Dynamics of intersecting brane systems – Classification and their applications –

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Based on collaboration with K. Maeda, K. Uzawa (+ Tanabe, Wakebe)

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1 Introduction

- **Check of the predictions of superstring theories**

**The situation where the effects of quantum gravity become important $\Rightarrow$ Black holes (singularity)
Early universe (singularity)**

It is urgent to see whether and how these problems are resolved and if superstrings can give realistic models of particles and their interaction including gravity

Here we consider a mixed problem of these subjects.

The dynamical systems in string theories are notoriously difficult to deal with. The main reason is that time dependence breaks supersymmetry and there are only few exact time-dependent solutions.

- **Time-dependent brane solution**

It is known that static black hole solutions are realized as intersecting branes in superstrings, or in their effective low-energy supergravities.

This has been acheived in rather general way, with and without (non-extreme) supersymmetry.

Unfortunately, so far, only some solutions are constructed with time-dependence, but there is no systematic study.

Here we give a quite systematic construction of such solutions and give a classification of these solutions.

It turns out that some of these describe black hole solutions in time-dependent cosmological setting.

- **Supersymmetric time-dependent solutions**

Simply including time-dependence breaks supersymmetry of the resulkting solutions.

It is known, however, that it is posible to keep supersymmetry if the time-dependence is introduced only through one of the light-cone coordinates.

We have constructed a class of such supersymmetric time-dependent brane solutions with some applications. (Maeda, NO, Tanabe, Wakebe)

This is the most general class of solutions constructed to date.

2 Branes and Black holes

I will first explain how brane solutions are related to black holes.
Example: 2-brane solution in $D = 11$

$$\begin{aligned} ds^2 &= H^{1/3}[H^{-1}(-dt^2 + dy_1^2 + dy_2^2) + dr^2 + r^2 d\Omega_7^2], \\ & A_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda} H^{-1}, \; H \equiv 1 + \frac{k}{r^6}. \end{aligned}$$

The metric is invariant in y_1, y_2 directons, and is asymptotically flat in r directions. This means that there is some object at $r = 0$ and extended in y_1, y_2.

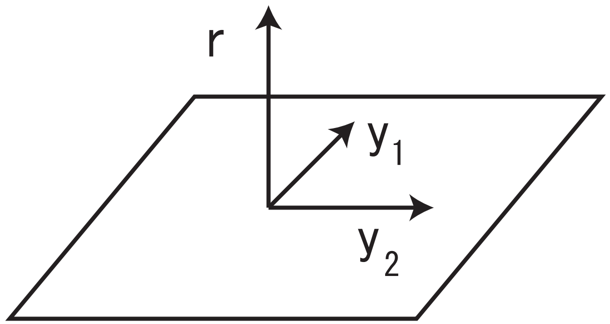

Figure 1: Membrane

Schwarzschild-type coordinates: $r = (R^6 - k)^{1/6}$

$$ds^2 = (1 - k/R^6)^{2/3}(-dt^2 + dy_1^2 + dy_2^2) + (1 - k/R^6)^{-2} dR^2 + R^2 d\Omega_7^2.$$

There is a horizon at $R = k^{1/6}$, but a time-like and space-like vectors are not interchanged upon crossing it.

Global structure $\sim$ Reissner-Nordstrom solution

$\Rightarrow$ See next Fig.

More general solutions can be constructed by combining these solutions. $\Rightarrow$ intersection rules

Fundamental objects

type IIA	**type IIB**
string	**string**
Dp-branes (p: even)	**Dp-branes (p: odd)**
NS5-brane	**NS5-brane**
KK-wave	**KK-wave**

Combining these, it is possible to make solutions that can be interpreted as $D = 4, 5$ black holes.

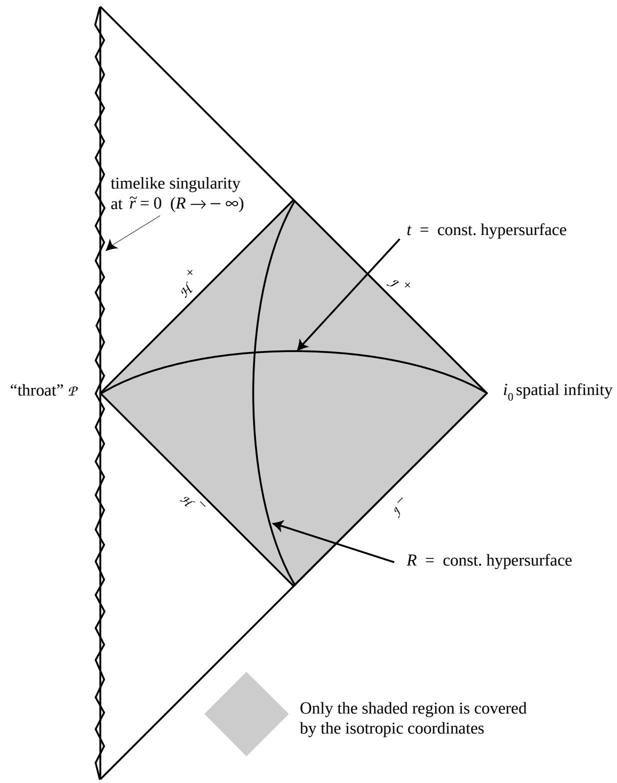

Figure 2: Penrose diagram for M2-brane

Example of intersecting solutions:

Type IIB D5-D1-KK-wave intersection:

$$\begin{aligned} ds^2 &= H_1^{-\frac{3}{4}} H_5^{-\frac{1}{4}} [-(1 - r_0^2/r^2) dt^2 + dx_9^2 + (r_0^2/r^2)(\cosh\sigma dt + \sinh\sigma dx_9)^2] \\ &\quad + H_1^{\frac{1}{4}} H_5^{-\frac{1}{4}} [dx_5^2 + dx_6^2 + dx_7^2 + dx_8^2] \\ &\quad + H_1^{\frac{1}{4}} H_5^{\frac{3}{4}} [(1 - r_0^2/r^2)^{-1} dr^2 + dr^2 \Omega_3^2]. \end{aligned}$$

Here $r^2 = x_1^2 + \cdots + x_4^2$, $H_i = 1 + \frac{r_0^2}{r^2} \sinh\alpha_i$**.**

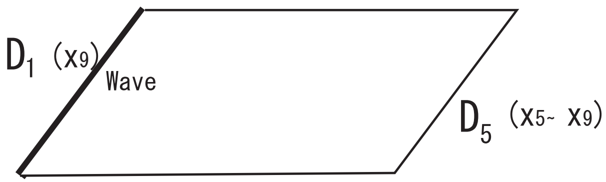

Figure 3: D1-D5-Wave intersecting solution

$x_5, \cdots, x_9$ **are compactified on a torus** T^5 **of radii** $R_5, \cdots, R_9$**, giving** $D = 5$ **black hole.**

These are static solutions. $\Rightarrow$ **Dynamical solutions?**

3 Solutions of dynamical intersecting branes

The low-energy effective action for the supergravity system coupled to dilaton and n_A- form field strength is given by

$$I = \frac{1}{16\pi G_D}\int d^D x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \sum_{A=1}^{m}\frac{1}{2n_A!}e^{a_A\phi}F_{n_A}^2\right],$$

G_D**: Newton constant in D dimensions**

g**: the determinant of the metric.**

The last term: includes both RR and NS-NS field strengths and $a_A = \frac{1}{2}(5-n_A)$ for RR field strength and $a_A = -1$ for NS-NS 3-form.

The field equations:

$$R_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \sum_A \frac{1}{2n_A!}e^{a_A\phi}\left[n_A\left(F_{n_A}^2\right)_{\mu\nu} - \frac{n_A-1}{D-2}F_{n_A}^2 g_{\mu\nu}\right],$$

$$\Box\phi = \sum_A \frac{a_A}{2n_A!}e^{a_A\phi}F_{n_A}^2, \quad \partial_{\mu_1}\left(\sqrt{-g}e^{a_A\phi}F^{\mu_1\cdots\mu_{n_A}}\right) = 0\,.$$

Bianchi identity: $\partial_{[\mu}F_{\mu_1\cdots\mu_{n_A}]} = 0.$

Metric ansatz:

$$ds^2 = \mathcal{A}(t,z)u_{ij}(z)dz^i dz^j - \mathcal{B}(t,z)dt^2 + \sum_{\alpha=1}^{p} \mathcal{C}^{(\alpha)}(t,z)(dx^\alpha)^2,$$

$u_{ij}(z)$**: the metric of the $(D-p-1)$-dimensional** Z **space which depends only on the $(D-p-1)$-dimensional coordinates z^i.**

The metric components u_0, u_α, v and the dilaton ϕ are assumed to be functions of t and z^i.

Field strength backgrounds:

$$F_{n_A} = \partial_i E_A(t,z)\, dt \wedge dy^{\alpha_1} \wedge \cdots \wedge dy^{\alpha_{q_A}} \wedge dz^i,$$

where $n_A = q_A + 2$.

The ansatz means that we have an electric background, but can include magnetic background by the replacement

$$g_{\mu\nu} \to g_{\mu\nu}, \qquad F_n \to e^{a\phi} * F_n, \qquad \phi \to -\phi.$$

The Einstein equations:

$$
e^{2(u_0-v)}(\Delta_z u_0 + u^{ij}\partial_i u_0 \partial_j U) - \sum_{\alpha=1}^{d}(\ddot{u}_\alpha + \dot{u}_\alpha^2 - \dot{u}_0\dot{u}_\alpha) - (D-d-1)(\ddot{v} + \dot{v}^2 - \dot{v}\dot{u}_0)
$$
$$
= \frac{1}{2}\dot{\phi}^2 + \sum_A \frac{D-q_A-3}{2(D-2)} S_A e^{2(u_0-v)} u^{ij}\partial_i E_A \partial_j E_A,
$$

$$
-(D-d-2)[\partial_i\dot{v} - \dot{v}\partial_i u_0] - \sum_{\alpha=1}^{d}\partial_i\dot{u}_\alpha + \sum_{\alpha=1}^{d}(\dot{u}_\alpha\partial_i u_0 - \dot{u}_\alpha\partial_i u_\alpha + \dot{v}\partial_i u_\alpha) = \frac{1}{2}\dot{\phi}\partial_i\phi,
$$

$$
e^{2(u_\alpha-u_0)}[\ddot{u}_\alpha + \dot{u}_\alpha(\dot{U} - 2\dot{u}_0 + 2\dot{v})] - e^{2(u_\alpha-v)}[\Delta_z u_\alpha + u^{ij}\partial_i u_\alpha \partial_j U]
$$
$$
= -\sum_A \frac{\delta_A^{(\alpha)}}{2(D-2)} S_A e^{2(u_\alpha-v)} u^{ij}\partial_i E_A \partial_j E_A,
$$

$$
R_{ij}(z) - D_i D_j U + \partial_i v\ \partial_j U + \partial_j v\ \partial_i U - \Big[\partial_i u_0 \partial_j u_0 + \sum_{\alpha=1}^{d}\partial_i u_\alpha \partial_j u_\alpha
$$
$$
+(D-d-3)\partial_i v \partial_j v\Big] - u_{ij}(\Delta_z v + u^{kl}\partial_k v \partial_l U) + u_{ij}e^{-2(u_0-v)}[\ddot{v} + \dot{v}(2\dot{v} - 2\dot{u}_0 + \dot{U})]
$$
$$
= \frac{1}{2}\partial_i\phi\partial_j\phi - \sum_A \frac{S_A}{2}\Big[\partial_i E_A \partial_j E_A - \frac{q_A+1}{D-2} u_{ij} u^{kl}\partial_k E_A \partial_l E_A\Big],
$$

The dot indicates derivatives with respect to t.

The dilaton and form field equations:

$$-e^{-U}\partial_t(e^{U-2u_0+2v}\dot{\phi}) + e^{-U}\frac{1}{\sqrt{u}}\partial_i(e^U\sqrt{u}\,u^{ij}\partial_j\phi) \;=\; -\frac{1}{2}\sum_A \epsilon_A a_A S_A u^{ij}\partial_i E_A \partial_j E_A,$$

$$\partial_t\left(e^U S_A\sqrt{u}\,\partial_i E_A\right) \;=\; 0,$$

$$\partial_i\left(e^U S_A\sqrt{u}\,u^{ij}\partial_j E_A\right) \;=\; 0,$$

where

$$U = u_0 + \sum_{\alpha=1}^{d} u_\alpha + (D-d-3)v,$$

$$S_A \equiv \exp\left[\epsilon_A a_A \phi - 2\left(u_0 + \sum_{\alpha\in q_A} u_\alpha\right)\right],$$

$$\delta_A^{(\alpha)} = \begin{cases} D-q_A-3 \\ -(q_A+1) \end{cases} \quad \text{for} \quad \begin{cases} y^\alpha \ \textbf{belonging to } q_A\textbf{-brane} \\ \text{otherwise} \end{cases}.$$

By a now standard (to me) procedure, we can systematically derive solutions to this system.

The intersection rule:
If q_A- and q_B-branes overlap over $\bar{q}$-brane, we have

$$\bar{q} = \frac{(q_A+1)(q_B+1)}{D-2} - 1 - \frac{1}{2}\epsilon_A \epsilon_B a_A a_B.$$

Take, for example, $D = 11$ supegravity without dilaton (M-theory):

There are 2 and 5 branes, called M2 and M5.

$q_A = q_B = 2 \quad \Rightarrow \quad \bar{q} = 0 \Rightarrow$ **no overlap.**

$q_A = 2, q_B = 5 \quad \Rightarrow \quad \bar{q} = 1 \Rightarrow$ **overlap over 1-brane.**

$q_A = q_B = 5 \quad \Rightarrow \quad \bar{q} = 3 \Rightarrow$ **overlap over 3-brane.**

Solution:

$$
\begin{aligned}
u_0 &= -\sum_A \frac{D-q_A-3}{\Delta_A} \ln h_A + g_0(t), \\
u_\alpha &= -\sum_A \frac{\delta_A^{(\alpha)}}{\Delta_A} \ln h_A + g_\alpha(t), \\
v &= \sum_A \frac{q_A+1}{\Delta_A} \ln h_A + g(t), \\
\phi &= \sum_A \epsilon_A a_A \frac{(D-2)}{\Delta_A} \ln h_A + g_\phi(t),
\end{aligned}
$$

$$
E_A = \frac{N_A}{e^{-U(t)} h_A}, \qquad h_A \equiv f_A(t) + H_A (\equiv \frac{Q_A}{r^{\tilde{d}}}),
$$

where $g(t)$'s and $f_A(t)$ are arbitrary functions of t, and N_A and Q_A are constants.

$$
N_A = \sqrt{\frac{2(D-2)}{\Delta_A}}, \qquad \Delta_A \equiv (D-q_A-3)(q_A+1) + \frac{1}{2}(D-2)a_A^2.
$$

$$2\tilde{d}g + \sum_{\alpha \notin q_A} 2g_\alpha + \epsilon_A a_A g_\phi = 0,$$

It turns out that only single brane can have time dependence with $f_A(t) = c_A t$, linear in t, others should be a constant.

The reason is probably that introducing time-dependence amounts to introducing gravity source, and this causes gravitationalwave, making the problem nonlinear and untractable.

It is a big challenge to circumvent this problem.

We now give examples of these solutions for $D = 11$, M-theory, called M-branes.

4 Dynamical intersecting M-branes

Intersections of two M-branes.

		0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
$(\mathrm{M2})^2$	M2	∘	∘	∘									(a)	–	$\surd$
	M2	∘			∘	∘									
M2M5	M2	∘	∘					∘					(a)	$\surd$	$\surd$
	M5	∘	∘	∘	∘	∘	∘						(b)	$\surd$	$\surd$
$(\mathrm{M5})^2$	M5	∘	∘	∘	∘	∘	∘						(a)	$\surd$	$\surd$
	M5	∘	∘	∘	∘			∘	∘						

The mark $\surd$ in the table shows which brane is time dependent

Concrete metrics for two M-branes.

M2M2		$\mathcal{A}=h_{\tilde{2}}^{1/3}H_2^{1/3}$	$g_{\tilde{0}}=h_{\tilde{2}}^{-1}H_2^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	**5**	$g_{\tilde{1}}=g_{\tilde{2}}=h_{\tilde{2}}^{-1}$	$g_3=g_4=H_2^{-1}$
M2M5		$\mathcal{A}=h_{\tilde{2}}^{1/3}H_5^{2/3}$	$g_0=h_{\tilde{2}}^{-1}H_5^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	**4**	$g_{\tilde{1}}=h_{\tilde{2}}^{-1}H_5^{-1}, g_{\tilde{6}}=h_{\tilde{2}}^{-1}$	$g_2=g_3=g_4=g_5=H_5^{-1}$
M2M5		$\mathcal{A}=h_{\tilde{5}}^{2/3}H_5^{1/3}$	$g_0=h_{\tilde{5}}^{-1}H_5^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(b)	**4**	$g_{\tilde{1}}=h_{\tilde{5}}^{-1}H_2^{-1}$ $g_{\tilde{2}}=g_{\tilde{3}}=g_{\tilde{4}}=g_{\tilde{5}}=h_{\tilde{5}}^{-1}$	$g_6=H_2^{-1}$
M5M5		$\mathcal{A}=h_{\tilde{2}}^{2/3}H_2^{2/3}$	$g_{\tilde{0}}=h_{\tilde{2}}^{-1}H_2^{-1}$
	dim(Z)	$g_{\tilde{\alpha}}$	g_α
(a)	**3**	$g_{\tilde{1}}=g_{\tilde{2}}=g_{\tilde{3}}=h_{\tilde{5}}^{-1}H_2^{-1}$ $g_{\tilde{4}}=g_{\tilde{5}}=h_{\tilde{2}}^{-1}$	$g_6=g_7=H_5^{-1}$

General solution:

The only one time-dependent brane is denoted by $\tilde{I}$:

$$h_{\tilde{I}} = h_{\tilde{I}}(t,z) \equiv A_{\tilde{I}}\, t + H_{\tilde{I}}(z)\,.$$

The solution is written as

$$ds^2 = \mathcal{A}(t,z)\left[-g_0(t,z)dt^2 + \sum_{\tilde{\alpha}\in\tilde{I}} g_{\tilde{\alpha}}(t,z)(dx^{\tilde{\alpha}})^2 + \sum_{\alpha\notin\tilde{I}} g_\alpha(z)(dy^\alpha)^2 + u_{ij}(z)dz^i dz^j\right],$$

with

$$\begin{aligned}
\mathcal{A} &= [A_{\tilde{I}}t + H_{\tilde{I}}(z)]^{a_{\tilde{I}}} \prod_{I\neq\tilde{I}} H_I(z)^{a_I} \quad , \quad g_0 = [A_{\tilde{I}}t + H_{\tilde{I}}(z)]^{-1} \prod_{I\neq\tilde{I}} H_I(z)^{-1}, \\
g_{\tilde{\alpha}} &= [A_{\tilde{I}}t + H_{\tilde{I}}(z)]^{-1} \prod_{I\neq\tilde{I}} H_I^{-\gamma_I^{(\tilde{\alpha})}} \quad , \quad g_\alpha = \prod_{I\neq\tilde{I}} H_I^{-\gamma_I^{(\alpha)}},
\end{aligned}$$

$$a_{\tilde{I}} = \frac{p_{\tilde{I}}+1}{D-2} \quad , \quad \text{and} \quad \gamma_I^{(\alpha)} = \begin{cases} 1 & \text{for} \quad \alpha\in I \\ 0 & \text{for} \quad \alpha\notin I \end{cases}.$$

Example: M5-M5

$$
\begin{aligned}
ds^2 \;=\; & (h_{\tilde 5} H_5)^{2/3}\left[(h_{\tilde 5} H_5)^{-1}\left(-dt^2+\sum_{\tilde\alpha=1}^{3}(dx^{\tilde\alpha})^2\right)+h_{\tilde 5}^{-1}\sum_{\tilde\alpha=4}^{5}(dx^{\tilde\alpha})^2\right.\\
& \left.+H_5^{-1}\sum_{\alpha=6}^{7}(dy^{\alpha})^2+u_{ij}dz^i dz^j\right],
\end{aligned}
$$

so

$$
\mathcal{A}=(h_{\tilde 5}H_5)^{2/3}, \quad g_0=g_{\tilde 1}=g_{\tilde 2}=g_{\tilde 3}=(h_{\tilde 5}H_5)^{-1}\;, \quad g_{\tilde 4}=g_{\tilde 5}=h_{\tilde 5}^{-1},
$$

$$
g_1=g_2=H_5^{-1}, \qquad h_{\tilde 5}=A_{\tilde 5}t+H_{\tilde 5}(z)\,.
$$

Intersections of three M-branes.

		0	1	2	3	4	5	6	7	8	9	10	$\tilde I$	cos	BH
(M5)^3	M5	○	○	○	○	○	○						(a)	√	–
	M5	○	○	○	○			○	○						
	M5	○	○	○	○					○	○				
	M5	○	○	○	○	○	○						(b)	–	√
	M5	○	○	○	○			○	○						
	M5	○	○			○	○	○	○						
	M5	○	○	○	○	○	○						(c)	–	–
	M5	○	○	○	○			○	○						
	M5	○	○	○		○		○		○					
M2(M5)^2	M2	○	○	○									(a)	√	√
	M5	○	○		○	○	○	○					(b)	√	√
	M5	○		○	○	○	○		○						
	M2	○	○	○									(a)	–	–
	M5	○	○		○	○	○	○					(b)	–	–
	M5	○	○				○	○	○	○					
(M2)^{2}M5	M2	○	○	○									(a)	√	√
	M2	○			○	○									
	M5	○	○		○		○	○	○				(b)	√	√
(M2)^3	M2	○	○	○									(a)	–	√
	M2	○			○	○									
	M2	○					○	○							

Intersections of four M-branes I.

		0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
$(\mathbf{M5})^4$	**M5**	○	○	○	○	○	○						(a)	–	–
	M5	○	○	○	○			○	○				(b)	–	–
	M5	○	○	○		○		○		○					
	M5	○	○		○	○			○	○					
	M5	○	○	○	○	○		○					(c)	–	–
	M5	○	○	○	○		○		○						
	M5	○	○	○		○	○			○					
	M5	○	○		○	○	○				○				
	M5	○	○	○	○	○	○						(d)	–	–
	M5	○	○	○	○			○	○				(e)	–	–
	M5	○	○	○		○		○		○					
	M5	○	○	○			○	○			○				
	M5	○	○	○	○	○	○						(f)	–	–
	M5	○	○	○	○			○	○						
	M5	○	○	○		○		○		○					
	M5	○	○	○			○		○	○					
M2 $(\mathbf{M5})^3$	**M2**	○	○	○									(a)	–	–
	M5	○	○		○	○	○	○					(b)	–	–
	M5	○	○		○	○			○	○					
	M5	○	○				○	○	○	○					
	M2	○	○	○									(c)	–	–
	M5	○	○		○	○		○	○				(d)	–	–
	M5	○	○		○	○	○			○					
	M5	○	○		○		○	○			○				
	M2	○	○	○									(e)	–	–
	M5	○	○		○	○	○		○				(f)	–	–
	M5	○	○		○	○		○		○					
	M5	○		○	○	○	○	○					(g)	–	–
$(\mathbf{M2})^2$ $(\mathbf{M5})^2$	**M2**	○	○							○			(a)	–	–
	M2	○		○							○				
	M5	○	○	○	○	○	○						(b)	–	–
	M5	○	○	○	○			○	○						
	M2	○	○							○			(c)	–	–
	M2	○				○		○					(d)	–	–
	M5	○	○	○	○	○	○								
	M5	○	○	○	○			○	○				(e)	–	–
	M2	○				○		○					(f)	√	√
	M2	○					○		○						
	M5	○	○	○	○	○	○						(g)	√	√
	M5	○	○	○	○			○	○						

Intersections of four M-branes II.

		0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$	cos	BH
$(\mathrm{M2})^3$ M5	M2	○	○	○									(a)	–	–
	M2	○			○	○									
	M2	○					○	○							
	M5	○	○		○		○		○	○			(b)	–	–
$(\mathrm{M2})^4$	M2	○	○	○									(a)	–	–
	M2	○			○	○									
	M2	○					○	○							
	M2	○							○	○					

One can continue this line up to intersecting eight branes, but it turns out that beyond four branes there is no interesting applications to physical system.

Near branes ($|\boldsymbol{z}| \sim 0$):

The spacetime structure is the same as that of the static solution unless the dimension of Z space is one.

Reason: the metric components diverge as $|\boldsymbol{z}| \to 0$ and the static harmonic parts dominate the time-dependent terms.

$$h_A \equiv c_A t + \frac{Q_A}{r^{\tilde{d}}}.$$

$\Rightarrow$ **M2-M2, M2-M5, M2-M2-M2, M5-M5-M5, M2-M2-M5-M5 systems are regular on the branes.**

Far away from horizon ($|\boldsymbol{z}| \to \infty$ and $H_{\tilde{I}} \to 0$):

The spacetime turns out to be time dependent and homogeneous. New time coordinate

$$\tau = \tau_0 (A_{\tilde{I}} t)^{(a_{\tilde{I}}+1)/2}, \qquad \tau_0 = \frac{2}{A_{\tilde{I}}(a_{\tilde{I}}+1)}$$

The asymptotic solution

$$ds^2 = -d\tau^2 + \left(\frac{\tau}{\tau_0}\right)^{2q_{\tilde{I}}} \sum_{\tilde{\alpha}} (dx^{\tilde{\alpha}})^2 + \left(\frac{\tau}{\tau_0}\right)^{2q_{\not\tilde{I}}} \left(\sum_{\alpha} (dy^{\alpha})^2 + u_{ij} dz^i dz^j \right) ,$$

where

$$q_{\tilde{I}} = \frac{a_{\tilde{I}}-1}{a_{\tilde{I}}+1} = -\frac{D-p_{\tilde{I}}-3}{D+p_{\tilde{I}}-1}, \quad q_{\not\tilde{I}} = \frac{a_{\tilde{I}}}{a_{\tilde{I}}+1} = \frac{p_{\tilde{I}}+1}{D+p_{\tilde{I}}-1}.$$

M-theory ($D = 11$):

$$q_{\tilde I} = -1/2,\ q_{\not{\tilde I}} = 1/4, \quad \text{for } \tilde I = \text{M2}\ (p_{\tilde I} = 2)\,,$$
$$q_{\tilde I} = -1/5,\ q_{\not{\tilde I}} = 2/5, \quad \text{for } \tilde I = \text{M5}\ (p_{\tilde I} = 5)$$

$\Rightarrow$ a Kasner-like expansion:

$$p_{\tilde I}\, q_{\tilde I} + p_{\not{\tilde I}} q_{\not{\tilde I}} = 1\,,$$
$$p_{\tilde I}(q_{\tilde I})^2 + p_{\not{\tilde I}}(q_{\not{\tilde I}})^2 = 1$$

$p_{\not{\tilde I}} = (D - p_{\tilde I} - 1)$ **is the dimension of the space volume perpendicular to the $\tilde I$-brane world-volume.**

5 Applications to cosmology and black holes

5.1 Cosmology

We suppose that our three-dimensional universe is a part of branes. Since our universe is isotropic and homogeneous, same branes must contain this whole three dimensions.

We find just six cases, i.e., M2-M5, M5-M5, M5-M5-M5, M2-M5-M5, M2-M2-M5, and M2-M2-M5-M5 brane systems can give an isotropic and homogeneous three space from the list of our solutions.

The scale factor of the universe is given by

$$a_{\tilde{I}} = \left(\frac{\tau}{\tau_0}\right)^{\beta_{\tilde{I}}},$$

in terms of the cosmic time with $\beta_{\tilde{I}} = 0, 1/7, 1/4$.

The power is too small to give a realistic expansion law.

In order to find a realistic expansion of the universe in this type of models, one have to include additional "matter" fields on the brane.

5.2 Time-dependent black holes

In this case, just as the case of a static black hole, we should compactify all brane world-volume, and the remaining d-dimension is our spacetime. (Note the difference from the cosmology.)

Example: M2-M2-M5-M5 brane system

We assume that one M2 brane is time-dependent.

$$d\bar{s}_4^2 = -(h_{\tilde{2}} H_2 H_5 H_{5'})^{-1/2} dt^2 + (h_{\tilde{2}} H_2 H_5 H_{5'})^{1/2} \left(dr^2 + r^2 d\Omega_2^2\right) ,$$

where

$$\begin{aligned} h_{\tilde{2}} &= \frac{t}{t_0} + \frac{Q_{\tilde{2}}}{r}, \\ H_2 &= 1 + \frac{Q_2}{r} , \quad H_5 = 1 + \frac{Q_5}{r} , \quad H_{5'} = 1 + \frac{Q_{5'}}{r} , \end{aligned}$$

This metric is rewritten as

$$d\bar{s}_4^2 = -(\tilde{H}_{\tilde{2}} H_2 H_5 H_{5'})^{-1/2} d\tau^2 + a_{\rm BH}^2(\tau)(\tilde{H}_{\tilde{2}} H_2 H_5 H_{5'})^{1/2} \left(dr^2 + r^2 d\Omega_2^2\right) ,$$

where

$$\tilde{H}_{\tilde{2}} = 1 + \frac{\tilde{Q}_{\tilde{2}}(\tau)}{r} , \quad \text{and} \quad a_{\rm BH} = \left(\frac{\tau}{\tau_0}\right)^{\frac{1}{3}} ,$$

with

$$\tilde{Q}_{\tilde{2}} \equiv \left(\frac{\tau}{\tau_0}\right)^{-\frac{4}{3}} Q_{\tilde{2}} \ , \quad \text{and} \quad \tau_0 \equiv \frac{4}{3} t_0 \, .$$

This gives an intersting exact solution describing black hole in an expanding universe.

Detailed causal structure ⇒ Maeda-Nozawa

6 Discussions

Summary: We have

- **given general intersecting dynamical brane solutions**
- **given the complete classification of the intersecting M-branes,**
- **discussed the dynamics of the higher-dimensional supergravity models with applications to cosmology and black hole physics.**

The solutions are the exact spacetime-dependent solutions.

These solutions were obtained by replacing a time-independent warp factor $h_{\tilde{I}} = H_{\tilde{I}}(z)$ of a supersymmetric solution by a time-dependent function $h_{\tilde{I}} = A_{\tilde{I}} t + H_{\tilde{I}}(z)$.

Our solutions can contain only one function depending on both time t and transverse space coordinates z^i.

Cosmology:

Supposing that our universe stays at a constant position in the bulk space, several four-dimensional effective theories on the branes give four-dimensional Minkowski space or FRW universe.

The power of the scale factor, however, is too small to give a realistic expansion law.

We have to consider additional matter on the brane in order to get a realistic expanding universe.

Black holes:

We can also discuss time-dependent black hole spacetimes which approach asymptotically the FRW universe, if we regard the bulk space as our universe.

The near horizon geometries of these black holes in the expanding universe are the same as the static solutions.

However the asymptotic structures are completely different, giving the FRW universe with scale factors same as the universe filled by stiff matter.

Consistency:

In the viewpoint of higher-dimensional theory, the dynamics of four-dimensional background are given by the solution of higher-dimensional Einstein equations.

If we start from the lower-dimensional effective theory for warped compactification, it sometimes happens that the solutions may not

be allowed in the higher-dimensional theory. (it is the case in M5-M5 brane, M5-M5-M5 and D2-D6 brane systems.)

Reason: the function of z in the metric is integrated out in the lower-dimensional effective action. and the information of the extra dimensions in the function h of the metric will be lost by the compactification.

We have to be careful when we use a four-dimensional effective theory to analyse the moduli stabilization problem and the cosmological problems in the framework of warped compactification of supergravity or M-theory.

Future direction:

Our solutions can contain only one function depending on both time and transverse space coordinates, and this seems to be a limitation on the applications of the solutions.

Recent study of similar systems depending on the light-cone time and space shows that it is possible to obtain solutions with more nontrivial dependence on spacetime coordinates. (Maeda, N.O., Tanabe, Wakebe)

It is interesting to study if similar more general solutions can be obtained by relaxing some of our ansätze.