



Inverse See-saw in Supersymmetry

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See-saw is perhaps the most elegant mechanism for neutrino mass generation, n_R is well motivated from $SO(10)$ and $SU(3)_H$. but..

$$y_\nu \overline{\ell}_L n_R H_u + M_R \overline{n}_R^c n_R + M_S \overline{s}_L n_R$$

Then, in the basis of (ν_L, s_L, n_R^c)

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & M_D \\ 0 & 0 & M_S \\ M_D & M_S & M_R \end{pmatrix}$$

Lightest mass eigenstate remain massless.....

- Is there any exact chiral symmetry to protect m_ν ?
- Why is there an additional singlet? E_6 ?
- What is the Lepton number violation scale Λ_ℓ ? Can it be within weak scale?



In this talk...

Two examples that tree-level masses are suppressed but only arise radiatively.

- generate neutrino mass in a modified Wyler-Wolfenstein model from radiative corrections. (*with Seong-chan Park, hep-ph/1010.xxxx*)
- generate charged lepton masses m_e and down-type quark masses m_d radiatively from $\langle H_u \rangle$ in MSSM (*large $\tan \beta$ limit, see for example, Dobrescu-Fox, upper-lifted MSSM, hep-ph/1001.3147*)

So no unbroken chiral symmetry....



$$\langle H_u \rangle \gg \langle H_d \rangle, m_e, m_d \text{ from } \langle H_u \rangle$$

Accidental symmetries in SM lagrangian

$$i\bar{Q}_L^i \not{D} Q_L^i + i\bar{u}_R^i \not{D} u_R^i + i\bar{d}_R^i \not{D} d_R^i + \dots$$

$$Q_L^i \rightarrow U_Q^{ij} Q_L^j, \quad u_R^i \rightarrow U_u^{ij} u_R^j, \quad d_R^i \rightarrow U_d^{ij} d_R^j$$

With three generations, $U(3)_Q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$

$$-y_u^{ij} \bar{Q}_L^i \epsilon H^\dagger u_R^j - y_d^{ij} \bar{Q}_L^i H d_R^j + \dots$$

break the above $[U(3)]^5$ into $U(1)_B \times U(1)_{\text{Lep}}$

$$Q_L^i \rightarrow e^{i\theta/3} Q_L^i, \quad u_R^i \rightarrow e^{i\theta/3} u_R^i, \quad d_R^i \rightarrow e^{i\theta/3} d_R^i$$

$$\ell_L^i \rightarrow e^{i\phi} \ell_L^i, \quad e_R^i \rightarrow e^{i\phi} e_R^i$$

Fermion mass is not only a electroweak symmetry breaking (EWSB) effect .

- If $y \rightarrow 0$, $U(3)$ symmetry will be restored and the corresponding fermion will be massless up to all loops.
- m_t (or m_u) $\neq 0 \rightarrow m_d, m_e$ must break the $U(3)$ s. (for instance, topcolor model)

To eliminate the tree-level contribution, tune the vevpossible in 2HDM (large $\tan \beta$)

$$\langle H_u \rangle \gg \langle H_d \rangle$$

Non-zero Yukawa couplings ensure that the chiral symmetries have been broken. The masses can be generated radiatively.



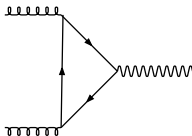
MSSM is a natural 2HDM

- Superpotential is holomorphic and ϵH^* is forbidden in superpotential.
- $\tilde{H}_u, \tilde{H}_d$ contributes to anomaly $[SU(2)_L]^2 U(1)_Y, \dots$ and Witten Anomaly

$$W = y_u Q u^c H_u + y_d Q d^c H_d + y_e \ell e^c H_d + \mu H_u H_d$$



2HDM has Peccei-Quinn symmetry (DFSZ axion, 1981).



$$\begin{aligned}
 A_{[SU(3)_C]^2 U(1)} &= 3\alpha + \frac{3}{2}(2(q - \alpha) + (u - \alpha) + (d - \alpha)) \\
 &= 3\alpha - \frac{3}{2}(h_u + h_d)
 \end{aligned}$$

$$q + u + h_u = 2\alpha, q + d + h_d = 2\alpha$$

$$h_u + h_d \neq 2\alpha \rightarrow A_3 \neq 0$$

$$M_{PQ} \sim M_{\text{Intermediate}}, \text{Kim-Nilles}$$



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$$10 \cdot 10 H_u + 10 \cdot \bar{5} H_d$$

Field	10	$\bar{5}$	H_u	H_d	θ
R -charge	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{6}{5}$	1
PQ	0	-1	0	1	0

$$10 \cdot \bar{5} H_u^* : \cancel{R} : \frac{1}{5} + \frac{3}{5} - \frac{4}{5} = 0 \neq 2, \cancel{PQ}$$



$$W = \mu H_u H_d \quad \cancel{\text{PQ}}$$

$$\begin{aligned} \mathcal{L}_{\text{soft}} \ni & m_{\tilde{f}}^2 |\tilde{f}|^2 \quad R - \text{invariant} \\ & + \quad M_{\frac{1}{2}} \lambda \lambda + A_u \tilde{Q} \tilde{u} H_u + \dots \quad \cancel{R} \\ & + \quad B \mu H_u H_d \quad \cancel{R}, \cancel{\text{PQ}} \end{aligned}$$

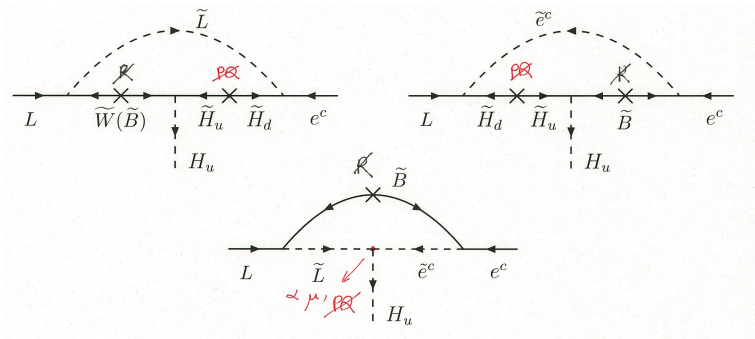
If $\cancel{\text{PQ}}, \cancel{R}$ and $[U(1)]^5$,

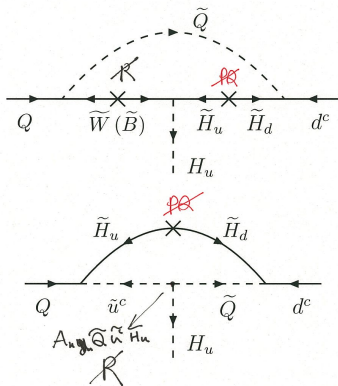
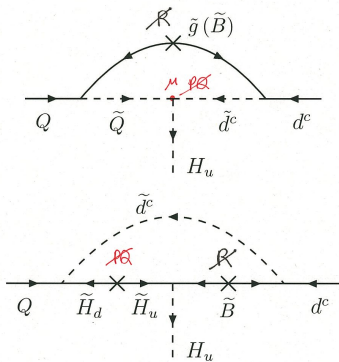
$$10 \cdot \bar{5} H_u^* \rightarrow m_e, m_d \neq 0$$

Another $\cancel{\text{PQ}}$ source, (proportional to μ)

$$\begin{aligned} F_{H_d} &= \frac{\partial W}{\partial H_d} = y_d Q d^c + y_e \ell e^c + \mu H_u \\ V \ni |F_{H_d}|^2 &= y_d \mu^* H_u^* \tilde{Q} \tilde{d} + y_e \mu^* H_u^* \tilde{\ell} \tilde{e} \end{aligned}$$







Physics Implications

- If all Yukawa couplings in MSSM are perturbative at M_{GUT} ,
 $2 \lesssim \tan \beta \lesssim 50$.

But what if m_b arise from $\langle H_u \rangle$ radiatively.....



ν_L does not carry any unbroken gauge symmetry
($SU(3)_C \times U(1)_{EM}$)....

$$-\frac{1}{2}M_\nu^{ij}\nu_L^{iT}C\nu_L^j$$

Type-I see-saw

$$y_\nu \overline{\ell}_L n_R H_u + M_R \overline{n}_R^c n_R + h.c. ,$$

For one generation y_ν break $U(1)_\ell \times U(1)_n \rightarrow U(1)_{\text{Lep}}$

$$M_R \rightarrow U(1)_{\text{Lep}}$$

$$m_\nu = M_D^T M_R^{-1} M_D$$

- Without tuning dimensionless y_ν , tiny m_ν from M_{GUT}
- $U(1)_{B-L}$ becomes anomaly free, easily embedded into $SO(10)$

Pati-Salam(Wyler-Wolfenstein)

$$y_\nu \overline{\ell}_L n_R H_u + M_S \overline{s}_L n_R + h.c.$$

In basis (ν_L, s_L, n_R^c)

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & M_D \\ 0 & 0 & M_S \\ M_D & M_S & 0 \end{pmatrix}$$

$$m_\nu = 0$$



Inverse see-saw (Mohapatra, Valle)

$$y_\nu \overline{\ell}_L n_R H_u + M_S \overline{s}_L n_R + \epsilon \overline{s}_L^c s_L$$

In basis (ν_L, s_L, n_R^c)

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & M_D \\ 0 & \epsilon & M_S \\ M_D & M_S & 0 \end{pmatrix}$$

$$m_\nu \simeq \epsilon \frac{M_D^2}{M_D^2 + M_S^2}$$



Tuning: Dimensionless y_ν or dimension-one M

$$y_e \sim 10^{-6}, y_\nu \sim 10^{-12}?$$

Dimension One: see-saw vs inverse

- In see-saw, M_R breaks $U(1)_{B-L}$ gauge symmetry at ultra-high scale, for instance, M_{GUT} .
- Now n, s are both SM gauge singlet..., the scale vanishes to restore the $U(1)_{\text{Lep}}$, can be identified as soft breaking of $U(1)_{\text{Lep}}$.



$$y_\nu \overline{\ell}_L n_R H_u + M_S \overline{s}_L n_R + M_R \overline{n}_R^c n_R$$

In basis (ν_L, s_L, n_R^c)

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & M_D \\ 0 & 0 & M_S \\ M_D & M_S & M_R \end{pmatrix}$$

$$\nu = -\frac{M_S}{\sqrt{M_D^2 + M_S^2}} \nu_L + \frac{M_D}{\sqrt{M_D^2 + M_S^2}} s_L$$

$$N_\pm = \frac{1}{\sqrt{M_\pm^2 + M_D^2 + M_S^2}} (M_D \nu_L + M_S s_L - M_\pm n_R^c)$$

with mass eigenvalues as

$$m_\nu = 0, \quad M_\pm = \frac{1}{2} \left(M_R \pm \sqrt{4M_D^2 + M_R^2 + 4M_S^2} \right)$$

$$U(1)_\nu \times U(1)_n \times U(1)_s$$

In basis (ν_L, s_L, n_R^c)

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & M_D \\ 0 & 0 & M_S \\ M_D & M_S & M_R \end{pmatrix}$$

With M_D, M_S

$$U(1)_\nu \times U(1)_n \times U(1)_s \rightarrow U(1)_{\text{Lep}}$$

Under $U(1)_{\text{Lep}}$

$$\begin{aligned} \nu_L &\rightarrow e^{i\alpha} \nu_L \\ s_L &\rightarrow e^{i\alpha} s_L \\ n_R &\rightarrow e^{-i\alpha} n_R^c \end{aligned}$$

With M_R

$$\cancel{U(1)_{\text{Lep}}} \rightarrow U(1)_{\nu-s}$$

Why no $M_R^* \bar{s}_L^c s_L$? SUSY

Supersymmetry does not forbid lepton number violation but only stabilize the model. So it is just “Technically Natural”.

$$W = y_\nu \ell n^c H_u + M_S s n^c + M_R n^c n^c$$

Field	ℓ	e^c	n^c	s	H_u	H_d	θ
R -charge	$\frac{1}{5}$	$\frac{3}{5}$	1	1	$\frac{4}{5}$	$\frac{6}{5}$	1
$U(1)_L$	1	-1	-1	1	0	0	0

W_{eff}	m	R -charge of m	$U(1)_L$ charge
$n^c n^c$	$n_R^c n_R$	$1 + 1 - 2\theta = 0$	-2
$\ell s H_u$	$\nu_L^c s_L$	$\frac{1}{5} + 1 + \frac{4}{5} - 2\theta = 0$	2
$\ell \ell H_u H_u$	$\nu_L^c \nu_L$	$\frac{1}{5} + \frac{1}{5} + \frac{4}{5} + \frac{4}{5} - 2\theta = 0$	2
ss	$\bar{s}_L^c s_L$	$1 + 1 - 2\theta = 0$	2



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New gauge interaction?

Under E_6

$$27 = 16 + 10 + 1$$

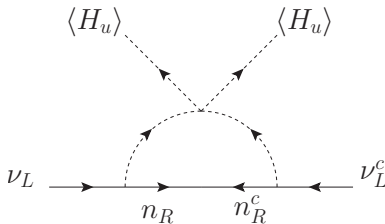
s_L is completely gauge singlet and any term involving s_L will be only gravitationally induced in

- Kähler potential
- Lepton number violation B -terms in soft-breaking lagrangian



R -invariant piece

$U(1)_{B-L}$ becomes anomalous so the leading is Yukawa interaction induced $\ell\ell H_u H_u$ Non-SUSY contribution



$$\begin{aligned}
 (M_\nu^{1-\text{loop}})_{ij} &= \sum_{k=1}^3 \frac{1}{16\pi^2} M_R^k \sum_{\phi=h,H} \frac{Y_{ik}^* Y_{jk} M_\phi^2}{M_\phi^2 - M_R^{k2}} \ln \left(\frac{M_\phi^2}{M_R^{k2}} \right) \\
 &\quad - \frac{Y_{ik}^* Y_{jk} M_A^2}{M_A^2 - M_R^{k2}} \ln \left(\frac{M_A^2}{M_R^{k2}} \right)
 \end{aligned}$$



$$M_R \frac{M_\phi^2}{M_\phi^2 - M_R^2} \ln \left(\frac{M_\phi^2}{M_R^2} \right)$$

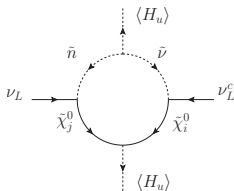
- $M_R \gg M_{h,H}$ restore the see-saw, $M_R < 10^{12}$ GeV
- $M_R \ll M_{h,H}$, inverse see-saw limit, $M_R \sim \text{KeV}$

We take the inverse see-saw limit (non-canonical Kähler potential) to ensure light neutrino mass.



$$\begin{aligned}
V &= \left| \frac{\partial W}{\partial n_R} \right|^2 = |(\ell H_u + M_s s_L + M_R n_R)|^2 \\
&= M_R^* \tilde{n}_R^* \tilde{\ell} H_u + M_R^* M_s \tilde{n}_R^* \tilde{s}_L
\end{aligned}$$

After M_{susy} ,



$$m_{\tilde{n}}^2 \sim M_{\text{susy}}^2 + M_R^2$$

- $M_R \gg M_{\text{susy}}$
- $M_R \ll M_{\text{susy}}$

$\mathcal{R}$ Contribution

The effective operators do not break R -symmetry

$$\mathcal{L}_{soft} \ni B_R M_R \tilde{n}_R \tilde{n}_R + B_S \epsilon_s \tilde{s}_L \tilde{s}_L + A_s M_R \tilde{\ell}_L \tilde{s}_L H_u + B_\nu M_\nu \tilde{\nu}_L \tilde{\nu}_L$$

m_ν only arise with gaugino mass insertion.

Soft SUSY breaking terms that also violate $U(1)_{\text{Lep}}$ but $1/M_{\text{Pl}}$ suppression.



Conclusions

- I present two examples that the fermion masses arise from radiative correction where the tree level masses are suppressed but all the chiral symmetries are broken.
- Supersymmetry plays an important role in stabilizing the suppressed tree level masses.

