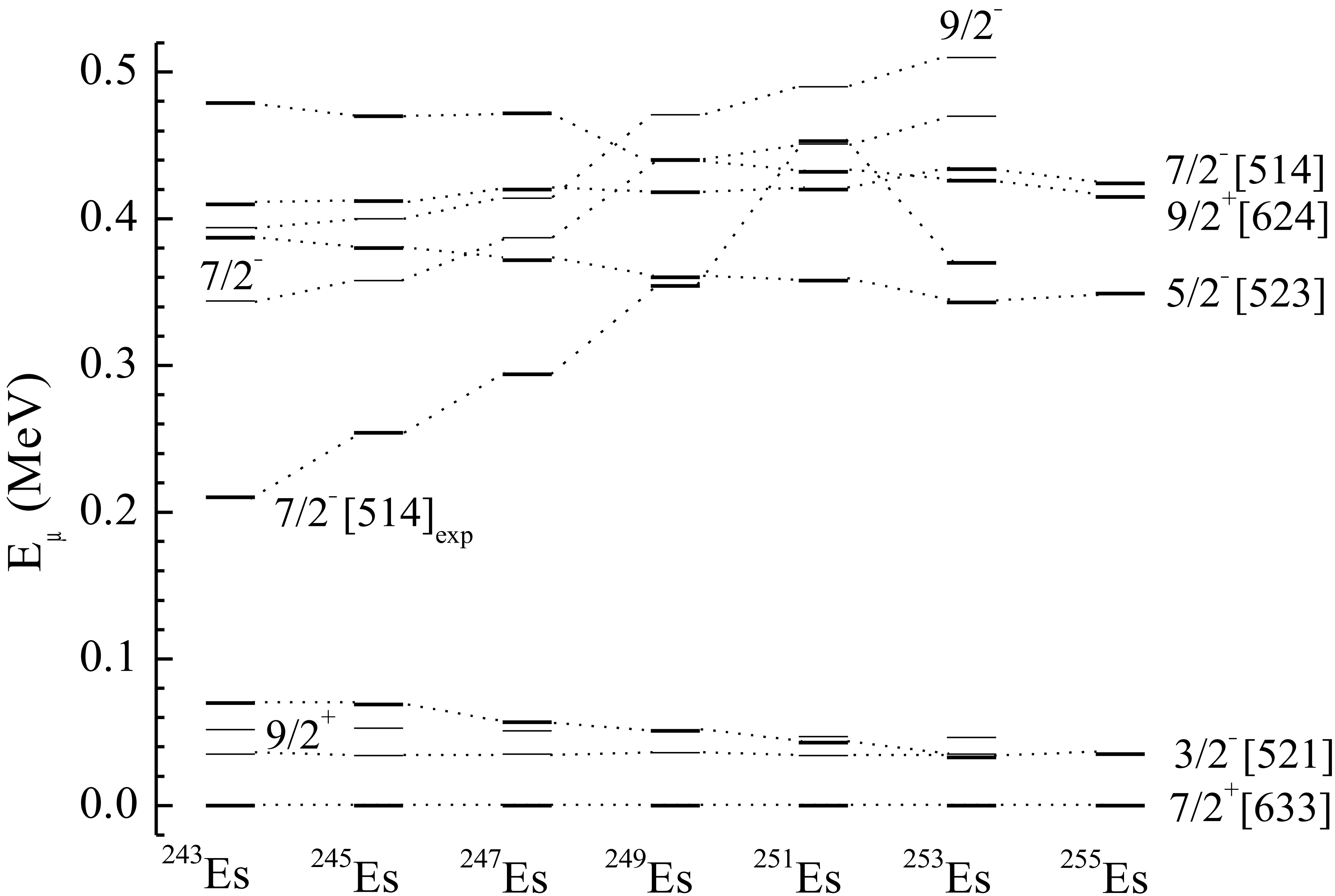
$$E_\mu = \sqrt{(e_\mu - e_F)^2 + \Delta^2} + \sqrt{(e_\mu{}' - e_F)^2 + \Delta^2}$$



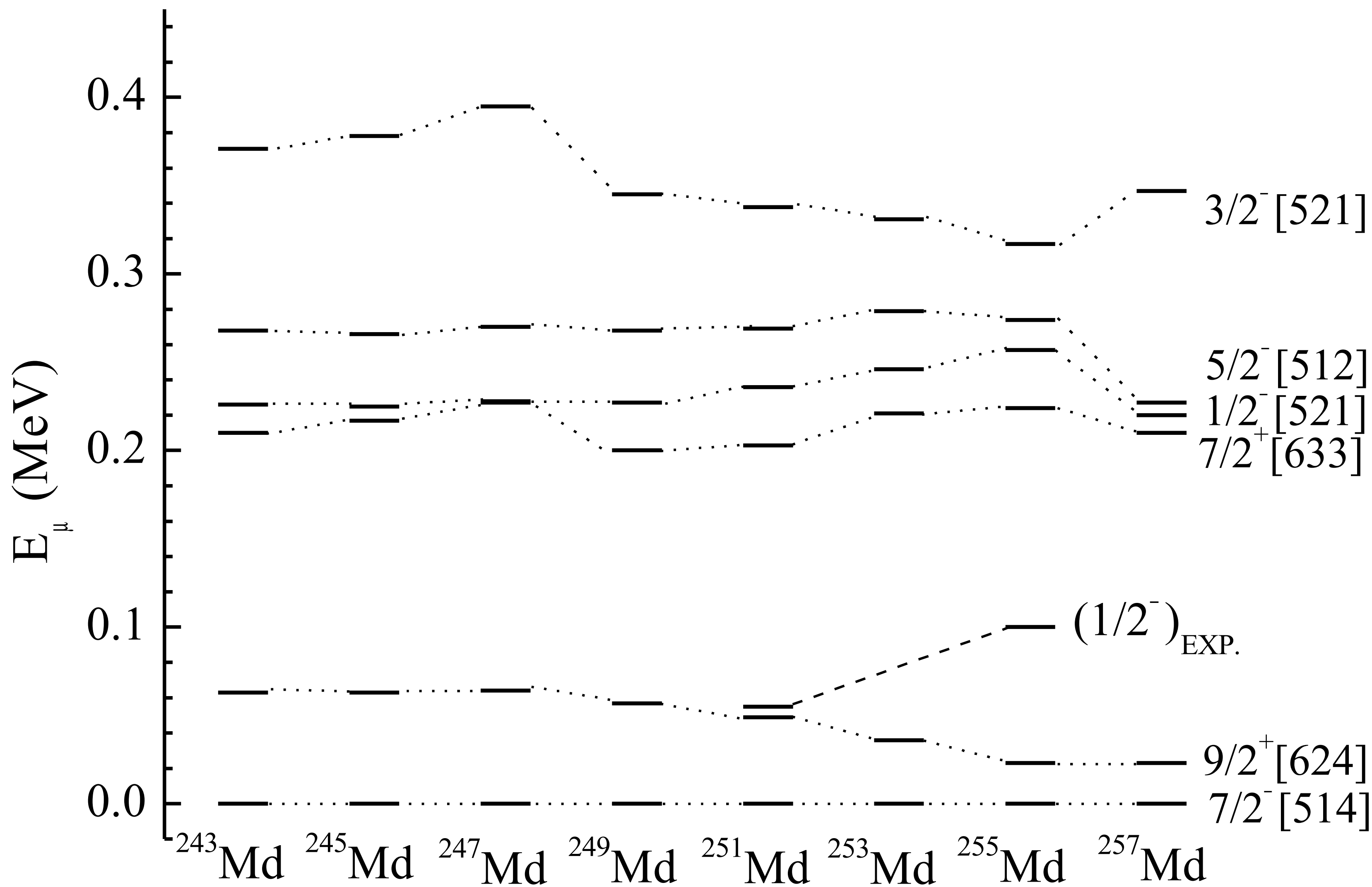
E_μ (MeV)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1/2⁻[770]
5/2⁻[752]
7/2⁺[613]
1/2⁺[620]
1/2⁺[631]
7/2⁻[743]
9/2⁻[734]
5/2⁺[622]
7/2⁺[624]
(1/2⁻)
(1/2⁺)
(7/2⁺)
(7/2⁻)
9/2⁻
1/2⁺
5/2⁺
7/2⁺
3/2⁺
5/2⁻
3/2⁺
245Cm
th
TCSM
QPM
exp
th
SLy4
SkP



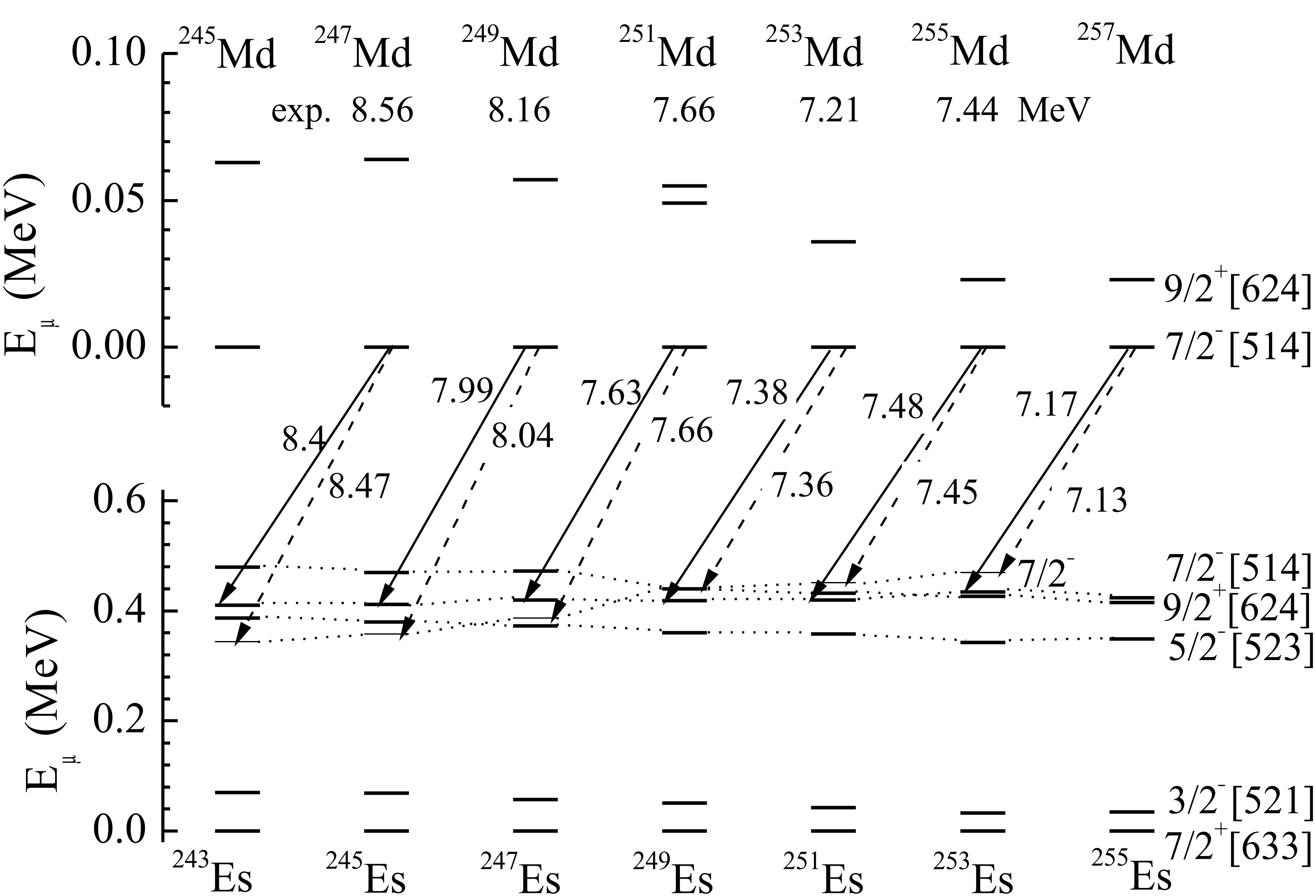
E_μ (MeV)
1.7
1.6
1.5
1.4
1.3
1.2
1.1
244No
246No
248No
250No
252No
254No
256No



0.5
0.4
0.3
0.2
0.1
0.0
E_μ (MeV)
9/2⁻
7/2⁻[514]
9/2⁺[624]
7/2⁻
5/2⁻[523]
7/2⁻[514]exp
9/2⁺
3/2⁻[521]
7/2⁺[633]
243Es
245Es
247Es
249Es
251Es
253Es
255Es

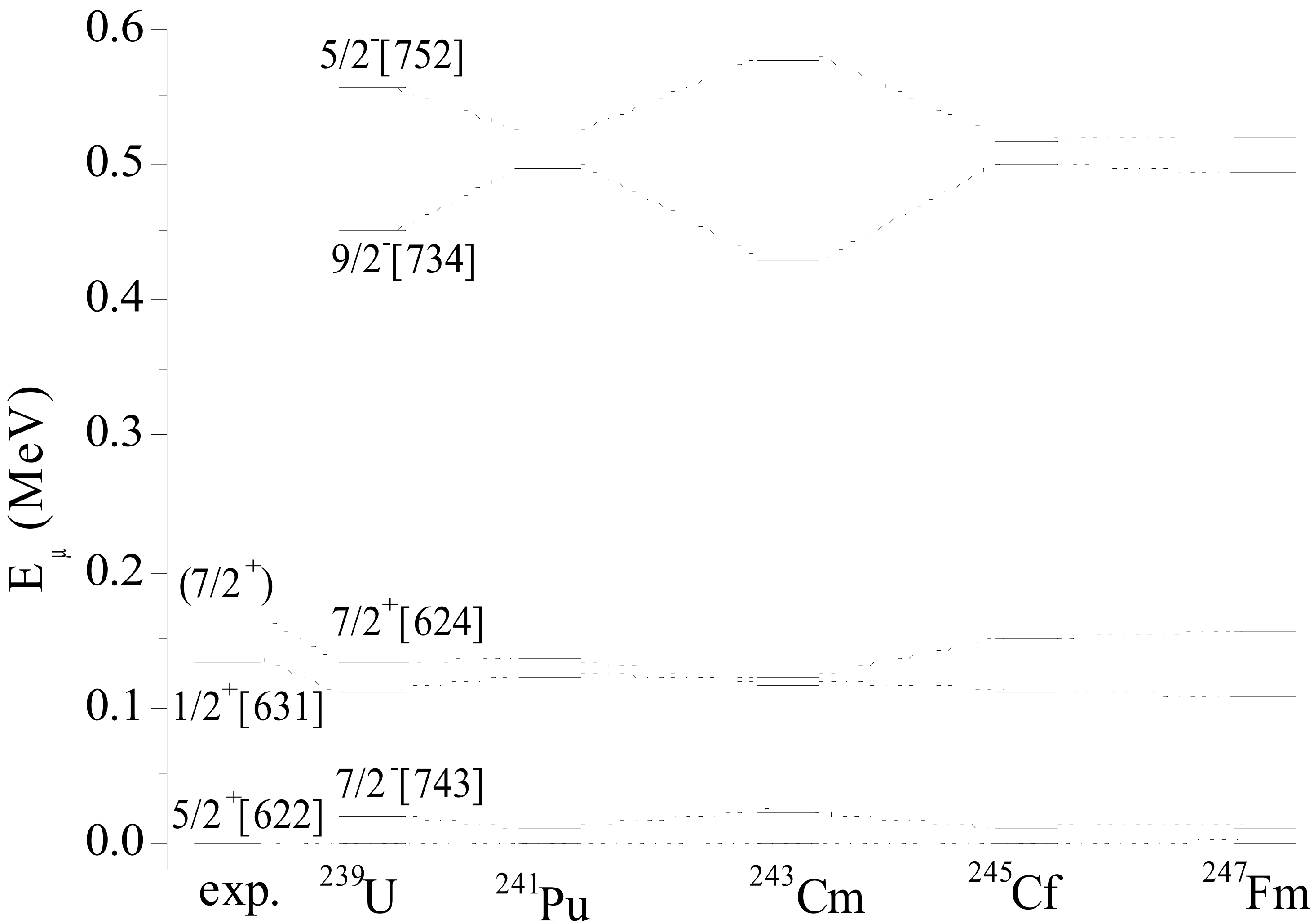


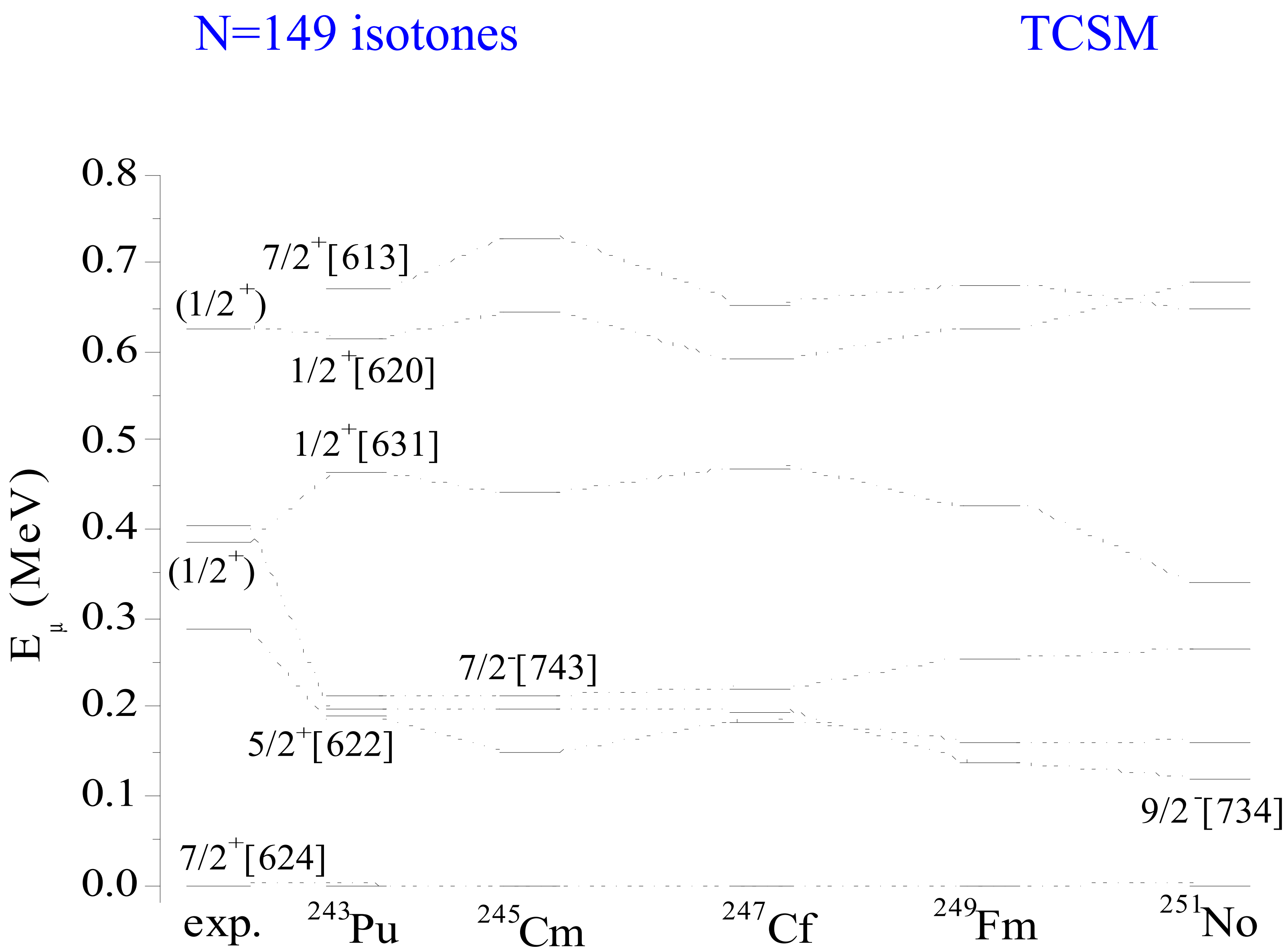
E$_\mu$ (MeV)
0.0
0.1
0.2
0.3
0.4
^{243}Md
^{245}Md
^{247}Md
^{249}Md
^{251}Md
^{253}Md
^{255}Md
^{257}Md
3/2$^-$[521]
5/2$^-$[512]
1/2$^-$[521]
7/2$^+$[633]
(1/2$^-$)$_{EXP.}$
9/2$^+$[624]
7/2$^-$[514]

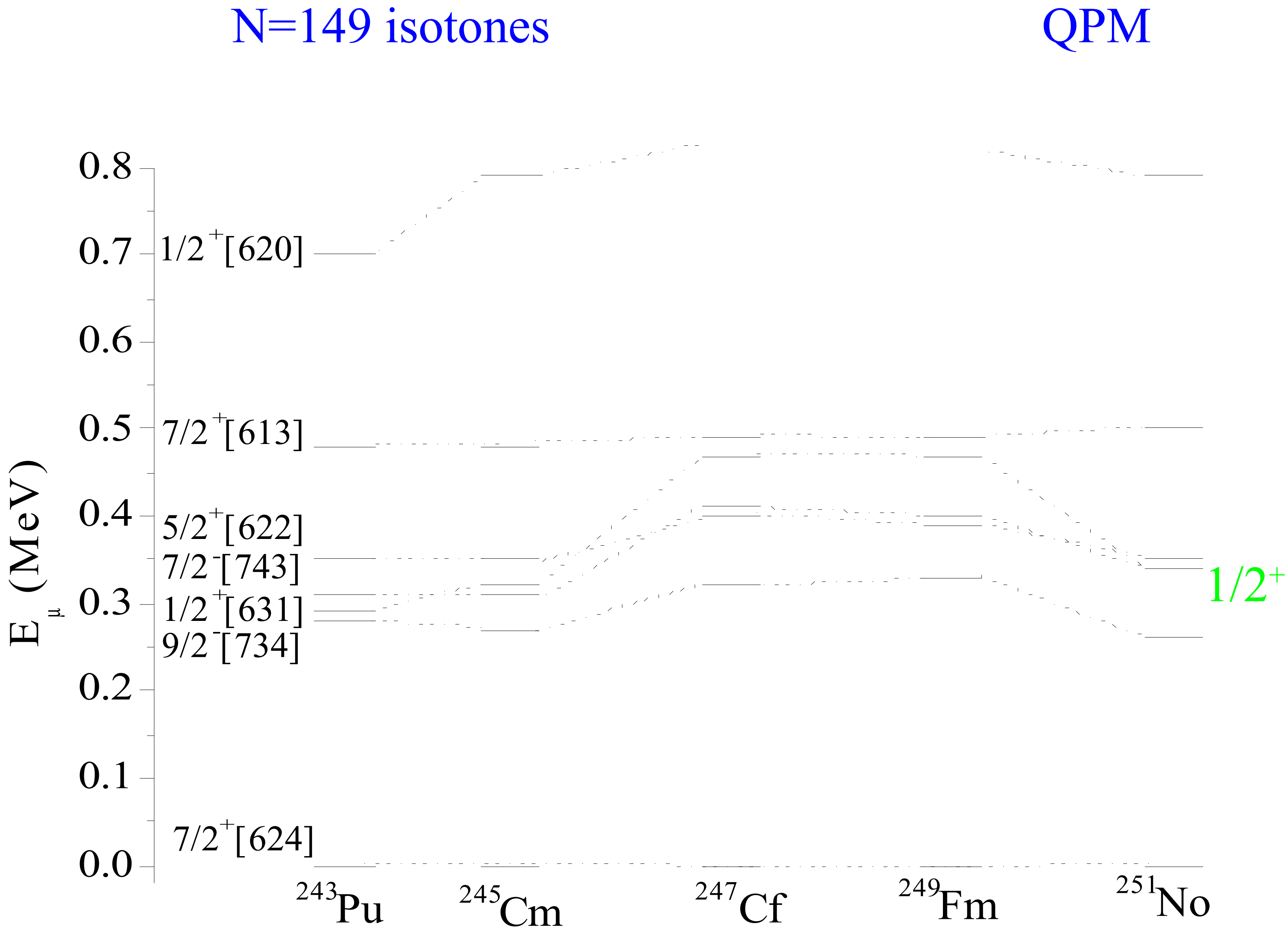


^{245}Md
^{247}Md
^{249}Md
^{251}Md
^{253}Md
^{255}Md
^{257}Md
exp. 8.56
8.16
7.66
7.21
7.44 MeV
0.10
0.05
0.00
0.6
0.4
0.2
0.0
E_μ (MeV)
E_μ (MeV)
8.4
8.47
7.99
8.04
7.63
7.66
7.38
7.36
7.48
7.45
7.17
7.13
7/2$^-$
9/2$^+$[624]
7/2$^-$[514]
7/2$^-$[514]
9/2$^+$[624]
5/2$^-$[523]
3/2$^-$[521]
7/2$^+$[633]
^{243}Es
^{245}Es
^{247}Es
^{249}Es
^{251}Es
^{253}Es
^{255}Es

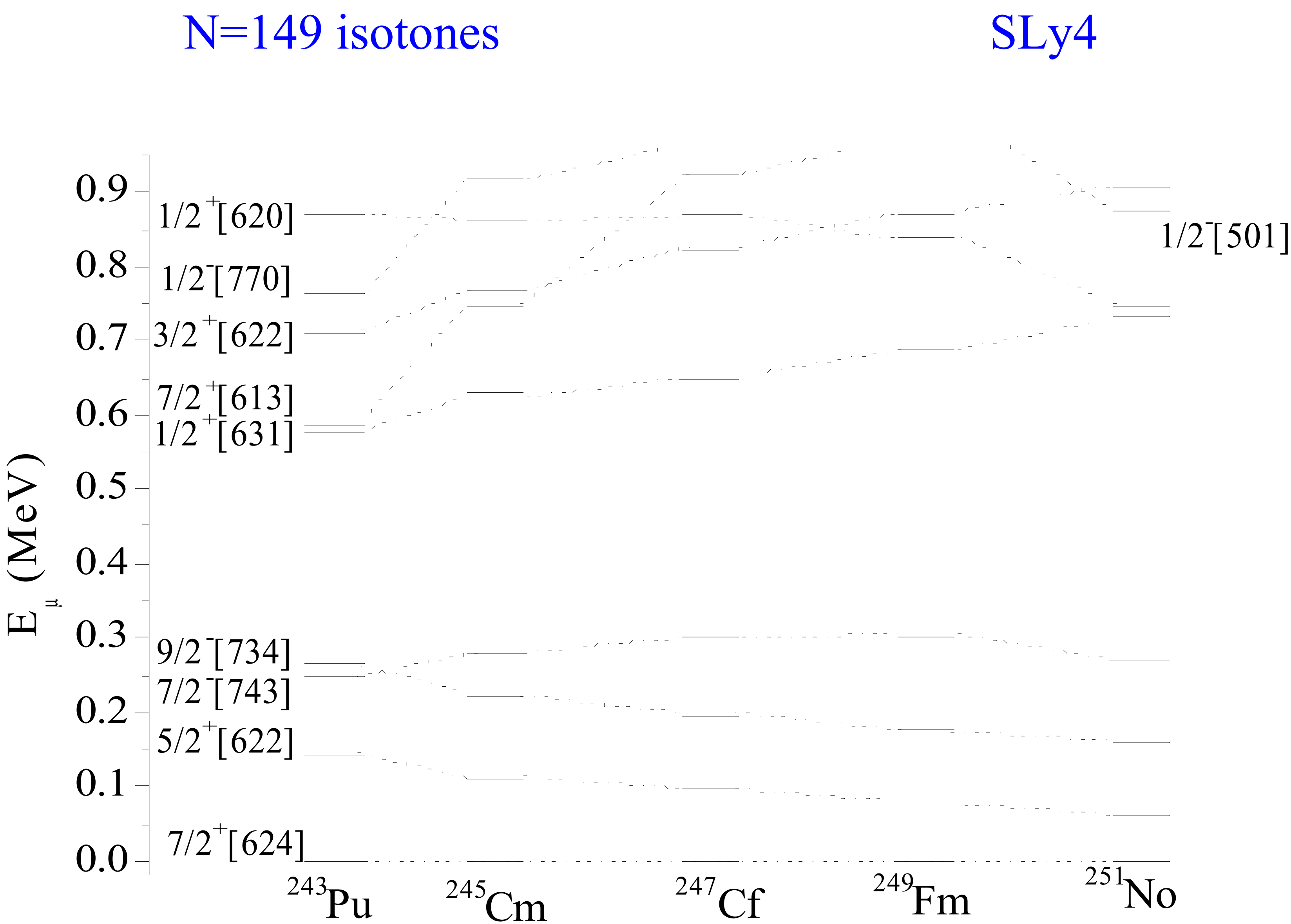
N=147 isotones



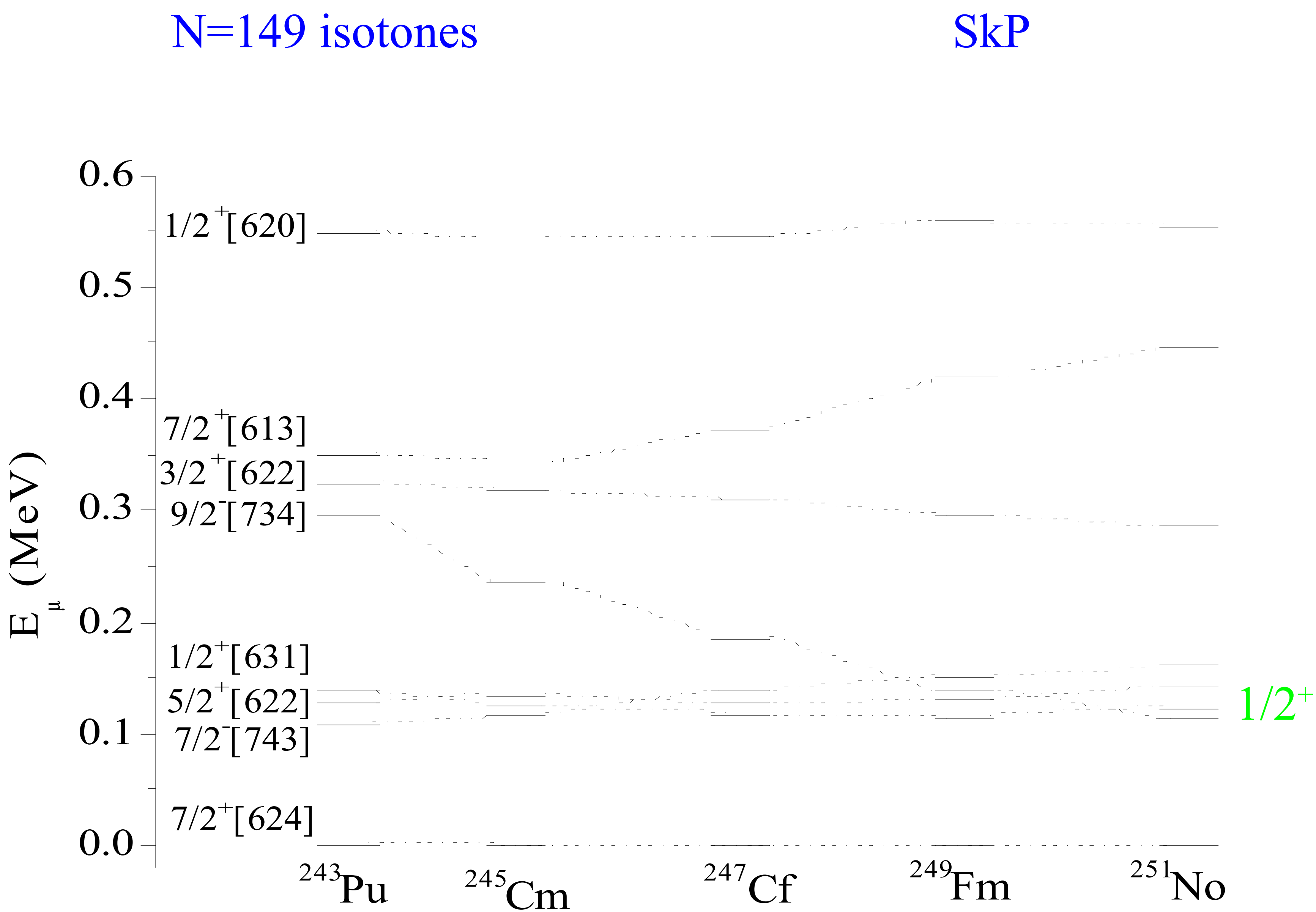

N=149 isotones
TCSM
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
E$_{\mu}$ (MeV)
7/2$^+$[613]
(1/2$^+$)
1/2$^+$[620]
1/2$^+$[631]
(1/2$^+$)
7/2$^-$[743]
5/2$^+$[622]
9/2$^-$[734]
7/2$^+$[624]
exp.
^{243}Pu
^{245}Cm
^{247}Cf
^{249}Fm
^{251}No



N=149 isotones
QPM
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
E_μ (MeV)
1/2+[620]
7/2+[613]
5/2+[622]
7/2-[743]
1/2+[631]
9/2-[734]
7/2+[624]
1/2+
^{243}Pu
^{245}Cm
^{247}Cf
^{249}Fm
^{251}No



N=149 isotones
SLy4
E_μ (MeV)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1/2+[620]
1/2-[770]
3/2+[622]
7/2+[613]
1/2+[631]
9/2-[734]
7/2-[743]
5/2+[622]
7/2+[624]
1/2-[501]
^{243}Pu
^{245}Cm
^{247}Cf
^{249}Fm
^{251}No

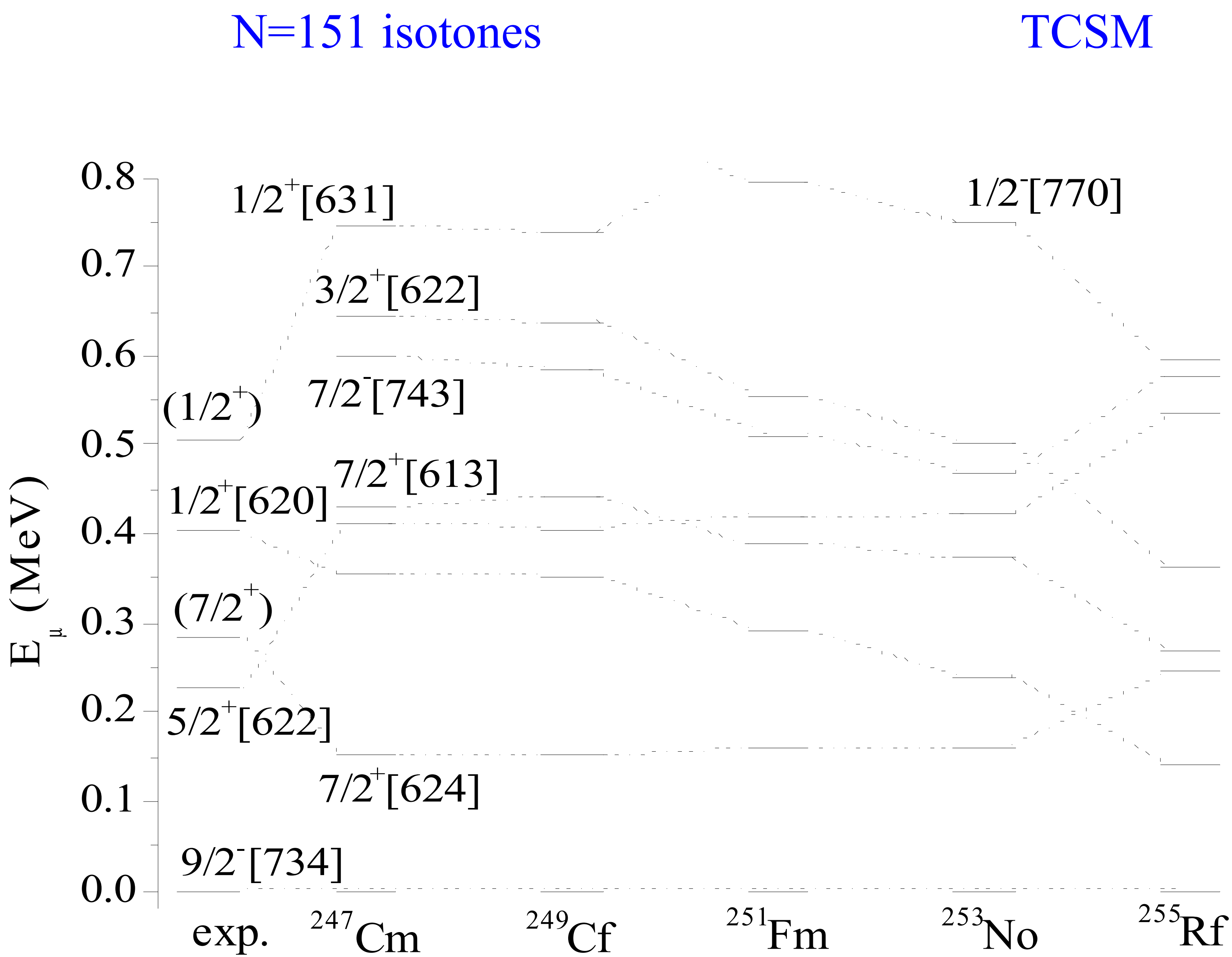


N=149 isotones
SkP
0.6
0.5
0.4
0.3
0.2
0.1
0.0
E_μ (MeV)
1/2+[620]
7/2+[613]
3/2+[622]
9/2-[734]
1/2+[631]
5/2+[622]
7/2-[743]
7/2+[624]
243Pu
245Cm
247Cf
249Fm
251No
1/2+

The smooth change of energies of almost all one-quasiparticle states in the isotone chain.

The deformation parameters of the N=149 nuclei treated are almost the same. Although different methods of calculations give various deformations of the ground state. For example, in the case of ^{251}No β_2=0.234 and β_4=0.057 are resulted from the TCSM, β_2=0.296 and β_4=0.01 from the HB with Sly4 parameterization.

Long-living isomer in ^{251}No: $1/2^+[631]$, about 1 s

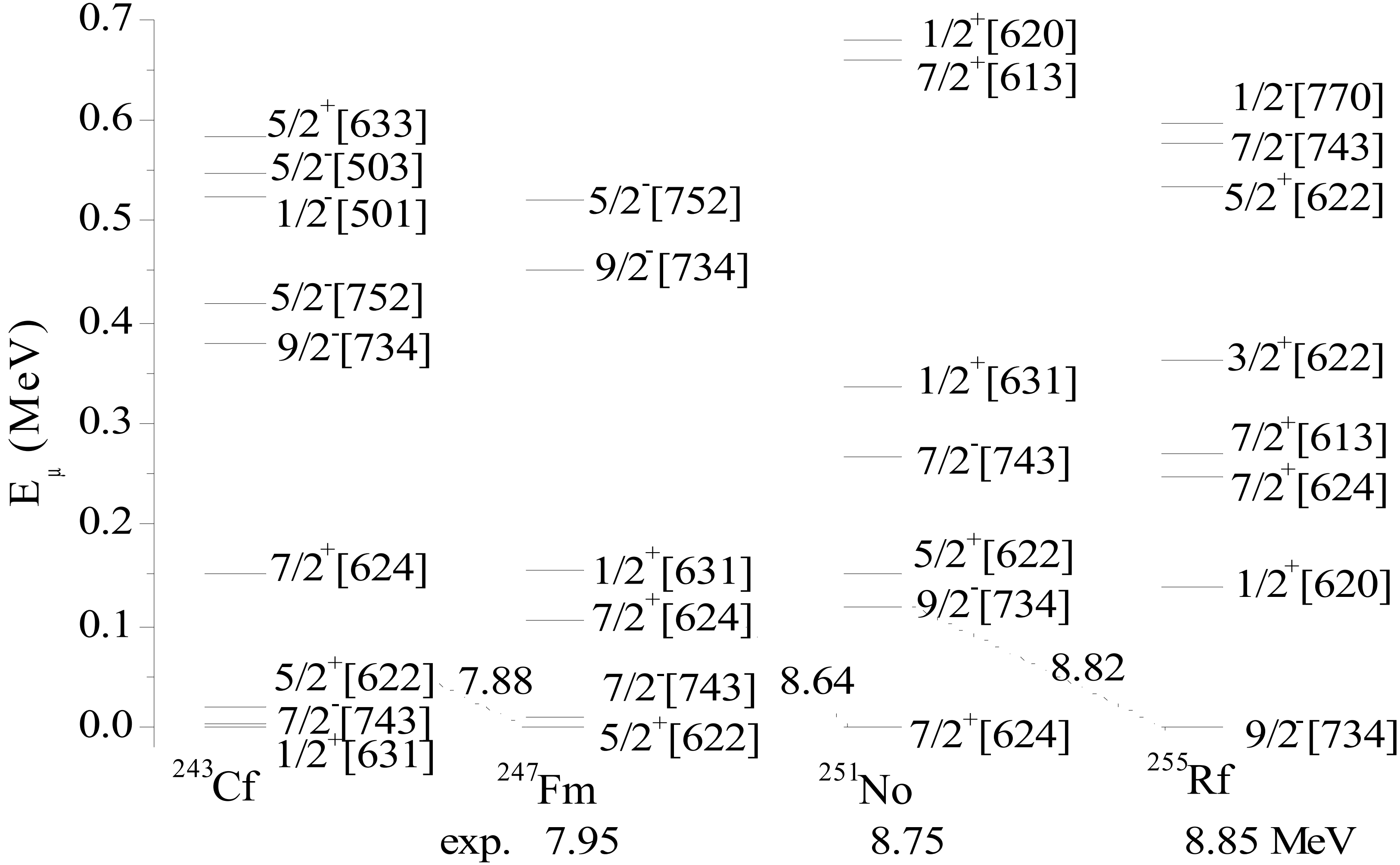

N=151 isotones
TCSM
E_μ (MeV)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
1/2+[631]
3/2+[622]
7/2-[743]
(1/2+)
7/2+[613]
1/2+[620]
(7/2+)
5/2+[622]
7/2+[624]
9/2-[734]
1/2-[770]
exp.
247Cm
249Cf
251Fm
253No
255Rf

For the N=151 nuclei, all our calculations with different methods give the $1/2^+[620]$ states below the $5/2^+[622]$ states.

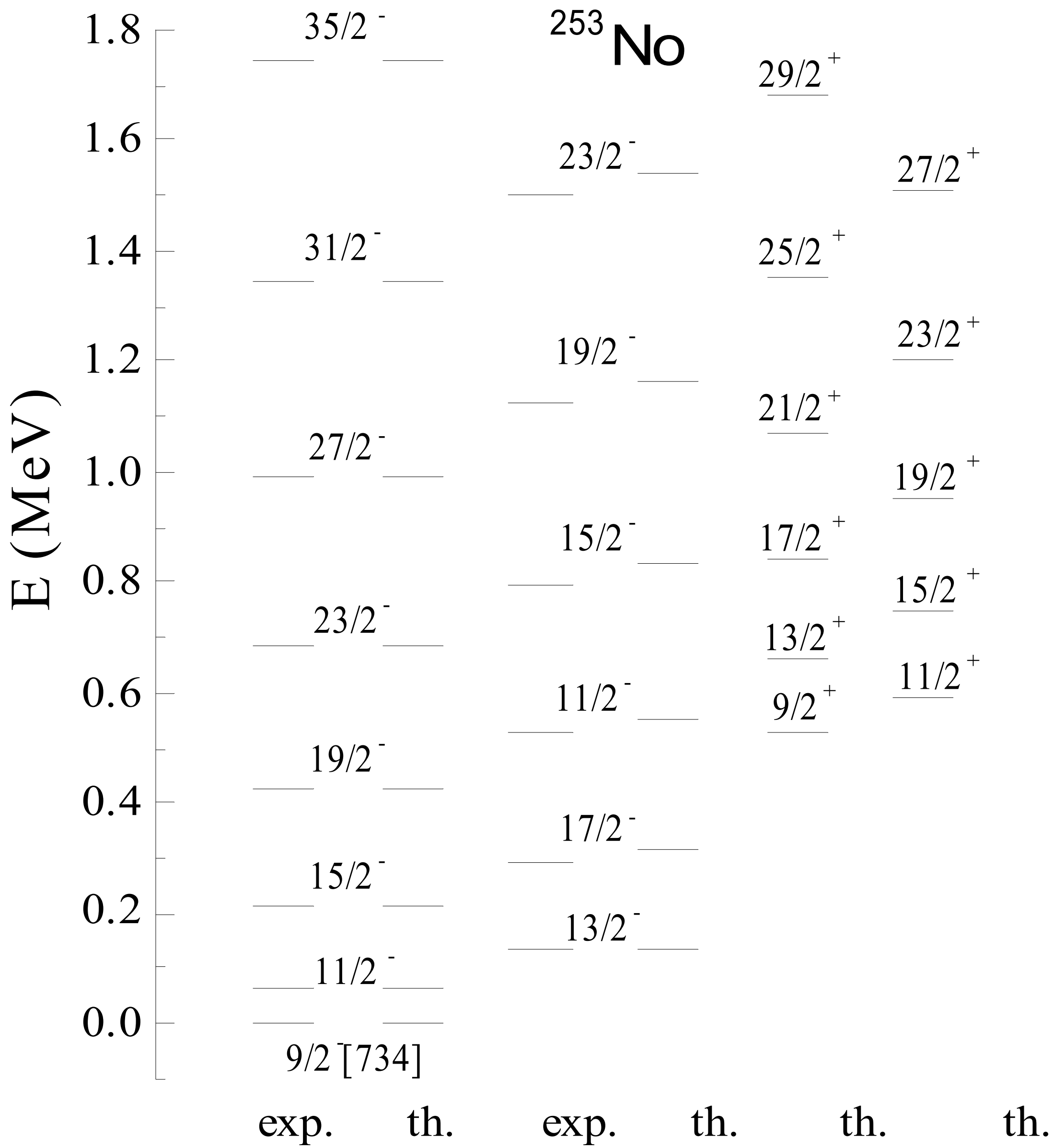
The $7/2^+[624]$ state is expected below the $5/2^+[622]$ state.

The $1/2^+[620]$ state goes down in heavier nuclei and could become the isomer state in ^{255}Rf.

In the N=149 and 151 nuclei the $1/2^+[620]$ single-particle state is above and the $5/2^+[622]$ and $1/2^+[631]$ single-particle states are below the corresponding Fermi levels. These levels go down with deformation. If one adjusts in ^{251}No the $1/2^+[631]$ level closer to the Fermi level than the $5/2^+[622]$ level, then in ^{253}No the $1/2^+[631]$ level would be below the $5/2^+[622]$ level as well. Thus, one can not simultaneously adjust the calculated one-quasiparticle states to the experimental assignments for ^{251}No and ^{253}No. This statement is true if the deformations of these nuclei are close as follows from our calculations.

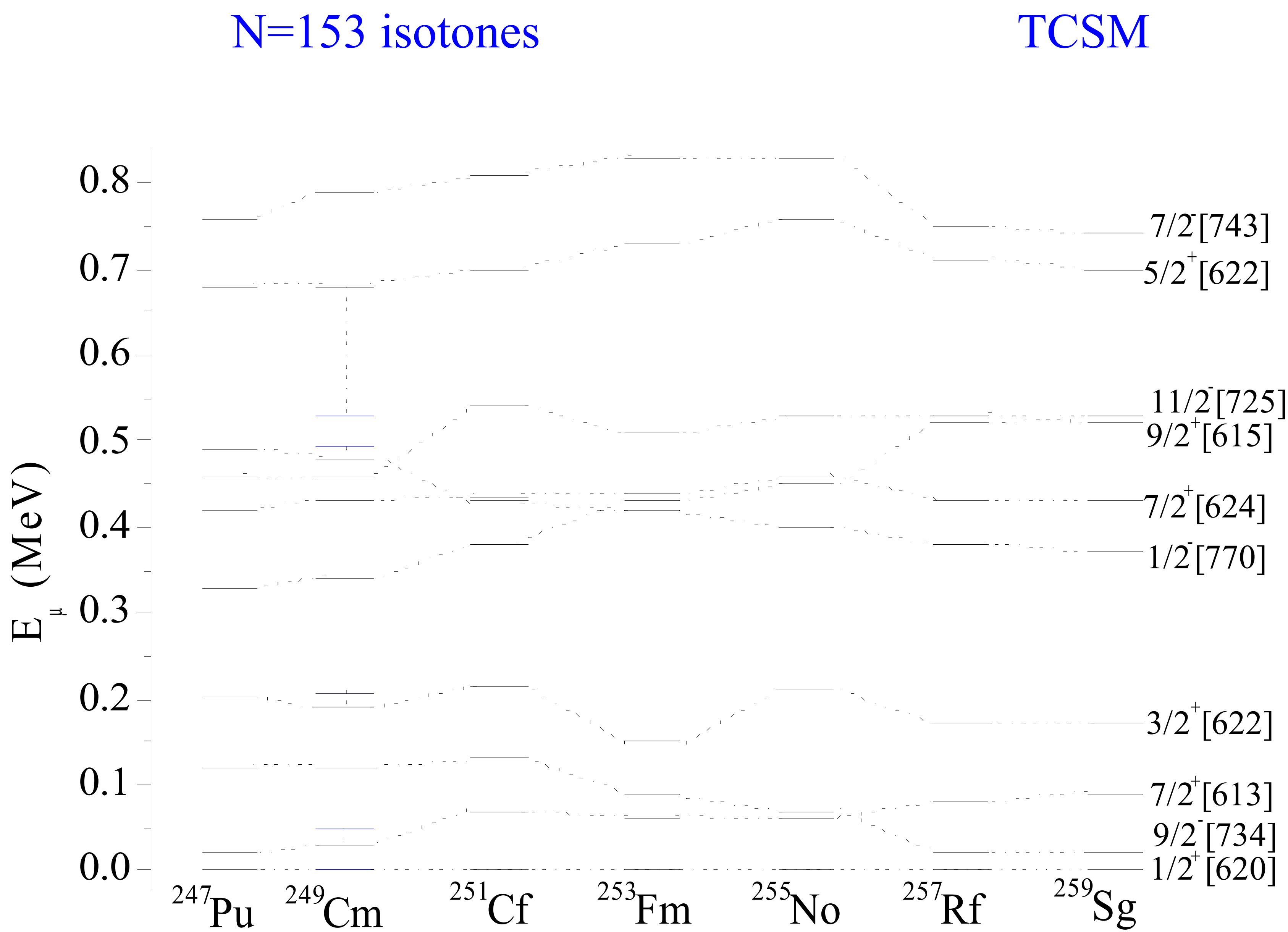


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
E_μ (MeV)
5/2+[633]
5/2-[503]
1/2-[501]
5/2-[752]
9/2-[734]
7/2+[624]
5/2+[622]
7/2-[743]
1/2+[631]
243Cf
5/2-[752]
9/2-[734]
1/2+[631]
7/2+[624]
7/2-[743]
5/2+[622]
247Fm
1/2+[620]
7/2+[613]
1/2+[631]
7/2-[743]
5/2+[622]
9/2-[734]
7/2+[624]
251No
1/2-[770]
7/2-[743]
5/2+[622]
3/2+[622]
7/2+[613]
7/2+[624]
1/2+[620]
9/2-[734]
255Rf
7.88
8.64
8.82
exp. 7.95
8.75
8.85 MeV

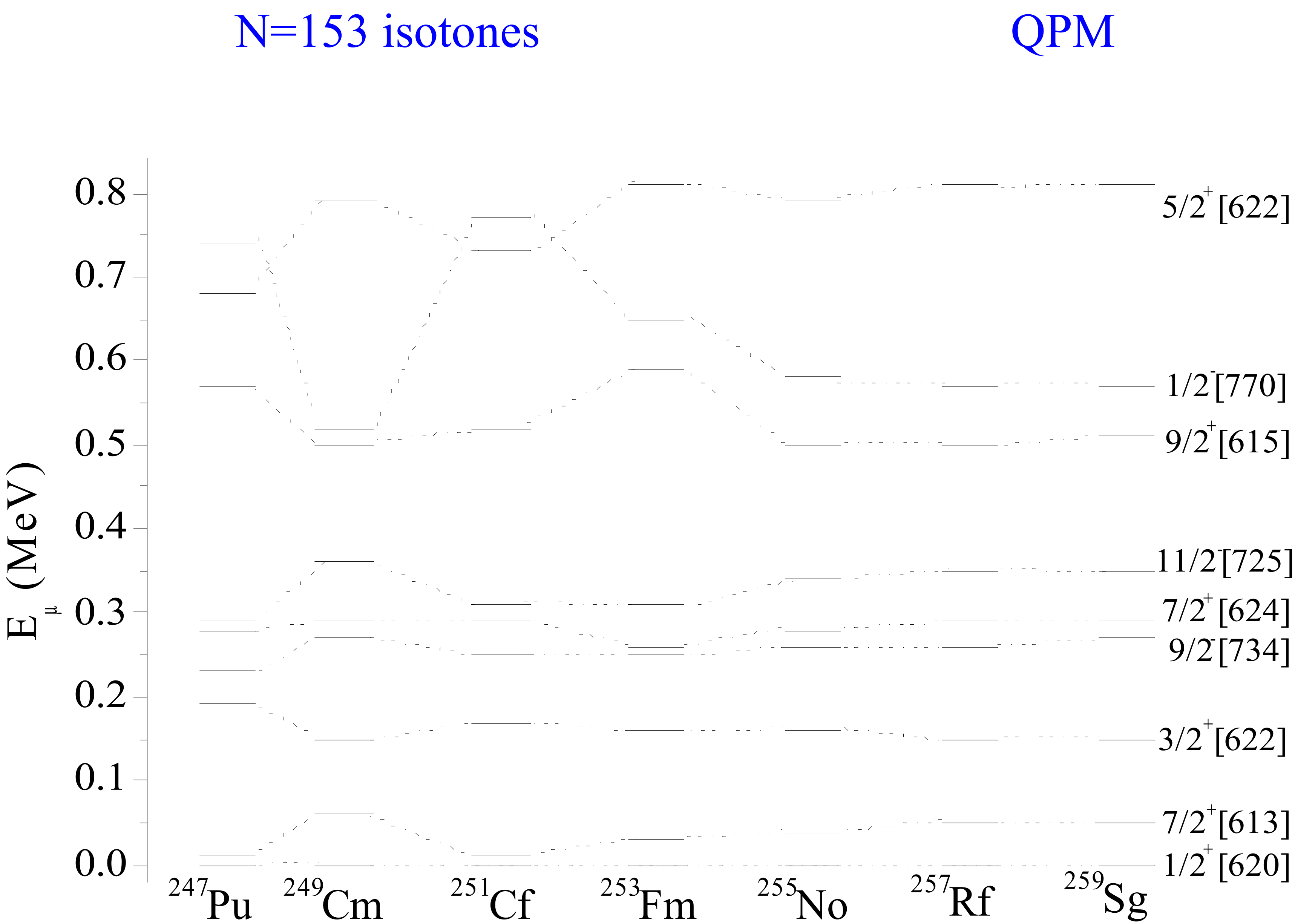


Rotational bands

The prediction of alternating parity bands at energy larger than 0.5 MeV.

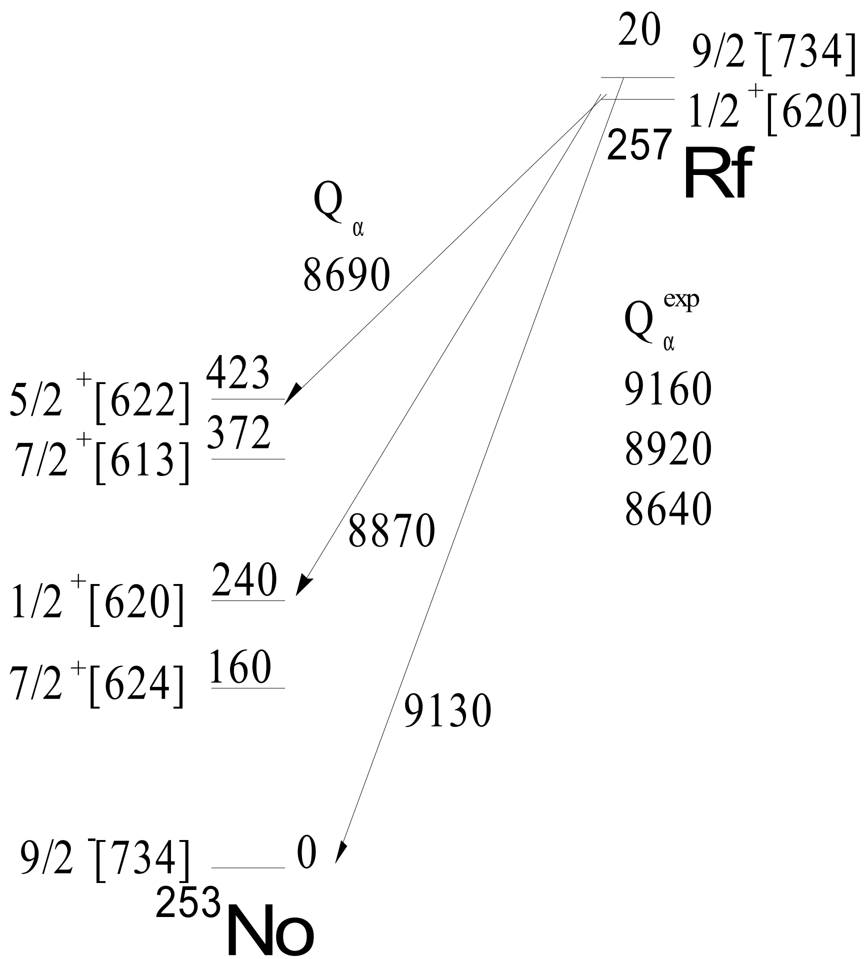


N=153 isotones
TCSM
E_μ (MeV)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
7/2⁻[743]
5/2⁺[622]
11/2⁻[725]
9/2⁺[615]
7/2⁺[624]
1/2⁻[770]
3/2⁺[622]
7/2⁺[613]
9/2⁻[734]
1/2⁺[620]
247Pu
249Cm
251Cf
253Fm
255No
257Rf
259Sg



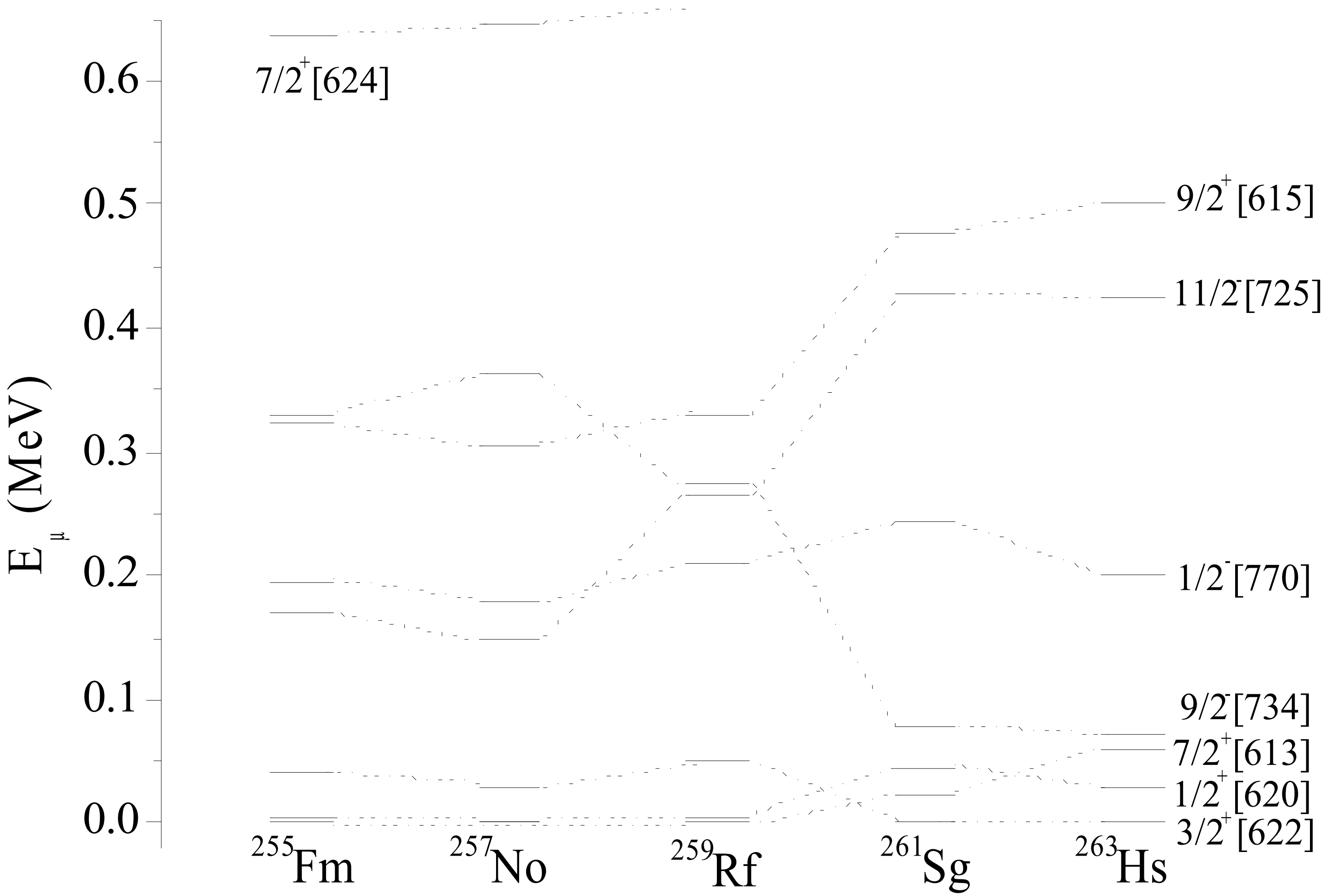
N=153 isotones
QPM
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
E_μ (MeV)
5/2+[622]
1/2-[770]
9/2+[615]
11/2-[725]
7/2+[624]
9/2-[734]
3/2+[622]
7/2+[613]
1/2+[620]
247Pu
249Cm
251Cf
253Fm
255No
257Rf
259Sg

The $7/2^+[613]$ state is expected to be the isomer at least for N=153 nuclei up to ^{255}No. In ^{257}Rf and ^{259}Sg the order of the $7/2^+[613]$ and $9/2^-[734]$ states changes and the $9/2^-[734]$ state can become the isomer. In ^{257}Rf the isomer state at 0.07 MeV has been found [EPJA 43, 175 (2010)] and tentatively assigned to the $11/2^-[725]$ state. However, all our calculations result the $11/2^-[725]$ state at energy larger then 0.3 MeV and with increasing Z there is no tendency for lowering the energy of this state below 0.1 MeV. The HB approach results the $11/2^-[725]$ states at energies above 0.6 MeV.



Using the one-quasiparticle spectra calculated with the TCSM for ^{257}Rf and ^{253}No, the possible α decay scheme of ^{257}Rf is suggested. The calculated Q_α values are consistent with the experimental values listed.

N=155 isotones TCSM



β_2=0.26 β_2=0.26

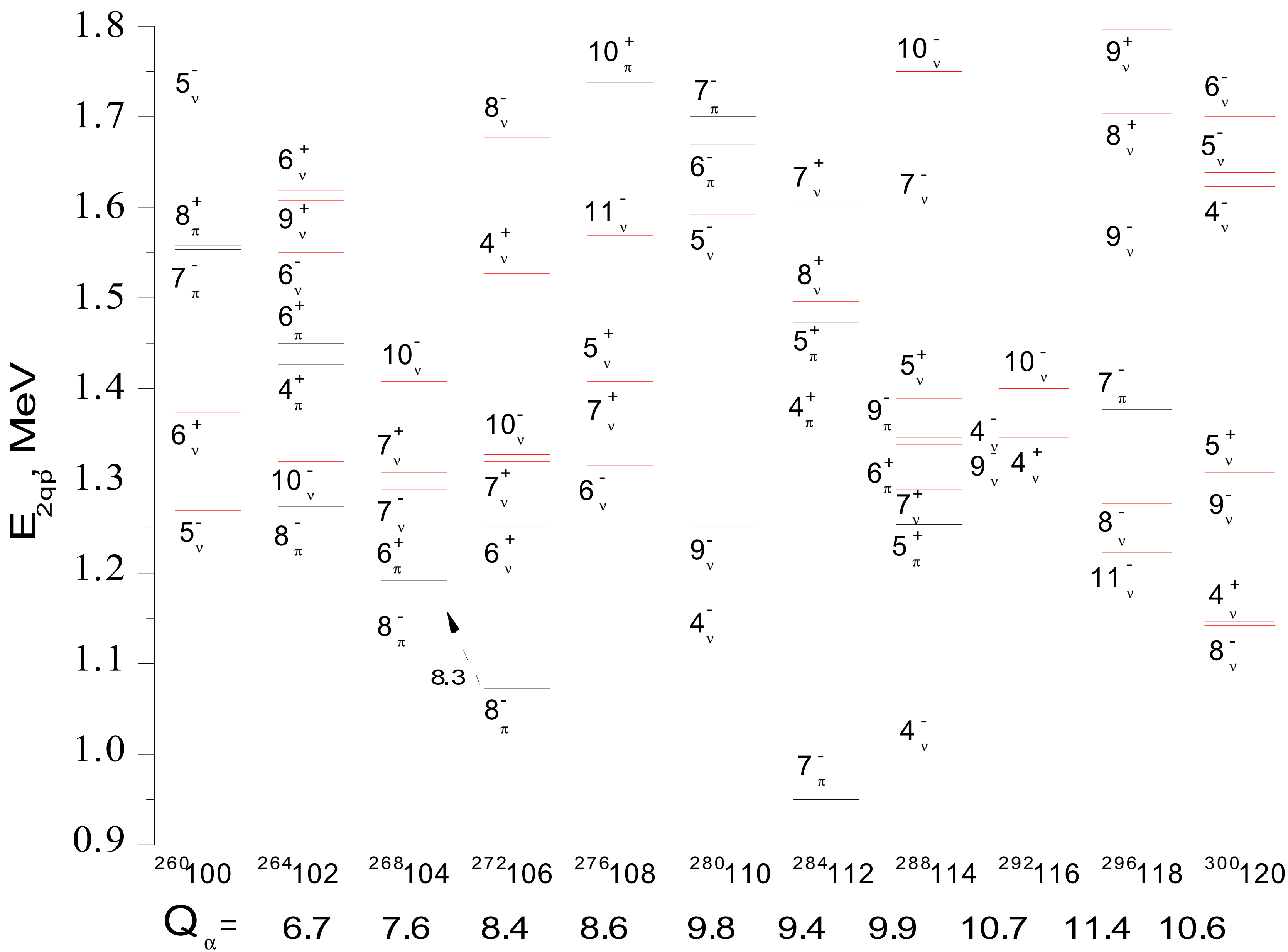
β_4=0.03 β_4=−0.01

The change of the deformation in isotone chain destroys the smooth dependence of the one-quasiparticle energies on *Z*.

Summary

- The energies of almost all low-lying one-quasiparticle states change rather smoothly in the isotone chains if there is no cross of the proton sub-shell, i.e. if the ground-state deformations of the isotones are close.
- The change of the deformation due to the proton shell effect (N=155 isotones) causes the rearrangement of the order of the one-quasiparticle states.

- The simultaneous good agreement of the calculated and experimental spectra for the N=149 and 151 nuclei can not be achieved without strong variation of the parameters.
- The calculations were performed with the TCSM and QPM which belong to the microscopic-macroscopic approach and with the self-consistent HB approach. All approaches qualitatively lead to the same conclusions.
- The used simple shape parametrization is suitable to describe some properties of heavy nuclei.



E_2qp, MeV
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
260 100
264 102
268 104
272 106
276 108
280 110
284 112
288 114
292 116
296 118
300 120
Qα= 6.7 7.6 8.4 8.6 9.8 9.4 9.9 10.7 11.4 10.6
8.3

$^{64}Ni+^{207}Pb \rightarrow ^{270}Ds+1n$

ground state: $E_{CN}^{*} \approx 14$ MeV

2qp isomer state: $E_{CN}^{*} \approx 14 - E_{\mu} \approx 12.8$ MeV

$W_{sur}(isomer)/W_{sur}(gs) \approx 2$

For $E_{\mu} \approx 1.2$ MeV, the population of isomer state is $\approx \exp(-(E_{\mu}-E_{rot})/T) \approx 0.32$.

$\sigma_{ER}(gs)/\sigma_{ER}(isomer) \approx 0.68:0.64$

exp.: $\approx 1:1$

1qp isomer state:

After the CN is cooled down by the neutron emission till $E^* < 8$ MeV

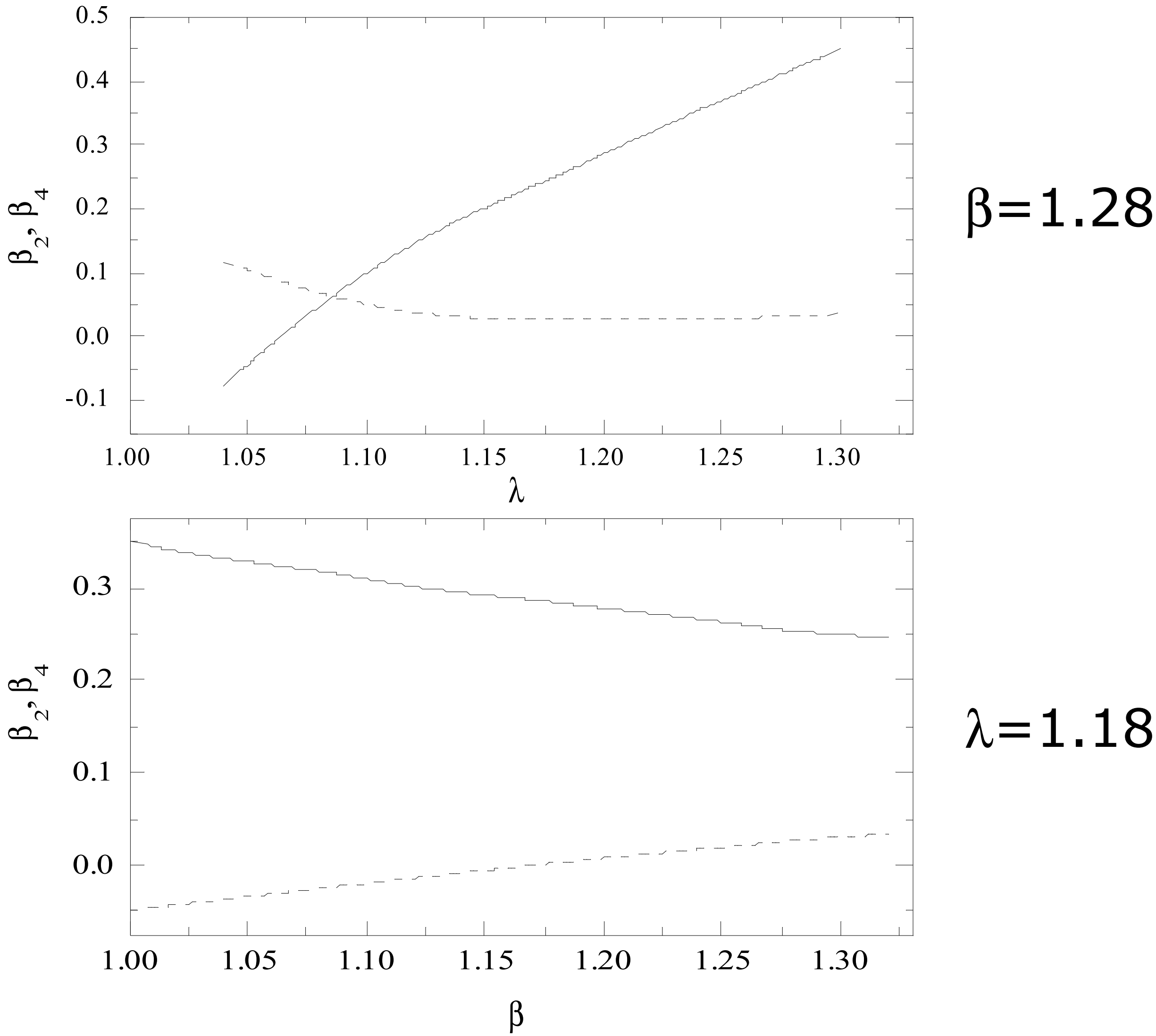
$p_{is} \approx \exp(-E_{is}/T)/[1+\exp(-E_{is}/T)]$

$T \approx 0.6$ MeV and $p_{is} > 0.35$ in the reaction treated.

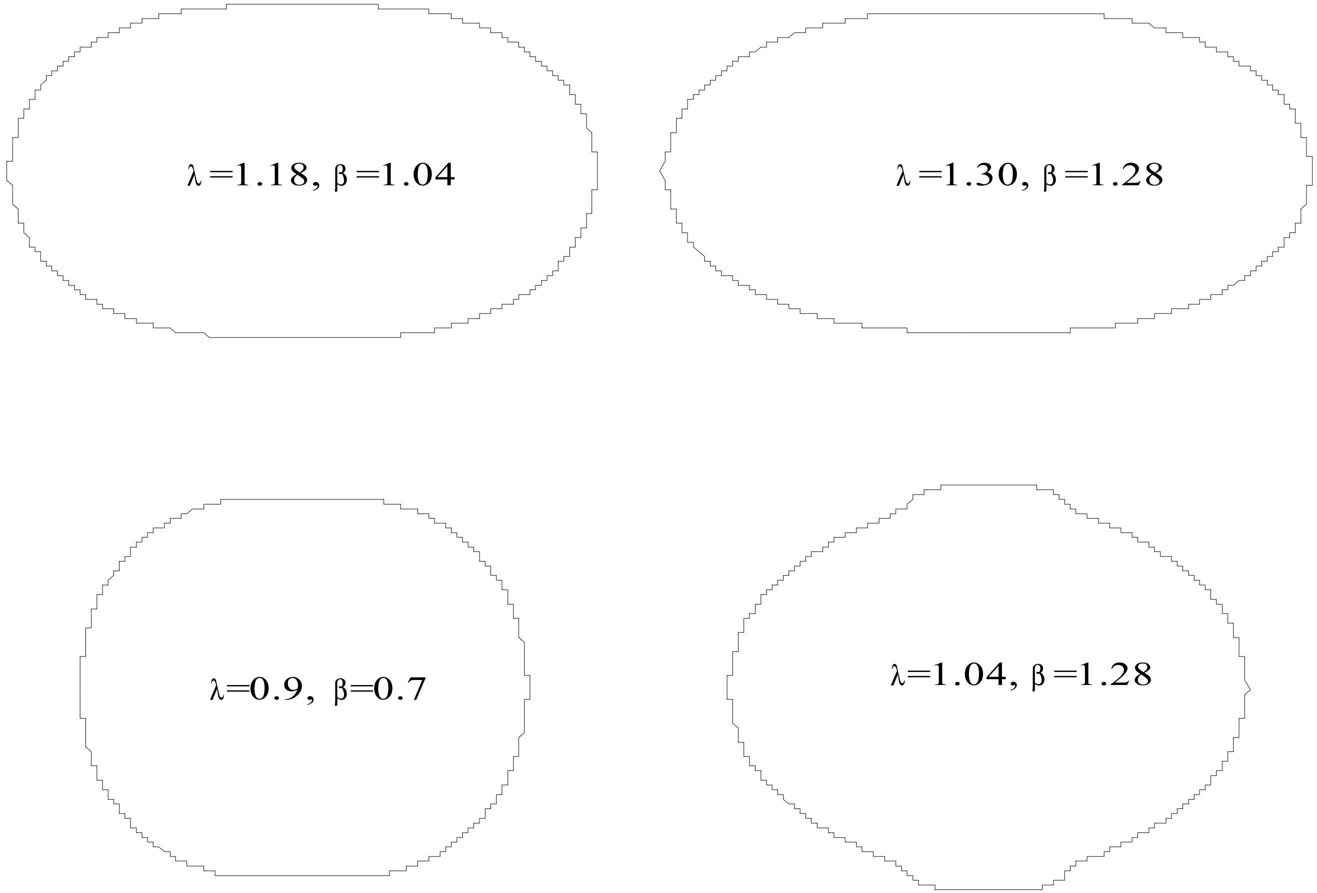
The population of isomer state is quite probable.

^{248}Fm

$\beta_2 -$ *solid lines* , $\beta_4 -$ *dashed lines*



^{248}Fm



$$H = T + V(\rho, z) + V_{LS} + V_{L^2}$$

$$V(\rho, z) = \begin{cases} \frac{1}{2} m\omega_z^2 (z - z_1)^2 + \frac{1}{2} m\omega_\rho^2 \rho^2, & z < z_1 \\ \frac{\varepsilon}{2} m\omega_z^2 (z - z_i)^2 (1 + c_i (z - z_i) + d_i (z - z_i)^2) + \frac{1}{2} m\omega_\rho^2 \rho^2, & z_1 < z < z_2 \\ \frac{1}{2} m\omega_z^2 (z - z_2)^2 + \frac{1}{2} m\omega_\rho^2 \rho^2, & z > z_2 \end{cases}$$

$$V_{LS} = -\frac{2\hbar\kappa_i}{m\omega_{0i}} (\nabla V \times \vec{p}) \vec{s}$$

$$V_{L^2} = -\frac{\hbar\kappa_i \mu_i}{m^2 \omega_{0i}^3} \hat{l}^2 + \hbar\kappa_i \mu_i \omega_{0i} N_1 (N_1 + 3)/2 \, \delta_{if}$$

$$\omega_{0i} = 41\, MeV / A_i^{1/3},\ A_i = a_i b_i^2 / 1.22^3,\ c_i = -2/z_i,\ d_i = -2/z_i^2,$$
$$\omega_\rho / \omega_z = a_i / b_i,\ z_2 - z_1 = 2R_0 \lambda - a_1 - a_2$$

Parameters

$$35 \le N - Z \le 56$$

for neutrons

$$\kappa_n = -0.076 + 0.0058(N-Z) - 6.53\times10^{-5}(N-Z)^2 + 0.002\,A^{1/3},$$

$$\mu_n = 1.598 - 0.0295(N-Z) + 3.036\times10^{-4}(N-Z)^2 - 0.095\,A^{1/3},$$

for protons

$$\kappa_p = 0.0383 + 0.00137(N-Z) - 1.22\times10^{-5}(N-Z)^2 - 0.003\,A^{1/3},$$

$$\mu_p = 0.335 + 0.01(N-Z) - 9.367\times10^{-5}(N-Z)^2 + 0.003\,A^{1/3},$$

The parts in front of the terms with $A^{1/3}$ vary: (0.05-0.053) for κ_n , (0.075-0.0768) for κ_p , (0.88-0.92) for μ_n , (0.58-0.61) for μ_p

Strength parameters of paring interaction

$$G_{\substack{n\\p}} = (19.2 \mp 7.4 \frac{N-Z}{A})\, A^{-1}\, MeV$$

$$A \approx 250 \rightarrow G_n \approx 0.075\, MeV \;, G_p \approx 0.085\, MeV$$

One-quasiparticle excitations

$$E_\mu = \sqrt{(e_\mu - e_F)^2 + \Delta^2} - \sqrt{(e_\mu{}' - e_F)^2 + \Delta^2}$$

Two-quasiparticle excitations

$$E_\mu = \sqrt{(e_\mu - e_F)^2 + \Delta^2} + \sqrt{(e_\mu{}' - e_F)^2 + \Delta^2}$$

$$\Delta \geq 0.35\, MeV \rightarrow BCS\ approximation$$

^{270}Hs

β_2=0.25 0.262 0.256

β_4=-0.026 -0.006 0.026