Peculiarities of the sub-barrier fusion with the quantum diffusion approach

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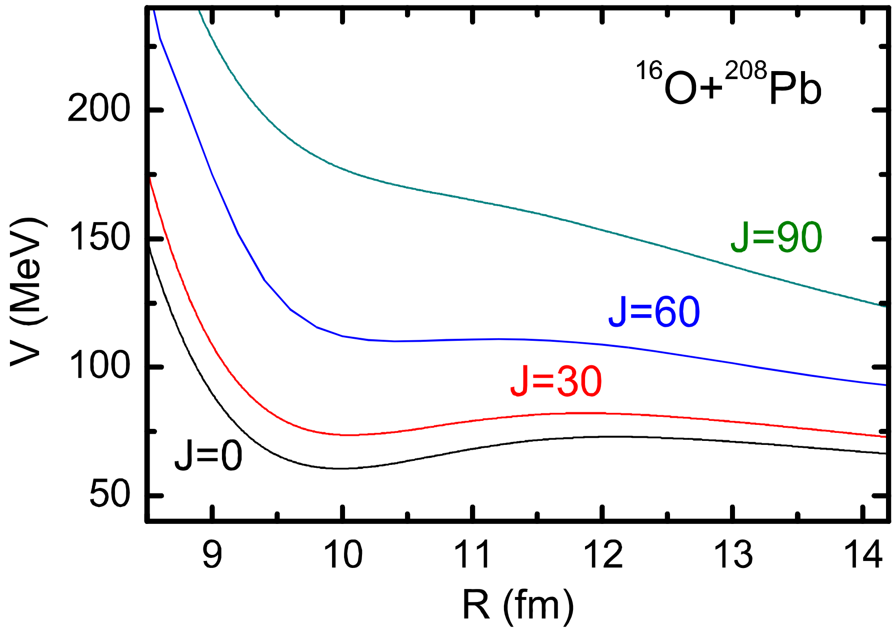
Introduction

- The behavior of capture cross sections at sub-barrier energies
- The astrophysical problems related to nuclear synthesis
- The conventional coupled-channel approach with realistic set of parameters is not able to describe the capture cross sections either below or above the Coulomb barrier
- Unexpected behavior of sub-barrier fusion can be related to the switching off the nuclear interaction at external turning point

Nucleus-Nucleus Potential



We use the double-folding formalism with density-depending effective nucleon–nucleon interaction and the nucleon densities of the projectile and target in the form of Woods-Saxon parameterization.

nuclear radius parameter: → r_0 = 1.15 fm.
diffuseness parameter: → a = 0.53–0.56 fm.

Only limited J has a contribution in capture !

$$V(R, J, \Omega_p, \Omega_T) = V_{nuc}(R, \Omega_p, \Omega_T) + V_{Coul}(R, \Omega_p, \Omega_T) + V_{rot}(R, J)$$

Nuclear term — Coulomb term — Centrifugal term

Ω_p, Ω_T --- the angles specifying the orientation of the nuclei with respect to the colliding axis

The capture cross section

The capture cross-section is a sum of partial capture cross-sections

$$\sigma(E_{c.m.}) = \sum_J \sigma_c(E_{c.m.}, J) = \pi\lambda\!\!\!^{-}\sum_J (2J+1) P_{cap}(E_{c.m.}, J),$$

$\lambda\!\!\!^{-\,2} = \hbar^2 / 2\mu E_{c.m.}$ --- the reduced de Broglie wavelength

P_{cap} --- the partial capture probability

The partial capture probability obtained by integrating the propagator *G* from the initial state (R_0, P_0) at time $t=0$ to the finite state at time *t*:

$$P_{cap} = \lim_{t\to\infty} \int_{-\infty}^{r_{in}} dR \int_{-\infty}^{\infty} dP\, G(R, P, t \mid R_0, P_0, 0) = \lim_{t\to\infty} \frac{1}{2} \operatorname{erfc}\left[\frac{\overline{R(t)}}{\sqrt{\Sigma_{RR}(t)}}\right].$$

$\overline{R(t)}$ --- first moment (the average of the coordinate *R*)

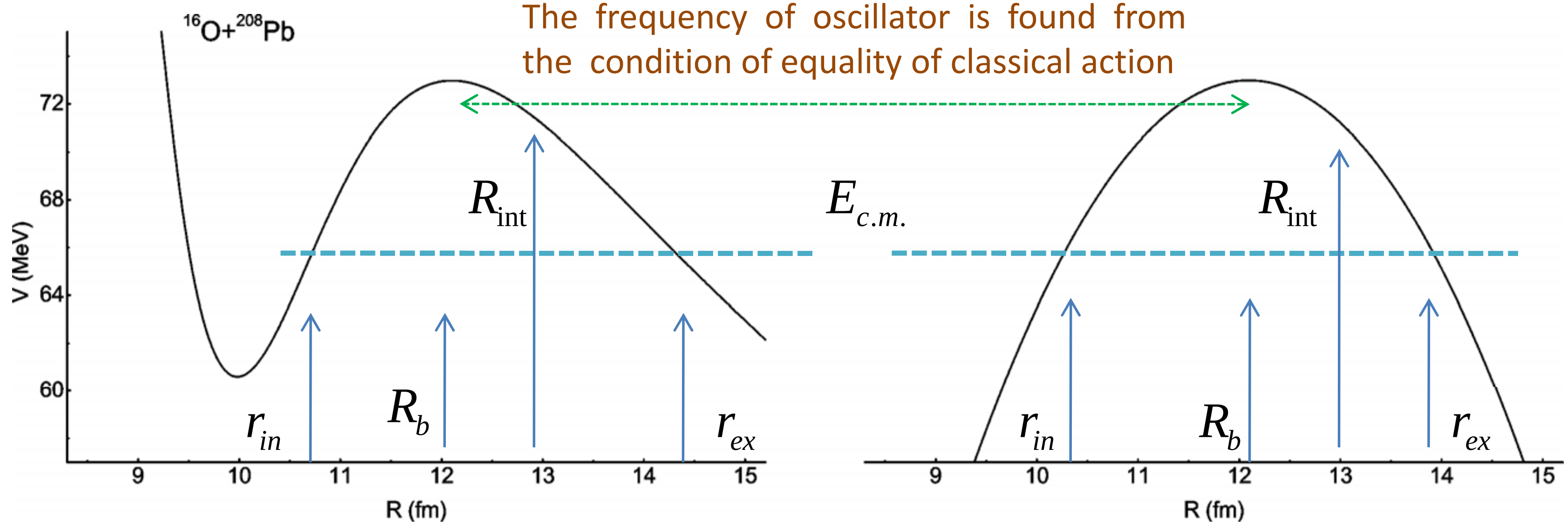
$\Sigma_{RR}(t)$ --- variance in the coordinate

G is calculated for the inverted oscillator which approximate the realistic nucleus-nucleus potential.

Approximation:

realistic nucleus-nucleus potential → inverted oscillator

The frequency of oscillator is found from the condition of equality of classical action



Many quantum mechanical, dissipative and non Markovian effects accompanying the passage through the potential barrier is taken into consideration in our formalism through $\overline{R(t)}$ and $\overline{\Sigma_{RR}(t)}$!

The parameters of the model are fixed for all reactions:

Friction coefficient	-----	$\hbar\lambda = 2$ MeV
Internal-excitation width	-----	$\hbar\gamma = 12$ MeV
The radius of interaction	-----	$R_{int} = R_b + 1.1$ fm (friction start to act)

The first moment and variance in coordinate

The expressions for the first and second moments:

The average of the coordinate *R*

$$\overline{R(t)} = A_t R_0 + B_t P_0$$

The variance in the coordinate

$$\Sigma_{RR}(t) = \frac{2\hbar^2\tilde{\lambda}\gamma^2}{\pi}\int_0^t d\tau' B_{\tau'}\int_0^t d\tau'' B_{\tau''}\int_0^\infty d\Omega \frac{\Omega}{\Omega^2+\gamma^2}\coth\left[\frac{\hbar\Omega}{2T}\right]\cos\left[\Omega(\tau'-\tau'')\right]$$

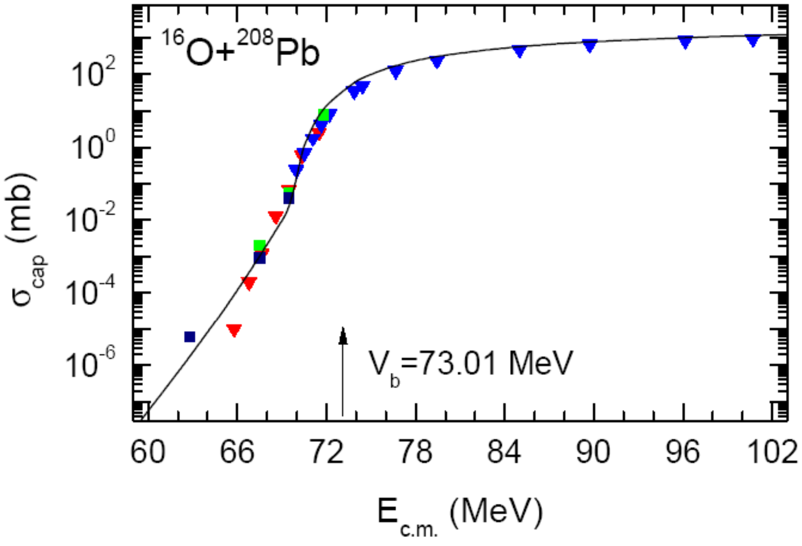
For the functions A_t and B_t one can find:

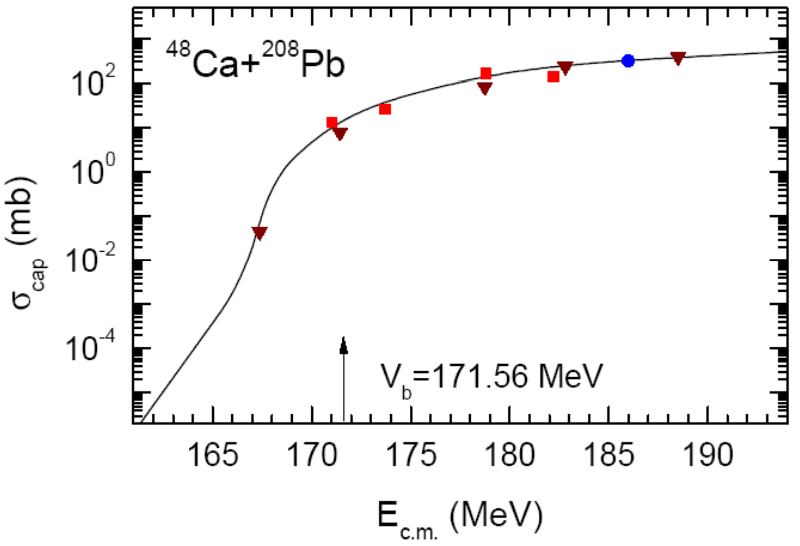
$$B_t = \frac{1}{\mu}\sum_{i=1}^{3}\beta_i (s_i+\gamma)e^{s_i t} \qquad A_t = \sum_{i=1}^{3}\beta_i\left[s_i(s_i+\gamma)+\hbar\tilde{\lambda}\gamma/\mu\right]e^{s_i t}$$

s_i are the real roots of the following equation: $(s+\gamma)(s^2-\omega_0^2)+\hbar\tilde{\lambda}\gamma s/\mu = 0$

The details of the used formalism are presented in:
V.V.Sargsyan *et al.*, Eur. Phys. J. A **45,** 125 (2010).

Lead-based reactions



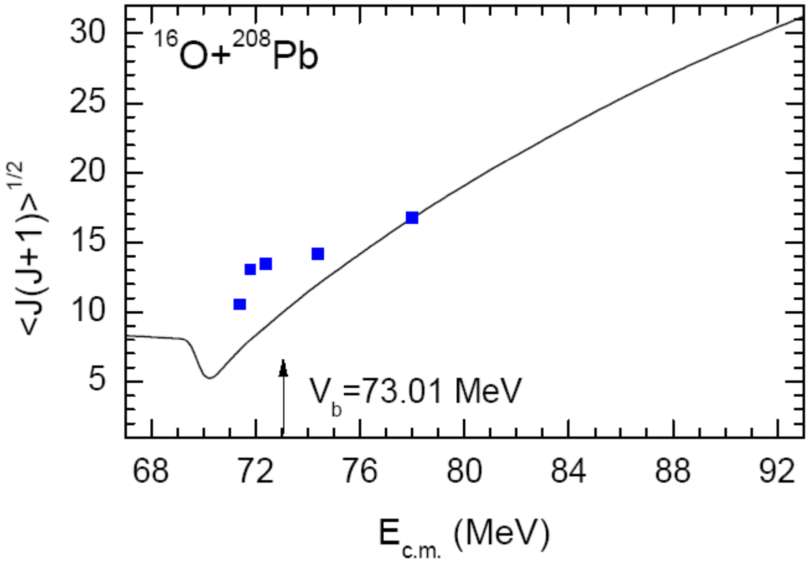


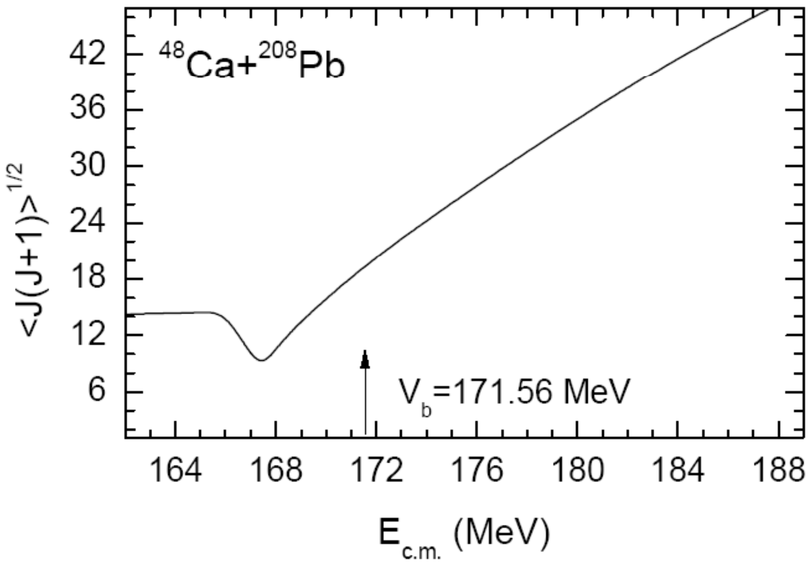
The calculated capture cross section versus $E_{c.m.}$ for the $^{16}O+^{208}Pb$ and $^{48}Ca+^{208}Pb$ reaction are compared with the experimental data.

There is sharp fall-off of the cross-sections just under the barrier. With decreasing $E_{c.m.}$ up to about 3.5–5.0 MeV below the Coulomb barrier the regime of interaction is changed because at the external turning point the colliding nuclei do not reach the region of nuclear interaction where the friction plays a role. As a result, at smaller $E_{c.m.}$ the cross-sections fall with smaller rate!!

With larger value of interaction radius the change of fall rate would occur at smaller $E_{c.m.}$ However, the uncertainty in the definition of R_{int} is rather small !!

Lead-based reactions





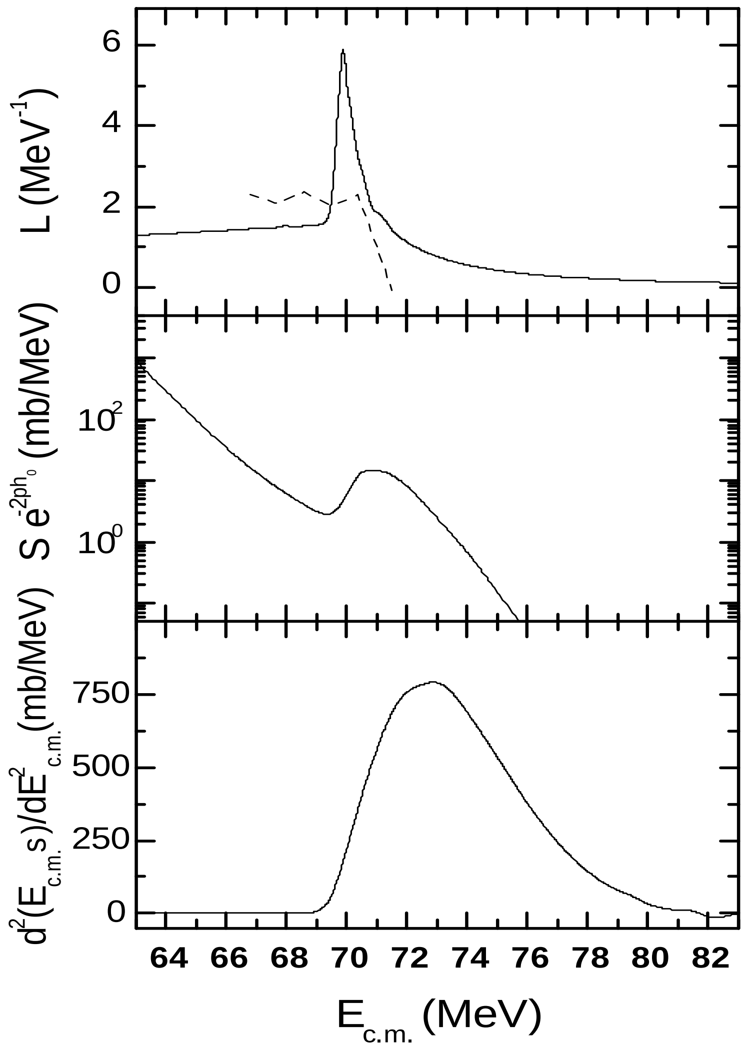
The calculated mean-square angular momentum of compound nucleus versus $E_{c.m.}$ for the $^{16}O+^{208}Pb$ reaction are compared with the experimental data. In lower part the calculated mean-square angular momentum versus $E_{c.m.}$ for the $^{48}Ca+^{208}Pb$ reaction.

$$\left\langle J^2 \right\rangle = \sum_J J\left(J+1\right)\sigma_c\left(E_{c.m.}, J\right)/\sigma\left(E_{c.m.}\right)$$

At energies of 3–4.5 MeV below the barrier $\langle J^2 \rangle$ has a minimum. The experimental data indicate the presence of the minimum as well. On the left-hand side of this minimum the dependence of $\langle J^2 \rangle$ on $E_{c.m.}$ is rather weak.

The found behavior of $\langle J^2 \rangle$, which is related to the change of the regime of interaction between the colliding nuclei, would affect the angular anisotropy of the products of fission following fusion.

S-factor and fusion barrier distribution



The calculated values (solid lines) of logarithmic derivative L (upper part), astrophysical *S*-factor (middle part) with $\eta_0 = \eta(E_{c.m.} = V_b)$, and fusion barrier distribution (lower part) for the $^{16}O+^{208}Pb$ reaction. Dashed line shows the values of L obtained only with the calculated cross sections at Ec.m. used in the experiment.

The expressions for the astrophysical *S*- factor and logarithmic derivative *L* are the following:

$$S = E_{c.m.}\sigma\exp\left(2\pi\eta\right) \quad \left[\eta\left(E_{c.m.}\right) = Z_1 Z_2 e^2\sqrt{\mu/\left(2\hbar^2 E_{c.m.}\right)}\right],$$

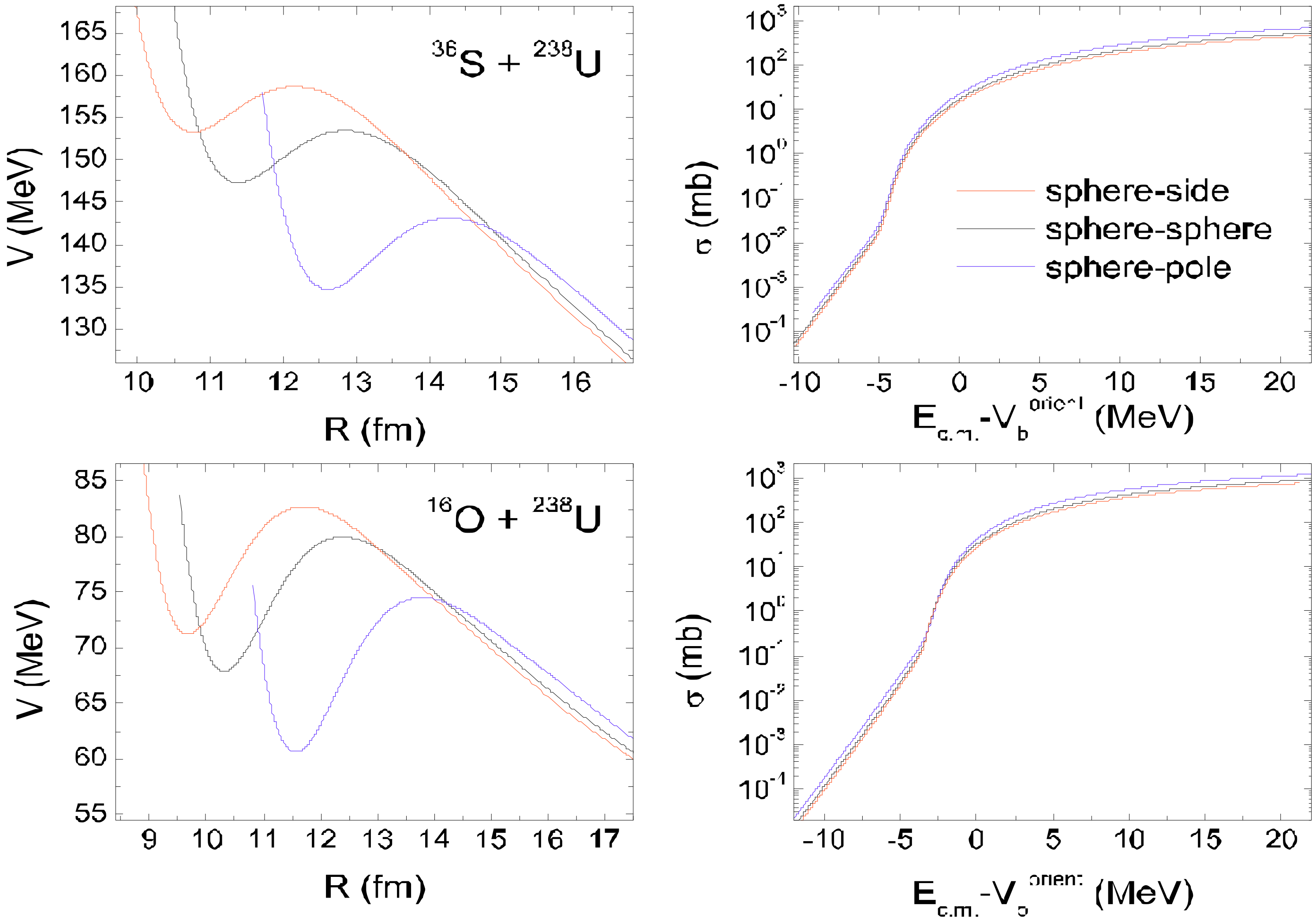
$$L\left(E_{c.m.}\right) = d\left(\ln\left(\sigma E_{c.m.}\right)\right)/dE_{c.m.}$$

The logarithmic derivative strongly increases just below the barrier and then has a maximum! This leads to the *S*-factor maximum which is seen in the experiments. After this maximum the *S*-factor decreases with $E_{c.m.}$ and then starts to increase. The same behavior has been revealed by extracting the *S*-factor from the experimental data.

If for finding the logarithmic derivative we use only the cross-sections calculated at the energies where the experimental cross sections are available, the function $L(E_{c.m.})$ would be similar to that obtained with the experimental data.

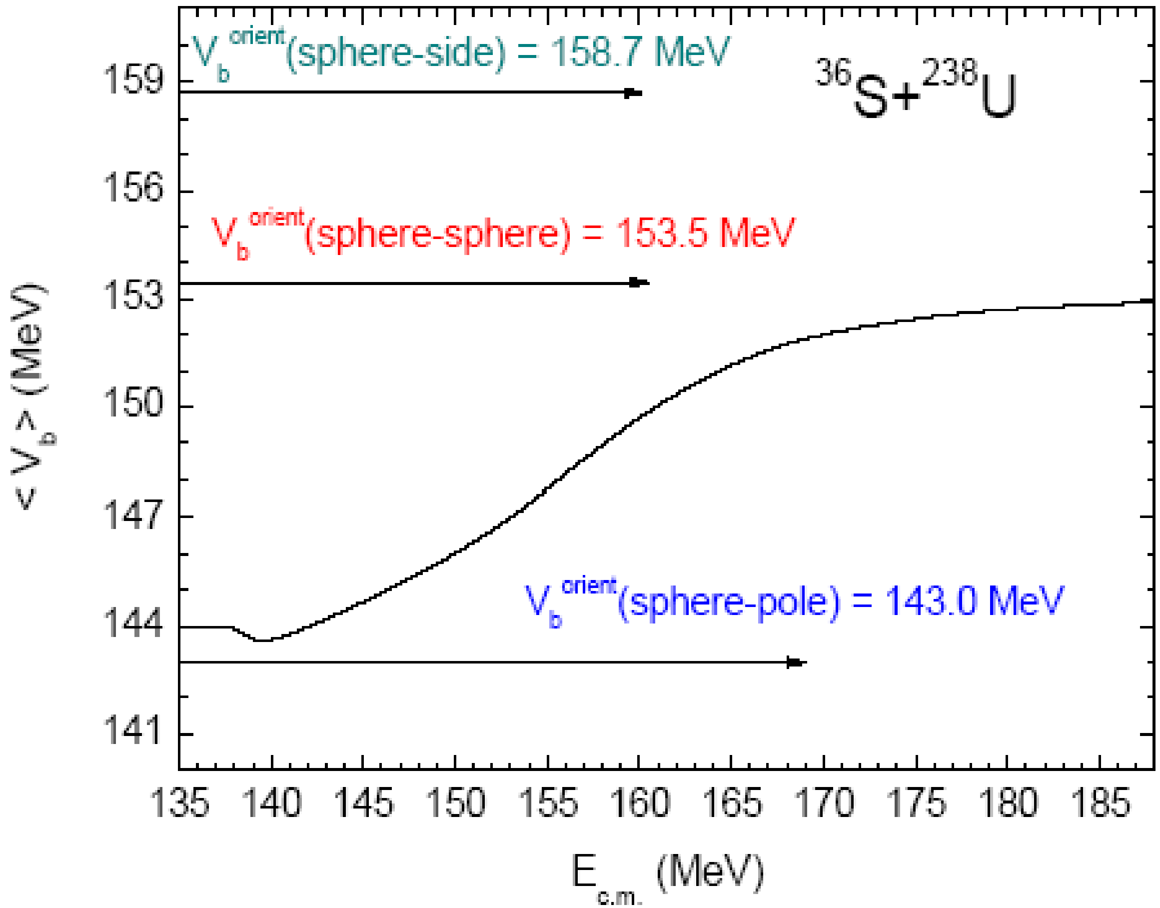
The energy increment should be at least 0.2 MeV to extract a function $L(E_{c.m.})$ with a very narrow maximum. The fusion barrier distribution calculated with this small energy increment has only one maximum. Using a larger energy increment of 0.6 MeV, one can get few oscillations in $d^2(E_{c.m.})/dE^2_{c.m.}$!

Orientation effect in capture



The capture cross section as a function of $E_{c.m.}$-V^{orient} is practically the same for all orientations !

The averaged Coulomb barrier



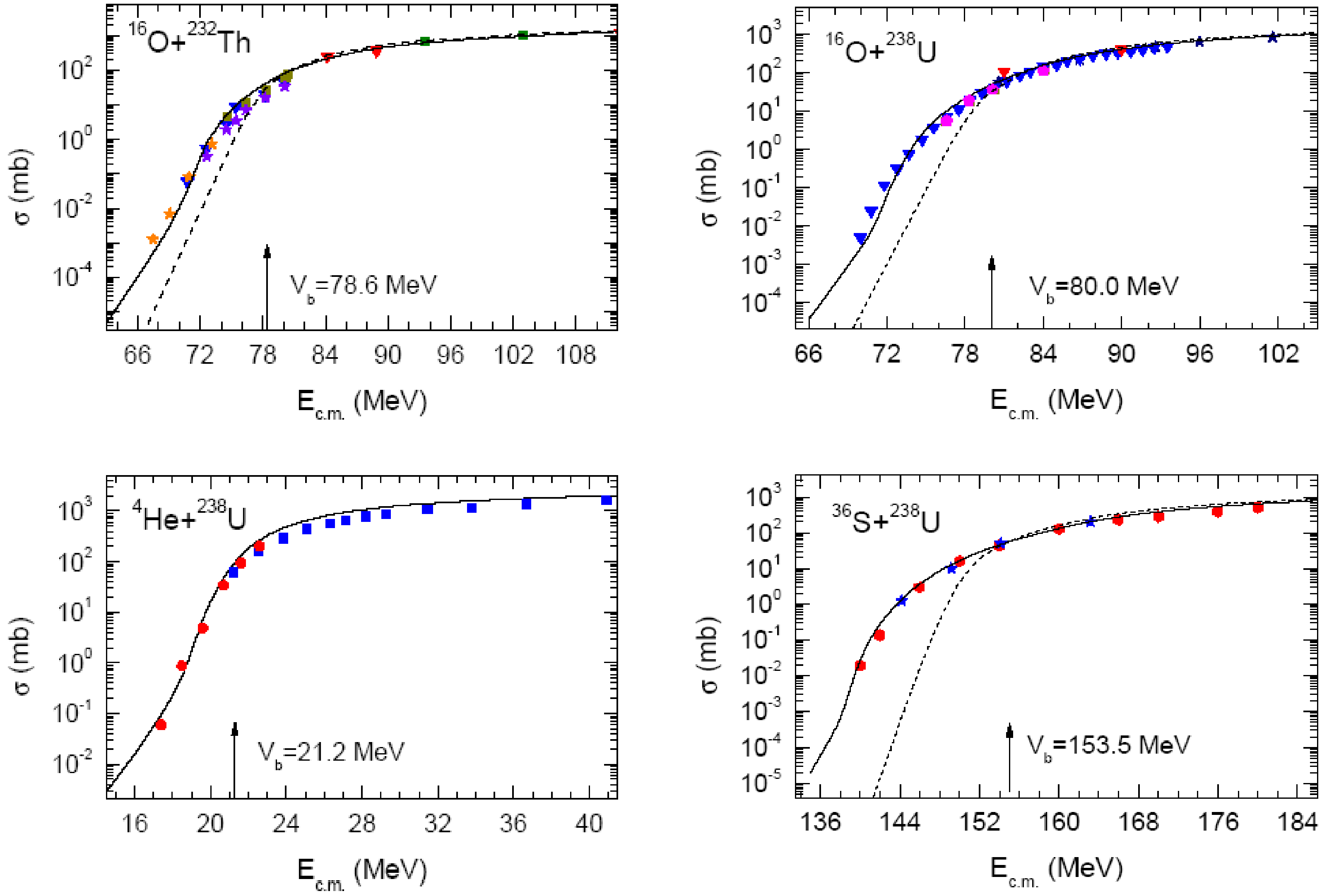
The calculated value *<V>* of averaged over the orientations of heavy deformed nucleus versus $E_{c.m.}$

With increasing of the $E_{c.m}$ the value of barrier $\langle V_b \rangle$ approaches to the value of the Coulomb barrier for the spherical nuclei.

The influence of deformation on the capture cross section is very weak already at bombarding energies about 15 MeV above the Coulomb barrier for the spherical nuclei!

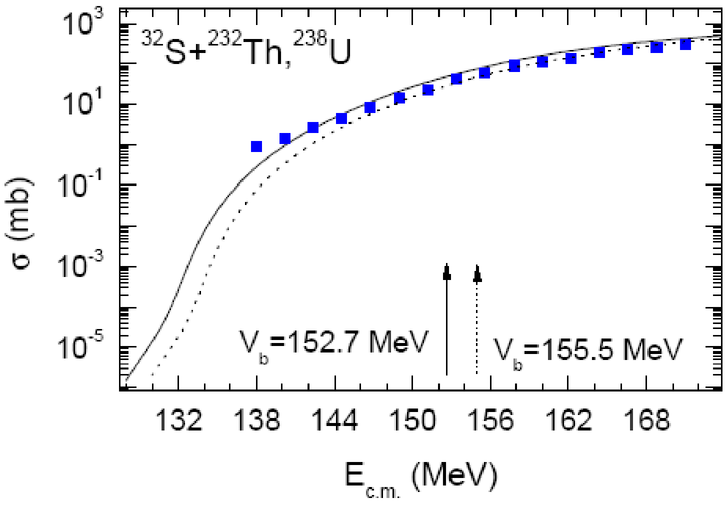
$$\langle V_b \rangle = \frac{\pi \lambda\!\!\!^{-2}}{\sigma_{cap}(E_{c.m.})} \sum_J (2J+1) \int_{-\infty}^{\pi/2} d\theta_2 \sin(\theta_2) P_{cap}(E_{c.m.}, J, \theta_1, \theta_2) V(R_b, Z_i, A_i, \theta_i, J)$$

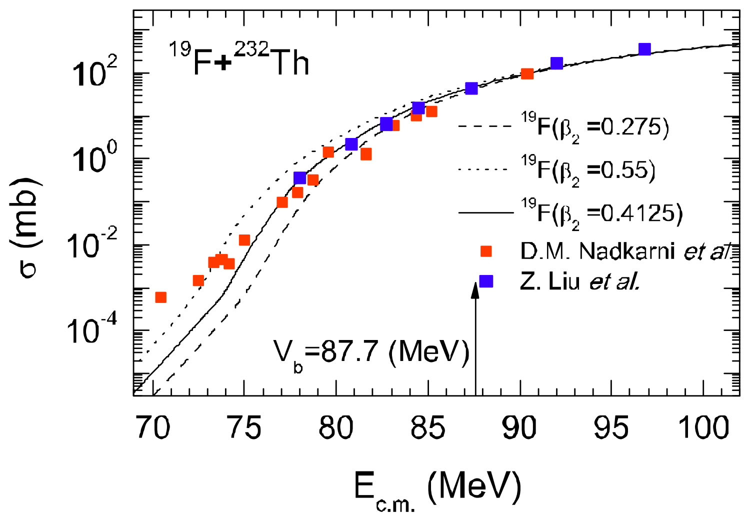
Actinide-based reactions

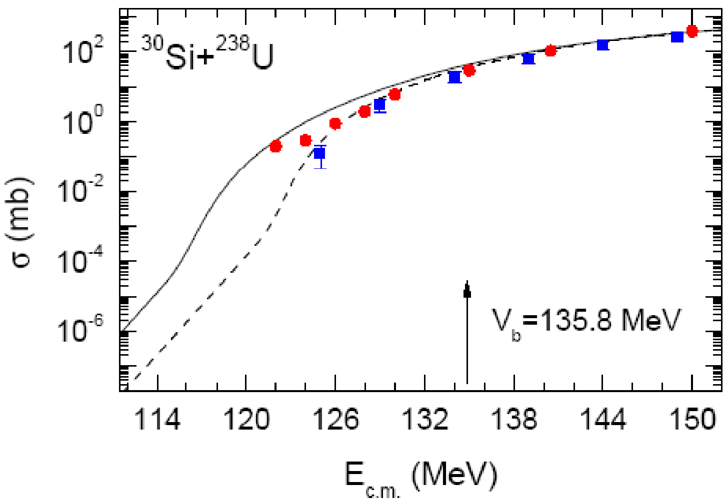


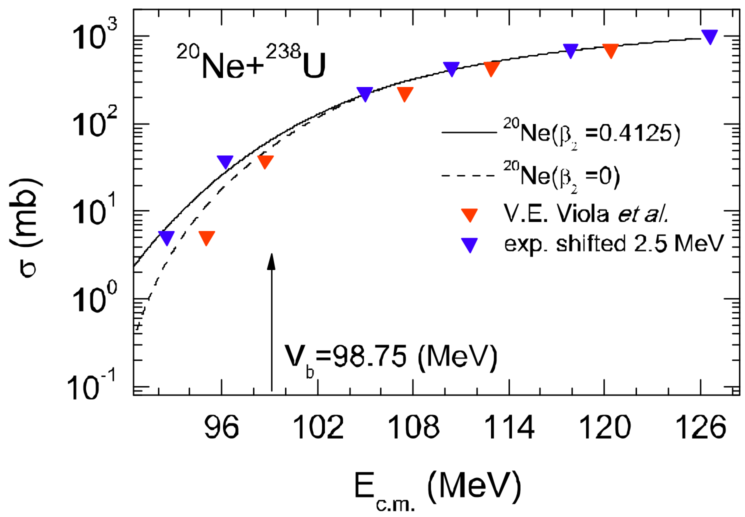
The calculated capture cross sections are in a rather good agreement with the available experimental data !!

Actinide-based reactions



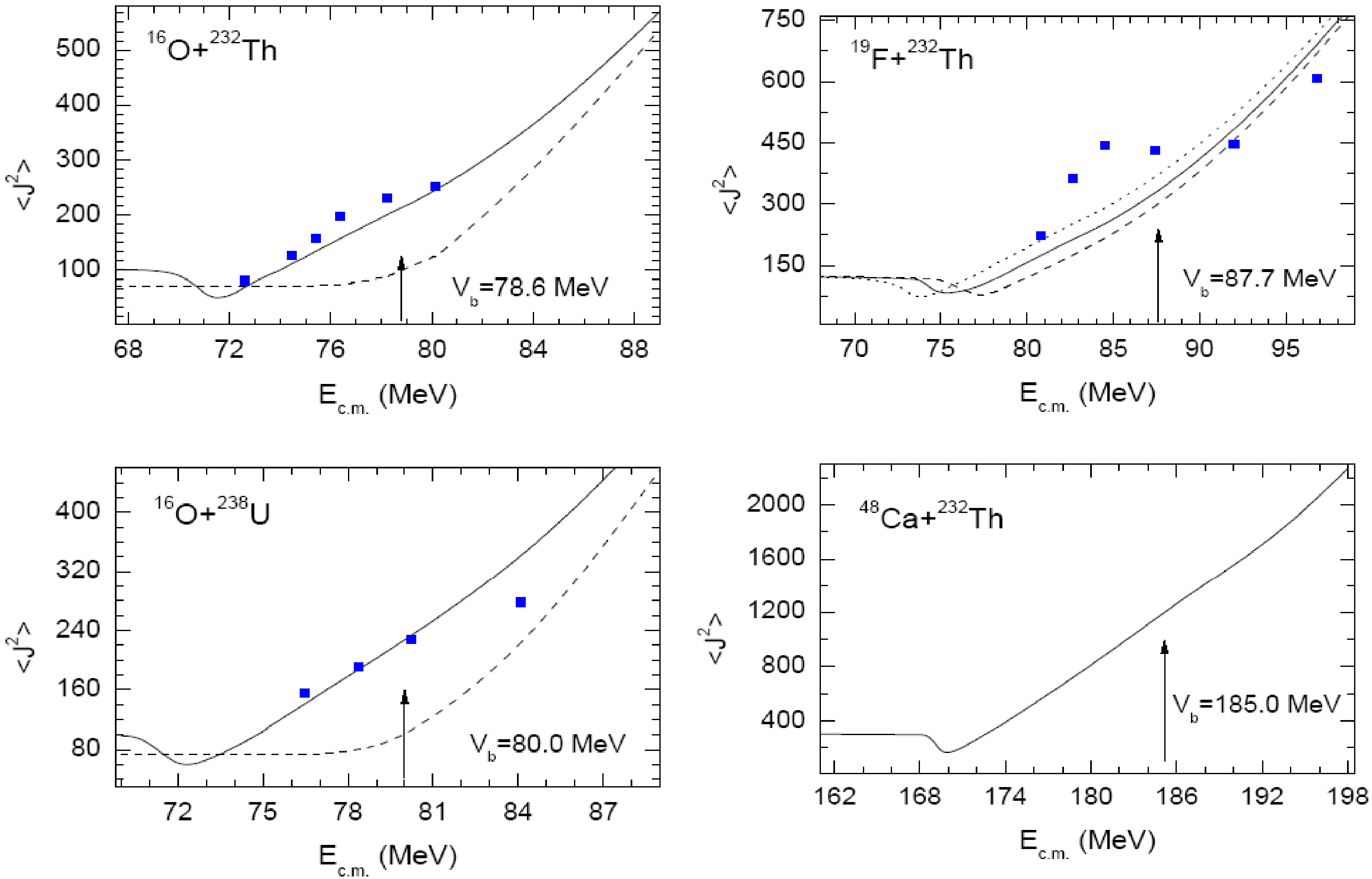






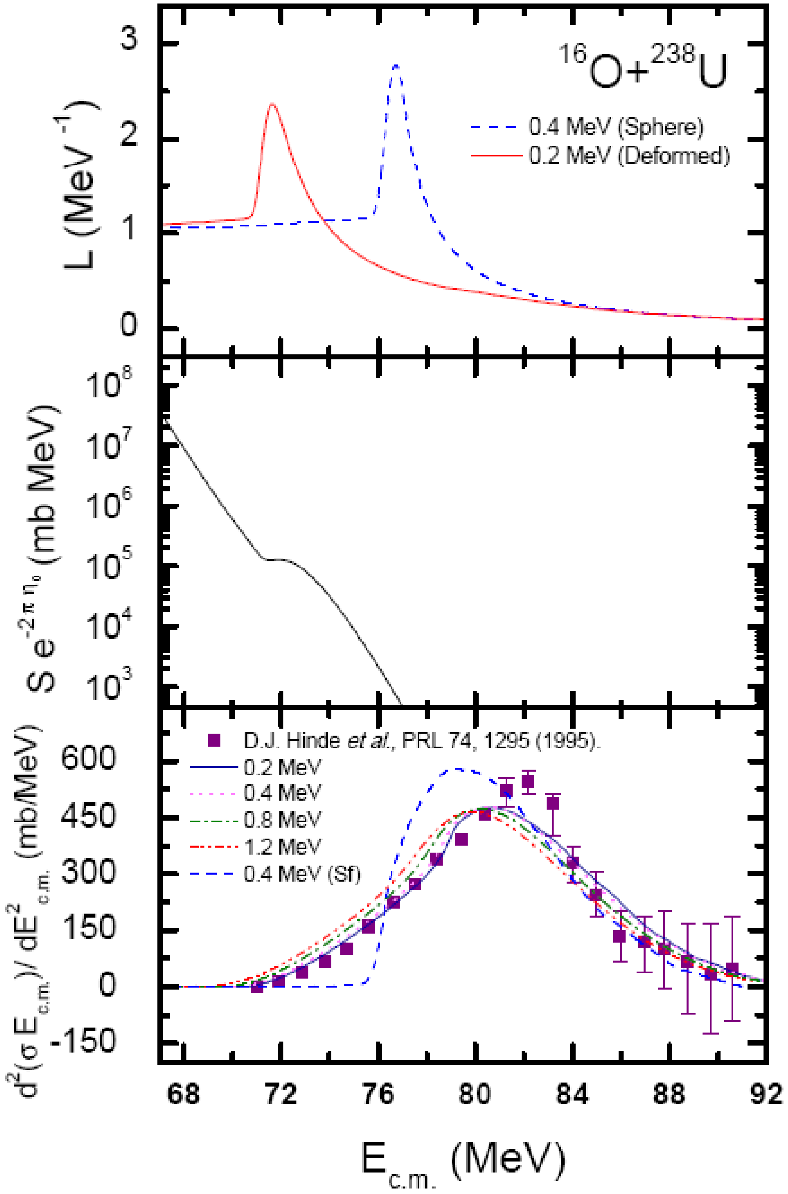
We are not able to reproduce the experimental data for the reaction ^{19}F+^{232}Th at $E_{c.m.} < 74$ MeV, even by varying static quadrupole deformation parameters.

Actinide-based reactions



At energies below the barrier $\langle J^2 \rangle$ has a minimum. This minimum depends on the deformations of nuclei and on the $Z_1 \times Z_2$.The experimental data indicate the presence ofthe minimum as well.

Fusion barrier distribution



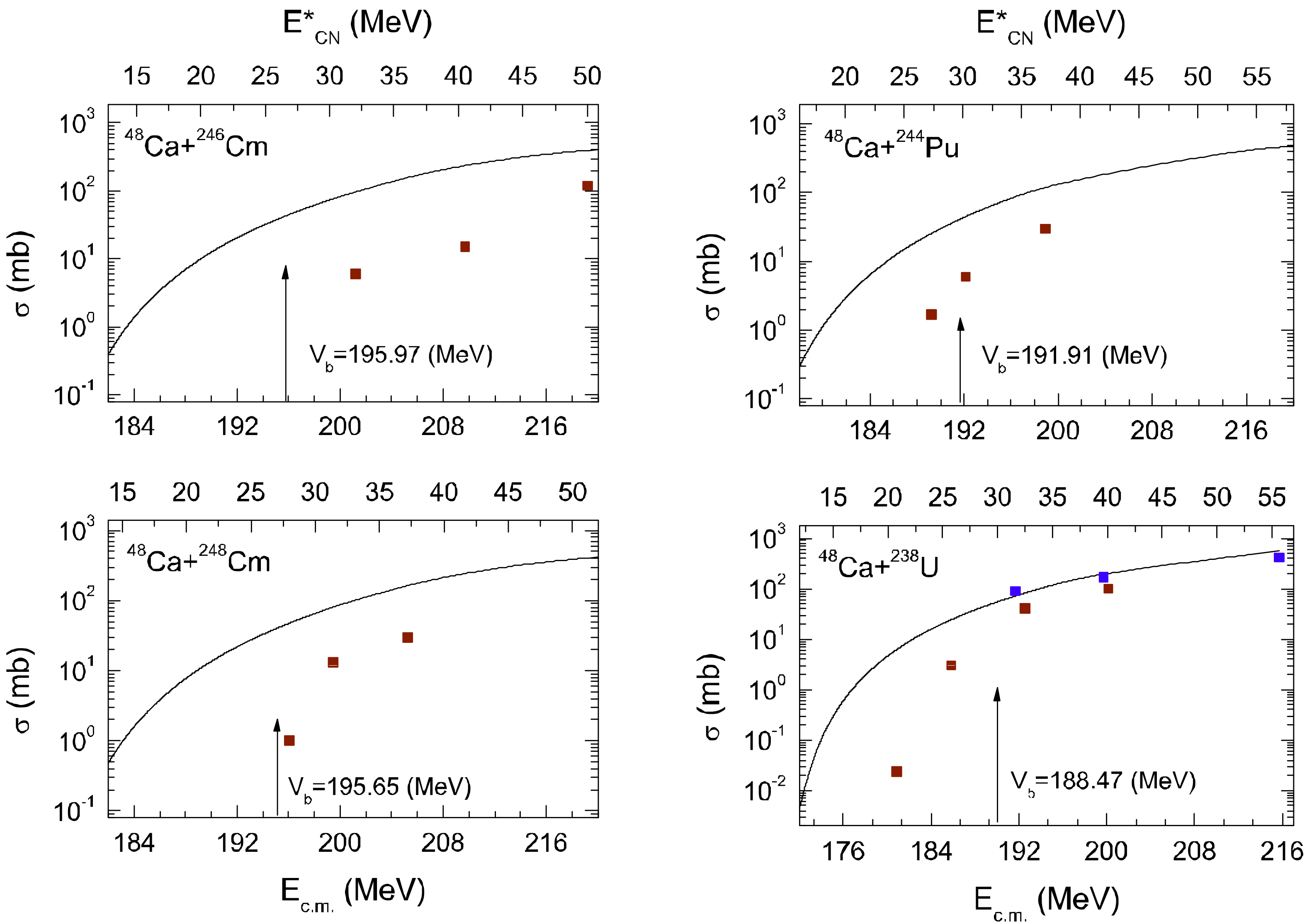
The calculated values of logarithmic derivative L (upper part), astrophysical *S*- factor (middle part) and fusion barrier distribution (lower part) for the $^{16}O+^{238}U$ reaction.

The logarithmic derivative strongly increases below the barrier and then has a maximum. The value of *L* shows a growth at $E_{c.m.}$ corresponding to the maximum of *S*-factor*.*

The barrier distributions calculated with the energy increment 0.2 MeV have only one maximum as in the experiment!!

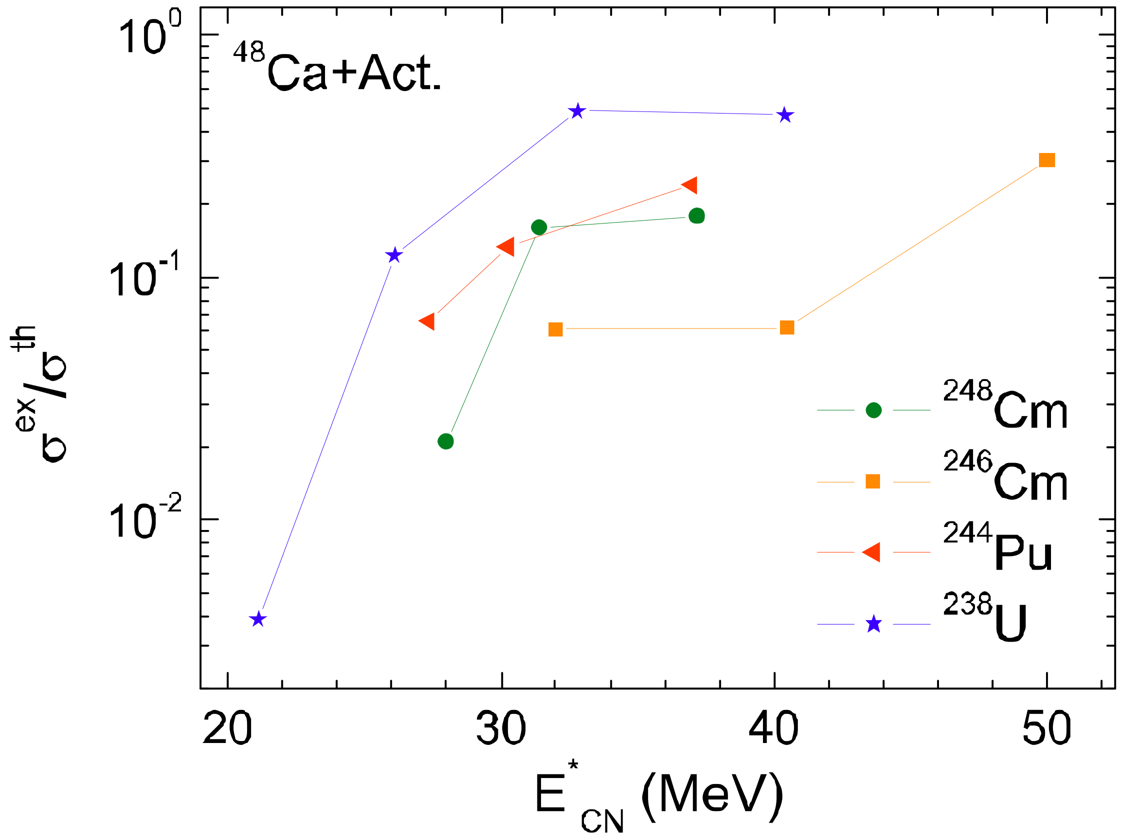
With increasing increment the barrier distribution is shifted to lower energies. Assuming the spherical target nucleus in calculations, we obtain more narrow barrier distribution.

Actinide-based reactions used in the synthesis of superheavies



The role of quasifission

The ratio of σ^{th}/σ^{ex} experimental and theoretical capture cross sections in the reactions with Ca projectile.

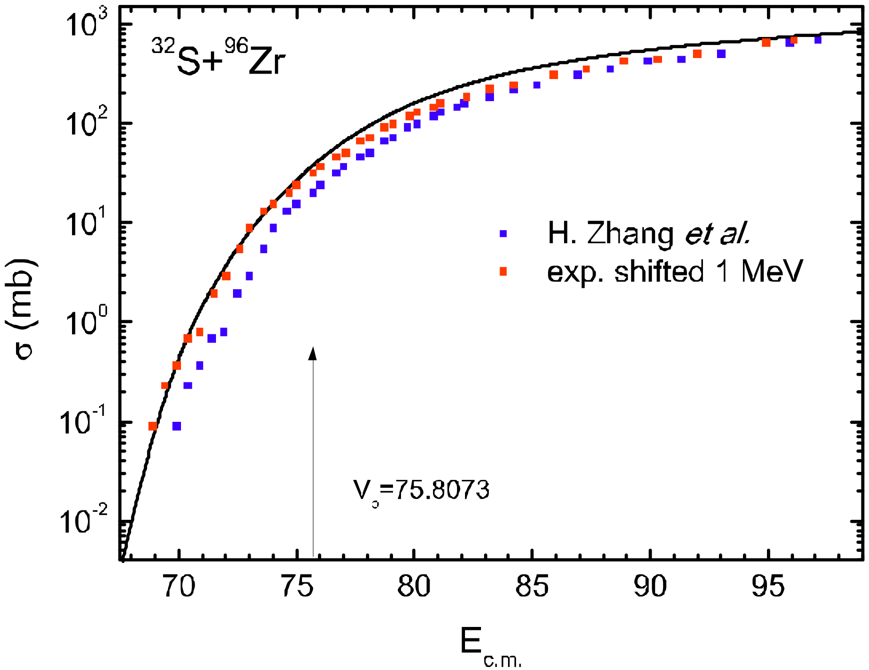


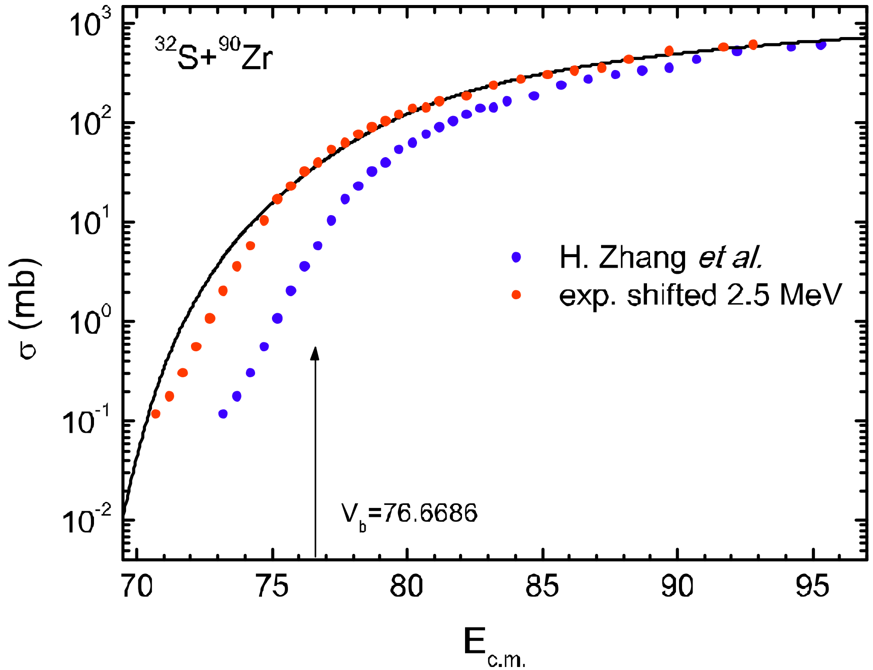
With increasing of excitation energy (fission cross section), the ratio tend to 1.

The contribution of quasifission near the entrance channel does not correctly taken into account in the experimental capture cross section !

The error bars in energy scale are quite large !

Reactions measured recently by *H. Zhang et al.*



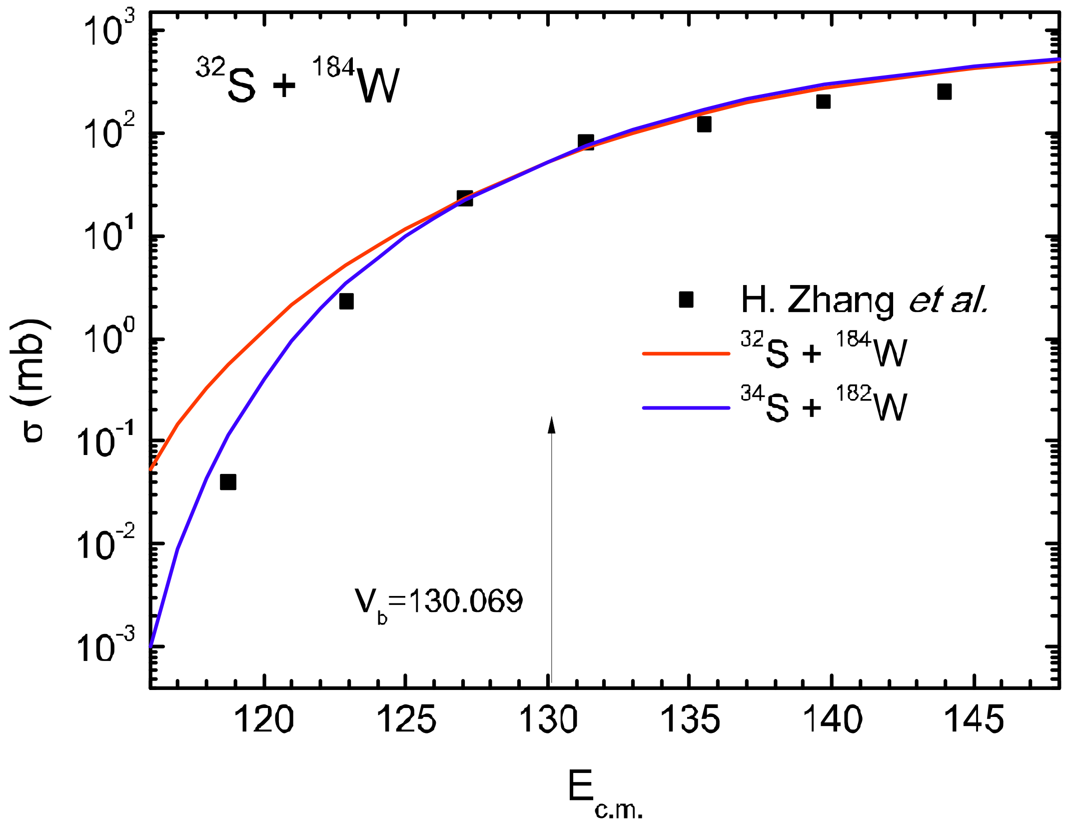


After shifting the energy scale we reproduce the slope of a the curve and the experimental data!

Possible reasons of disagreement of theoretical calculation and experimental data:

1. Correct collibration ?
2. Transition coefficient ?
3. The purity of the target ?

Reactions measured recently by *H. Zhang et al.*



After the substitution of reactions

$^{32}S+^{184}W \rightarrow ^{34}S+^{182}W$

In the entrance channel, then we have good agreement with experimental data !

Is there any neutron transfer before capture stage ?

Two arguments of neutron transfer:

1. The positive Q-value !
2. Correct reproduction of experimental data !

Conclusions

- The quantum diffusion approach has been applied to study the capture or fusion cross sections at sub-barrier energies. The available experimental data at energies above and below the Coulomb barrier are well described.
- Due to the change of the regime of interaction (the turning-off the nuclear forces and friction) at sub-barrier energies, the decrease rate of the cross sections is changed below the barrier.
- The average angular momentum of compound nucleus versus $E_{c.m.}$ would have a minimum and then saturation at sub-barrier energies. One can suggest the experiment.
- The importance of quasifission near the entrance channel has been shown for actinide-based reactions leading to the superheavies.