FORMATION OF STRONGLY DEFORMED NUCLEAR STATES

A.S. Zubov

Joint Institute for Nuclear Research, 141980 Dubna, Russia

Kavli Institute fot Theoretical Physics China, CAS,

Bejing, China, October 26, 2010

Co-authors

V.V. Sargsyan,

G.G. Adamian,

N.V. Antonenko,

Joint Institute for Nuclear Research, 141980 Dubna, Russia,

and

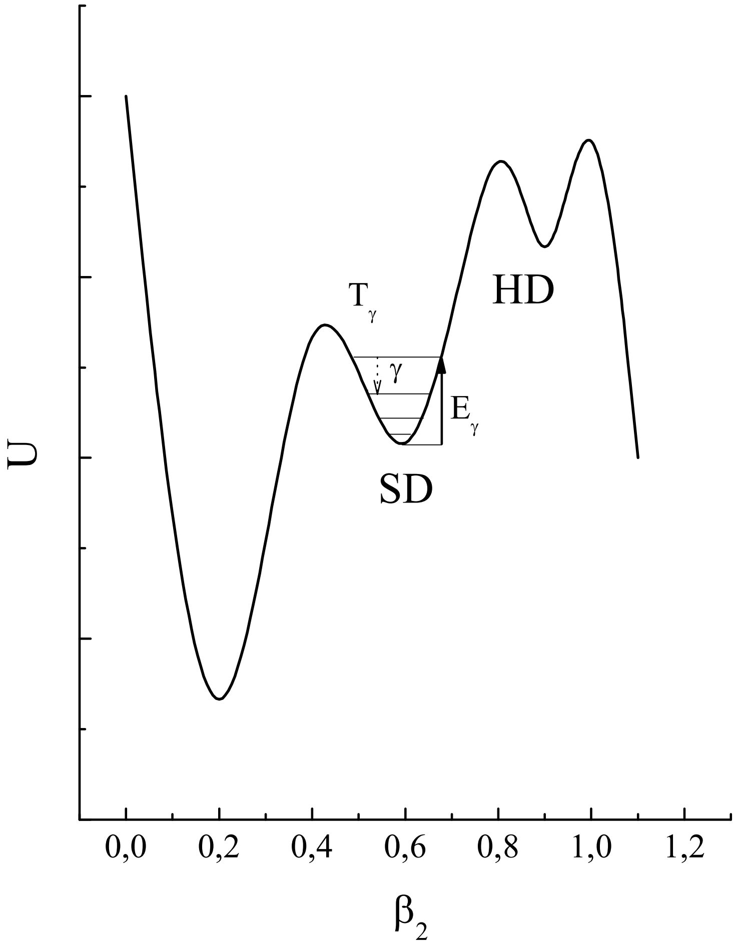
W. Scheid,

Institut für Theoretische Physik der Justus–Liebig–Universität, D–35392 Giessen, Germany

Plan of the talk

- Introduction. Cluster interpretation of superdeformed (SD) and hyperdeformed (HD) states.
- Formation of HD states by neutron emission from dinuclear system (DNS).
  - Model.
  - Potential energy of DNS.
  - Neutron emission.
  - Method of identification of HD band.
  - Results of calculations.
- Formation of HD states in the entrance channel at sub-barrier energies.
  - Model.
  - Results of calculations.
- Summary.
- Outlook.

Hyperdeformed (HD) and superdeformed (SD) nuclear states



Experimental identification.

1) Spectroscopy of E2-transitions.

$$E_\gamma(L) = L(L+1)/(2\Im),$$
$$T_\gamma(L) = \frac{408.1}{5/(16\pi)(Q_2^{(c)})^2(E_\gamma(L\to L-2))^5}.$$

L, $\Im$, and $Q^{(c)}$ can be obtained from the spectroscopy.

2) Analysis of resonances in fission and elastic scattering.

Hyperdeformed (HD) and superdeformed (SD) nuclear states

Experimental observation of HD states:

(t, pf) and (d, pf) reactions for actinides (PRL 28, 1707 (1972); NPA 502, 271 (1989); PLB 461, 15 (1999); PRL 80, 2073 (1998)).

Experimental observation of SD states:

1) A=150 region. ^{108}Pd(^{48}Ca,$4n$)^{152}Dy, ^{124}Sn(^{30}Si,$5n$)^{149}Gd, etc. (PRL 57, 811 (1986); PRL 60, 503 (1988)).

2) A=190 region. ^{160}Gd(^{36}S,$5n$)^{191}Hg, ^{173}Yb(^{24}Mg,$5n$)^{192}Pb, etc. (PRL 63, 360 (1989); Z. Phys. A338, 469 (1991)).

3) Light $N = Z$ nuclei. ^{24}Mg(^{20}Ne,2α)^{36}Ar, ^{28}Si(^{20}Ne,2α)^{40}Ca, etc. (PRL 85, 2693 (2000), PRC 67, 041303(R) (2003)).

Cluster interpretation of HD and SD states

TCSM calculations – S. Cwiok *et al.,* Phys. Lett. B 322, 304 (1994):
HD state of ^{232}Th is interpreted as cluster configuration of ^{132}Sn and ^{100}Mo.
DNS model calculations – T.M. Shneidman *et al.,* Nucl. Phys. A 671, 119 (2000):
The moment of inertia of DNS in the sticking limit:

$$\Im = (\Im_1 + \Im_2 + \mu R^2),$$

$\Im_i = \frac{1}{5} m_0 A_i (a_i^2 + b_i^2)$ – rigid body approximation, where

$$a_i = R_{0i}(1 - \frac{\beta_i^2}{4\pi})(1 + \sqrt{\frac{5}{4\pi}}\beta_i),\ b_i = R_{0i}(1 - \frac{\beta_i^2}{4\pi})(1 - \sqrt{\frac{5}{16\pi}}\beta_i).$$

The charge multipole moment

$$Q_{\lambda\mu}^{(c)} = \sqrt{\frac{16\pi}{2\lambda+1}} \int \rho^{(c)}(\mathbf{r}) r^{\lambda} Y_{\lambda\mu}(\Omega) d\mathbf{r}.$$

Taking the total charge density $\rho^{(c)}(\mathbf{r}) = \rho_1^{(c)}(\mathbf{r}) + \rho_2^{(c)}(\mathbf{r})$, the electric quadrupole moment of DNS:

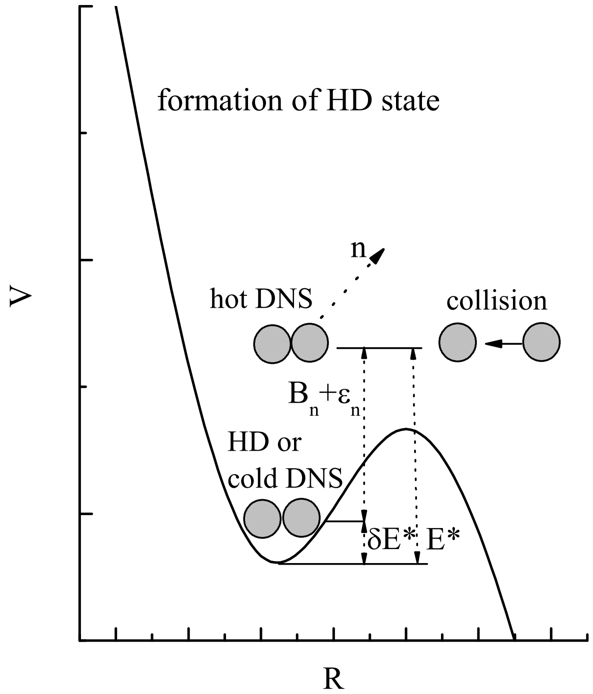
$$Q_2^{(c)} = 2e\frac{A_2^2 Z_1 + A_1^2 Z_2}{A^2} R^2 + Q_2^{(c)}(1) + Q_2^{(c)}(2),$$

where $Q_2^{(c)}(i)$ are quadrupole moments of DNS nuclei.

Results for ^{232}Th, ^{234}U, ^{240}Pu (HD) and ^{152}Dy, ^{149}Gd (SD) are in a good agreement with the experiment.

Formation of HD states by neutron emission from dinuclear system (DNS)

Model



HD state can be treated as configuration of two touching nuclei, cold dinuclear system (DNS), in local potential minimum.

HD state formation cross section:

$$\sigma_{\rm HD}(E_{\rm c.m.}) = \sum_{L=L_{min}}^{L_{max}} \frac{\pi\hbar^2}{2\mu E_{\rm c.m.}} \times (2L+1) P_{\rm cap}(E_{\rm c.m.}, L) P_{\rm HD}(E_{\rm c.m.}, L).$$

The probability of formation of the HD state by neutron emission from the excited initial DNS:

$$P_{\rm HD}(E_{\rm c.m.}, L) = \sum_{k=1}^{2} P_{n_k}(E^*, B_{n_k}, L) w_{n_k}(E^*, B_{n_k}, \delta E^*),$$

where P_{n_k} is the probability of neutron emission from DNS, w_{n_k} is the probability to emit the neutron with a kinetic energies in the interval from $E^* - B_{n_k} - \delta E^*$ to $E^* - B_{n_k}$ and to cool the excited DNS to $E^*_{\rm HD} < \delta E^*$.

Capture probability

The formalism of reduced density matrix is used for the calculation of $P_{\rm cap}$. One can visualize a capture as a process in which a part of the initial Gaussian wave packet populates the nucleus-nucleus potential pocket behind the Coulomb barrier. By solving the quantum master equation

$$\frac{d}{dt}\rho=-\frac{i}{\hbar}[H_c,\rho]-\frac{i\lambda_p}{2\hbar}[R,\{P,\rho\}_+]-\frac{D_{PP}}{\hbar^2}[R,[R,\rho]]+\frac{D_{RP}}{\hbar^2}[P,[R,\rho]]+[R,[P,\rho]]\frac{D_{RP}}{\hbar^2}$$

for the R degree of freedom, we find the diagonal elements $\rho(t,R)$ of the reduced density matrix in the coordinate representation. $H_c=\frac{1}{2\mu}P^2+V$, D_{PP}, D_{RP} and λ_p are diffusion and friction coefficients. The capture probability is defined with the ratio

$$P_{\rm cap}(E_{\rm c.m.},L)=\frac{\int_{-\infty}^{R_b}\rho(\tau,R)dR}{\int_{R_b}^{\infty}\rho(t{=}0,R)dR},$$

where R_b defines the position of the Coulomb barrier, and the projectile is assumed to approach the target from the right side. The value of τ determines the time of capture.

Details of calculation can be found in V.V. Sargsyan *et al.,* Phys. Rev. C 80, 034606 (2009).

Potential energy of DNS

Collective coordinates: the relative distance R, the mass (charge) asymmetry coordinate $\eta = (A_1 - A_2)/(A_1 + A_2)$ ($\eta_Z = (Z_1 - Z_2)/(Z_1 + Z_2)$).

The potential energy of DNS:

$U(R, \eta, \eta_Z, \beta_1, \beta_2, L) = B_1 + B_2 + V(R, \eta, \eta_Z, \beta_1, \beta_2, L)$,

where B_1 and B_2 are the mass excesses of the fragments at their ground states.

Nucleus-nucleus potential

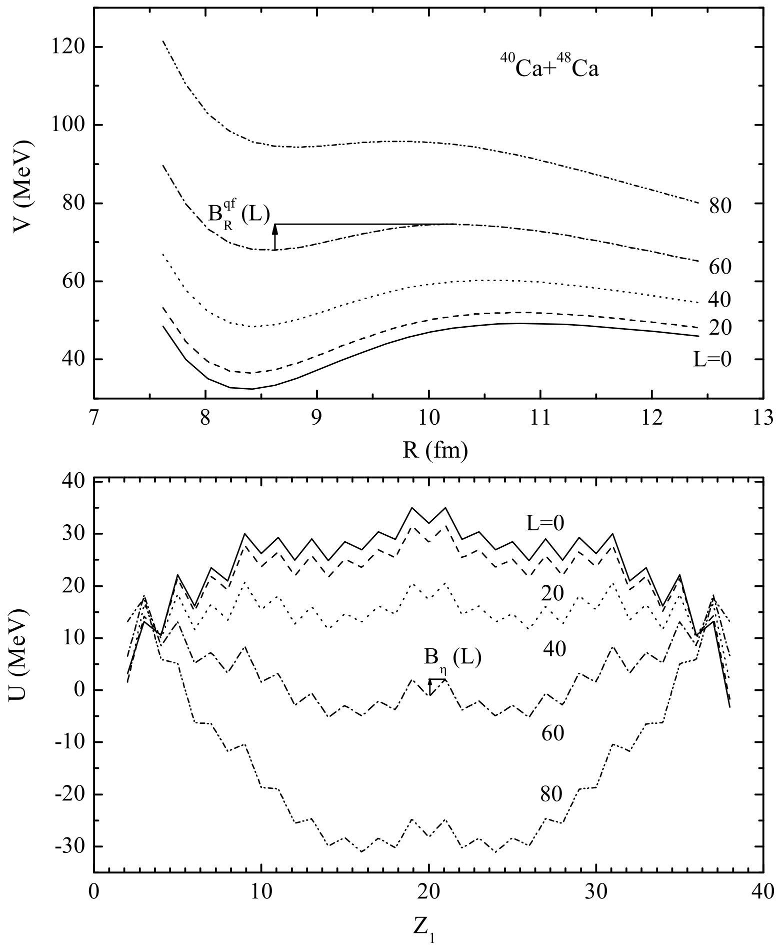
$V(R, \eta, \eta_Z, \beta_1, \beta_2, L) = V_C(R, \eta_Z, \beta_1, \beta_2) + V_N(R, \eta, \beta_1, \beta_2) + V_{rot}(\eta, \beta_1, \beta_2, L)$,

is the sum of the Coulomb potential V_C, the nuclear potential V_N, and the centrifugal potential

$V_{rot} = \hbar^2 L(L+1)/(2\Im)$ – for the trapped DNS,

$V_{rot} = \hbar^2 L(L+1)/(2\mu R^2)$ – before capture.

Potential energy of DNS



The excitation energy of initial DNS:

$$E^* = E_{\mathrm{c.m.}} - V(R_m, \eta_0, \eta_{Z_0}, \beta_1, \beta_2, L).$$

The evolution of the initial excited DNS is prescribed by the competition between the neutron emission from the system and the DNS transition over the quasifission barrier B_R^{qf} in R or over the barriers $B_{\eta_Z}^{sym}$ and $B_{\eta_Z}^{asym}$ in η_Z, in the direction to more symmetric and more asymmetric configurations.

Neutron emission

The probability of formation of the HD state by neutron emission from the excited initial DNS is

$$P_{n_k} = \frac{\Gamma_{n_k}(E^*,\eta_{Z_0},L)}{\Gamma_n(E^*,\eta_{Z_0},L)+\Gamma_R^{qf}(E^*,\eta_{Z_0},L)+\Gamma_{\eta_Z}^{sym}(E^*,\eta_{Z_0},L)+\Gamma_{\eta_Z}^{asym}(E^*,\eta_{Z_0},L)}.$$

The widths of different processes are calculated using the statistical approach (Zubov *et al.*, Eur. Phys. J. A 33, 223 (2007)).

DNS level density:

$$\rho_{DNS}(E^*,A_1,A_2,J_1,J_2) = \int_0^{E^*} \rho_1(E_1,A_1,J_1)\rho_2(E^*-E_1,A_2,J_2)dE_1.$$

Probability of transition through barriers B_i ($i = R, \eta_Z$, $j =$qf, sym, asym):

$$R_i^j(E^*,\eta_{Z_0},L) = \int_0^{E^*-B_i^j(L)} \frac{\rho_{DNS}(E^*-B_i^j(L)-\epsilon,A_1,A_2,J_1,J_2)d\epsilon}{1+\exp[2\pi(\epsilon+B_i^j(L)-E^*)/(\hbar\omega_i^j(L))]}.$$

The corresponding widths:

$$\Gamma_i^j(E^*,\eta_{Z_0},L) = \frac{R_i^j(E^*,\eta_{Z_0},L)}{2\pi\rho_{DNS}(E^*,A_1,A_2,J_1,J_2)}.$$

Neutron emission

The probability of neutron emission

$$R_{n_k}(E^*,\eta_{Z_0},L)=\sum_{J_k^d}\int\limits_0^{E^*-B_{n_k}}\rho_{DNS}(E^*-B_{n_k}-\epsilon,A_k-1,A_{k'},J_k^d,J_{k'})T_{J_k^d}(A_k-1,\epsilon)d\epsilon,$$

where J_k^d is the spin of "k"-th nucleus of the DNS after emission of neutron, $k,k'=1,2$.

Neutron emission width:

$$\Gamma_{n_k}(E^*,\eta_{Z_0},L)=\frac{R_{n_k}(E^*,\eta_{Z_0},L)}{2\pi\rho_{DNS}(E^*,A_1,A_2,J_1,J_2)}.$$

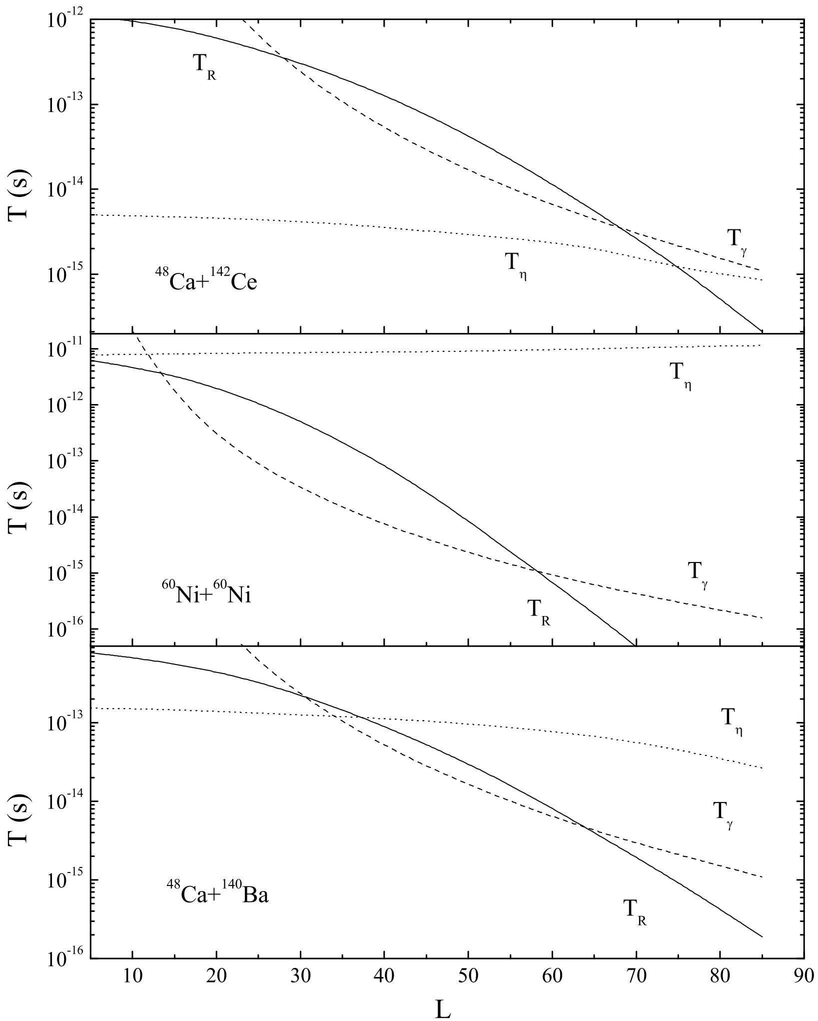
Using the Maxwellian form of neutron spectrum, the probability and to cool the excited DNS to $E^*_{\rm HD}<\delta E^*$ is

$$w_{n_k}(E^*,B_{n_k},\delta E^*)=\frac{\int\limits_{E^*-B_{n_k}-\delta E^*}^{E^*-B_{n_k}}\epsilon_n\exp[-\epsilon_n/T_{n_k}(E^*)]/[T_{n_k}(E^*)]^2d\epsilon_n}{\int\limits_0^{E^*-B_{n_k}}\epsilon_n\exp[-\epsilon_n/T_{n_k}(E^*)]/[T_{n_k}(E^*)]^2d\epsilon_n},$$

where $T_{n_k}(E^*)$ is nuclear temperature, $\delta E^*=0.2$ MeV.

Method of identification of HD band

Measuring rotational $\gamma-$quanta in coincidence with decay fragments of the dinuclear system.



Condition to the range of L:

$$T_\gamma(L) \lesssim T_R(L) \lesssim T_\eta(L).$$

The tunneling times through the barrier in R and η_Z ($i = R, \eta_Z$):

$$T_i = \frac{2\pi}{\Omega_i^i}(1 + \exp[2\pi B_i^i/(\hbar\omega_i^i)]),$$

Energy and time of collective $E2-$transition between the rotational states with angular momenta L and $L-2$:

$$E_\gamma(L \to L-2) = L(L+1)/(2\Im) - (L-2)(L-1)/(2\Im),$$

$$T_\gamma(L) = \frac{408.1}{5/(16\pi)(Q_2^{(c)})^2(E_\gamma(L\to L-2))^5},$$

where E_γ is in units of keV, $Q_2^{(c)}$ in $10^2(e$ fm$^2)$ and T_γ in s.

HD state identification cross section

The probability of the emission of x γ-quanta from the HD state just before its decay in R

$$P_{x\gamma R} = \frac{\Lambda_R(L-2x)}{\Lambda_{tot}(L-2x)} \prod_{k=0}^{x-1} \frac{\Lambda_\gamma(L-2k)}{\Lambda_{tot}(L-2k)},$$

where $\Lambda_{\gamma,R,\eta_Z} = \hbar/T_{\gamma,R,\eta_Z}$ are the rates of different competing processes, $\Lambda_{tot} = \Lambda_\gamma + \Lambda_R + \Lambda_{\eta_Z}$.

Then the cross section of emission of at least x γ-quanta(um) from the HD state before its decay in R (*HD state identification cross section*)

$$\sigma_{x\gamma R} = \sum_{L=L_{min}}^{L_{max}} \sigma_{\rm HD}(E_{\rm c.m.}, L) \sum_{x'=x}^{[L/2]} P_{x'\gamma R}.$$

The cross section of the HD state decay in R without γ-emission (x=0)

$$\sigma_{0\gamma R} = \sum_{L=L_{min}}^{L_{max}} \sigma_{\rm HD}(E_{\rm c.m.}, L) \frac{\Lambda_R(L)}{\Lambda_{tot}(L)}.$$

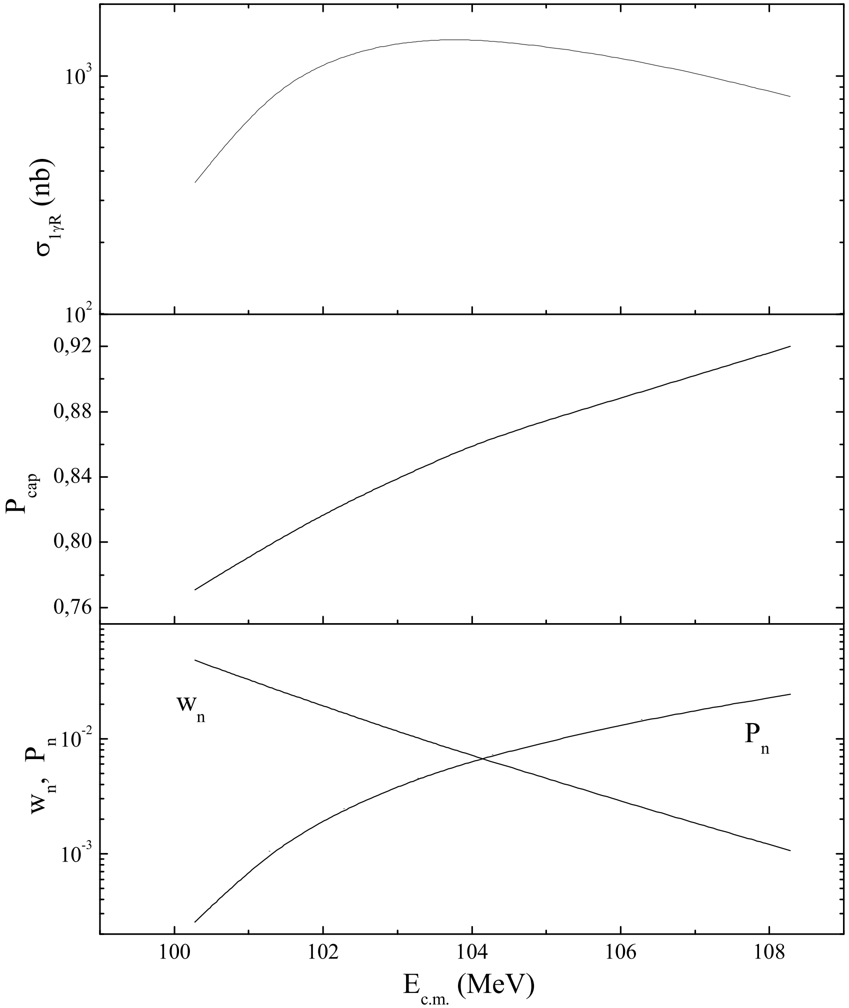
Results of calculation

Optimal reactions for the formation and identification of HD state.

- In the entrance channel of the reaction the DNS should have a local potential minimum which is populated, and after neutron emission a cold quasibound state, treated here as hyperdeformed, is formed.
- There should exist the range of angular momenta satisfying the time conditions.
- The neutron-rich projectile and targets are more preferable to increase the probability of neutron emission from the initial DNS. However, only those of these nuclei are considered which can be experimentally accelerated with quite a large intensity.
- The estimated identification cross section of the HD state should be suitable for the present experimental setups.

The reactions ^{48}Ca+^{124,128,130,132,134}Sn, ^{48}Ca+^{136,138}Xe, ^{48}Ca+^{137,138,140}Ba, ^{40}Ca+^{83,84}Kr, ^{48}Ca+^{83,84,86}Kr, ^{40,48}Ca+^{40,48}Ca, ^{58,60}Ni+^{58,60}Ni, and ^{40}Ca+^{58}Ni are suggested for the population and identification of hyperdeformed states.

Results of calculation



^{60}Ni+^{60}Ni, L=30–40.

The maximum of the dependence $\sigma_{1\gamma R}(E_{\rm c.m.})$ corresponds to the optimal bombarding energy.

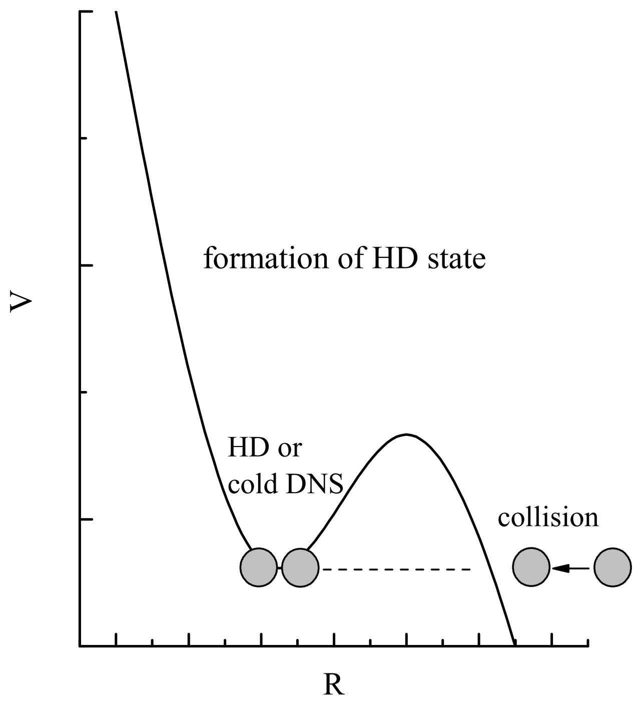
Results of calculation

Reactions	$\Im$ $\hbar^2$/MeV	$Q_2^{(c)}$ $10^2(e$ fm$^2)$	L	σ_{HD} nb	$\sigma_{1\gamma R}$ nb	$\sigma_{2\gamma R}$ nb
^{48}Ca+^{83}Kr	90.6	28.4	$60 < L < 70$	78	1.1	0.5
			$70 < L < 80$	32	2.4	1.3
^{40}Ca+^{48}Ca	44.6	14.8	$50 < L < 60$	3.4$\times 10^3$	200	72
			$60 < L < 70$	1.5$\times 10^3$	40	24
^{60}Ni+^{60}Ni	89.3	32.9	$20 < L < 30$	2.2$\times 10^3$	1.4$\times 10^3$	1.2$\times 10^3$
			$30 < L < 40$	2$\times 10^3$	1.4$\times 10^3$	1.4$\times 10^3$
			$40 < L < 50$	1$\times 10^3$	770	650
			$50 < L < 60$	510	280	180
^{48}Ca+^{134}Sn	129.8	33.8	$50 < L < 60$	13	4.9	4.4
			$60 < L < 70$	5.6	2.7	2.3
			$70 < L < 80$	2.1	1.3	0.97

In the reaction ^{60}Ni+^{60}Ni at $L = 40 - 50$, $\sigma_{10\gamma R} = 260$ nb.

Formation of HD states in the entrance channel at sub-barrier energies

Model



HD state formation cross section
$\sigma_{\rm HD}(E_{\rm c.m.},L)=\frac{\pi\hbar^2}{2\mu E_{\rm c.m.}}(2L+1)P_{\rm cap}(E_{\rm c.m.},L)$.

The optimal bombarding energy
$E_{\rm c.m.}(L)=V(R_m,\eta,\eta_Z,\beta_1,\beta_2,L)$.

The sub-barrier tunneling is considered using the quantum diffusion approach with the formalism of reduced density matrix, developed in [V.V. Sargsyan *et al.*, submitted to Eur. Phys. J. A (2010).]

The partial identification cross section: $\sigma_{x\gamma R}(L)=\sigma_{\rm HD}(L)\sum_{x'=x}^{[L/2]}P_{x'\gamma R}$.
$B_R^{cap}=V'(R_b',\eta,\eta_Z,\beta_1,\beta_2,L)-V(R_m,\eta,\eta_Z,\beta_1,\beta_2,L)$, where
$V_{rot}=\hbar^2L(L+1)/(2\Im)$ for the trapped DNS, $V'_{rot}=\hbar^2L(L+1)/(2\mu R^2)$ before capture.

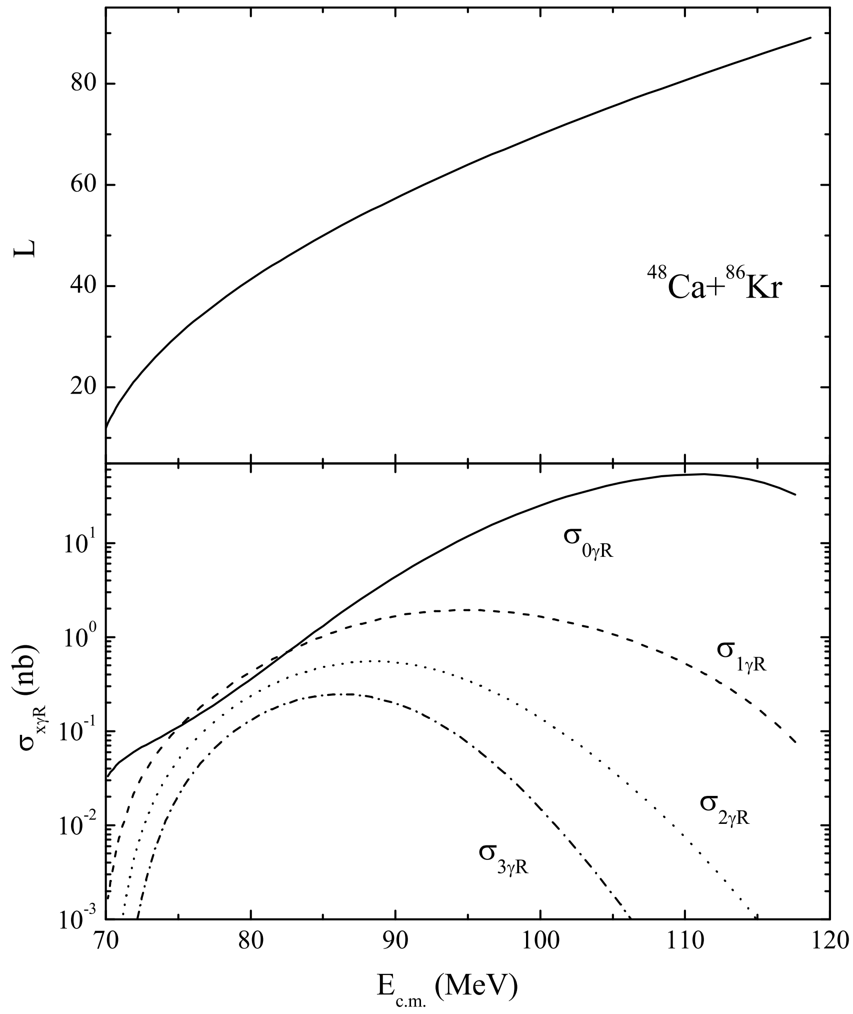
Results of calculation

Optimal reactions for the formation and identification of HD state.

- In the entrance channel of the reaction the DNS should have a local potential minimum which is populated by tunneling through the entrance barrier.
- If projectile or target are deformed, at the capture stage of the process all orientations in addition to pole-pole are possible. In this case, the DNS formed in the minimum of potential pocket with pole-pole orientation, will be excited. This can sufficiently decrease life-time of populated quasibound state. The spherical nuclei in the reactions for production and identification of HD states at sub-barrier energies are preferable.
- Additional excitation of trapped DNS (with $\eta_Z = \eta_{Z_0}$ and $\eta = \eta_0$) can also take place, if N/Z-equilibrium for initial DNS before capture (with $\eta_Z = \eta_{Z_0}$ and $\eta = \eta_{in}$) has to be reached, in the case of $\eta_0 \neq \eta_{in}$. The reactions for which the condition of N/Z-equilibrium is satisfied before capture ($\eta_0 = \eta_{in}$) are preferable.

The reactions ^{48}Ca+^{124}Sn, ^{48}Ca+^{136}Xe, ^{48}Ca+^{138}Ba, ^{48}Ca+^{140}Ce, ^{48}Ca+^{86}Kr, ^{58}Ni+^{58}Ni, ^{40}Ca+^{40}Ca, and ^{48}Ca+^{48}Ca are suggested for the population and identification of hyperdeformed states.

Results of calculation



Summary

- Using the cluster approach, we proposed a model of the HD state formation in the entrance channel of heavy-ion reaction at bombarding energies near and below the Coulomb barrier. The initial excited DNS then can be de-excited by the emission of a neutron to the cold quasibound state which is identical to the HD state. Another mechanism for the population of HD state is the direct sub-barrier tunneling.
- The neutron emission from the initial excited DNS, which competes with the quasifission and the diffusion of the initial DNS to more symmetric or asymmetric configurations, is described by using a statistical approach. Tunneling through Coulomb barrier is considered using the quantum diffusion approach with the formalism of reduced density matrix.
- One can identify the HD state by measuring the consecutive collective rotational $E2$-transitions in coincidence with the decay fragments of the DNS constituting the HD configuration.
- The optimal reactions and conditions for the identification of HD states are proposed and the HD state formation and identification cross sections are estimated.

  The details are available in the papers: A.S. Zubov, V.V. Sargsyan, G.G. Adamian, N.V. Antonenko, and W. Scheid, Phys. Rev. C 81, 024607 (2010); Phys. Rev. C 82, 034610 (2010).

Outlook

- Analysis of formation of SD states in particle evaporation reactions.
  ^{108}Pd(^{48}Ca,$4n$)^{152}Dy, ^{24}Mg(^{20}Ne,2α)^{36}Ar, etc.
  Are light particles emitted from compound nucleus, final SD cluster configuration or some intermediate cluster configuration?
- Comparing the experimental data (band intensities, population cross sections) with the predictions of our model.
- Proposal for the optimal conditions in these reactions.
- Possibility of existence of HD states in light $N = Z$ nuclei (^{36}Ar, ^{40}Ca)?