Flavoured Holographic Duals of 3D Chern-Simons-Matter Theories

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JHEP 0911:125,2009 (ArXiv:0909.3845) (with M. Ammon, J. Erdmenger, A. O'Bannon and T. Wrase)

Outline

1. Motivation: Holographic "Quarks" in AdS5/CFT4
2. Review of the ABJM Construction
3. General Aspects of Flavour in ABJM
4. Codimension Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence
5. Codimension One: $\mathcal{N} = (0, 6)$ Chiral Flavour
6. Codimension One: $\mathcal{N} = (3, 3)$ Nonchiral Flavour
7. Codimension Two: $\mathcal{N} = 4$ Flavour
8. Conclusions & Outlook

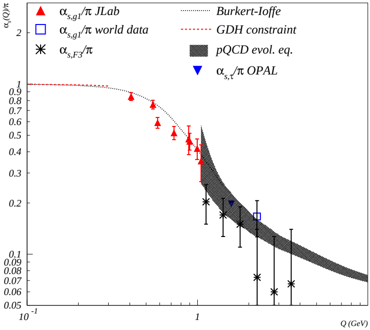
Outline

Motivation: Conformal Window in QCD

New tools for strongly coupled gauge theories?

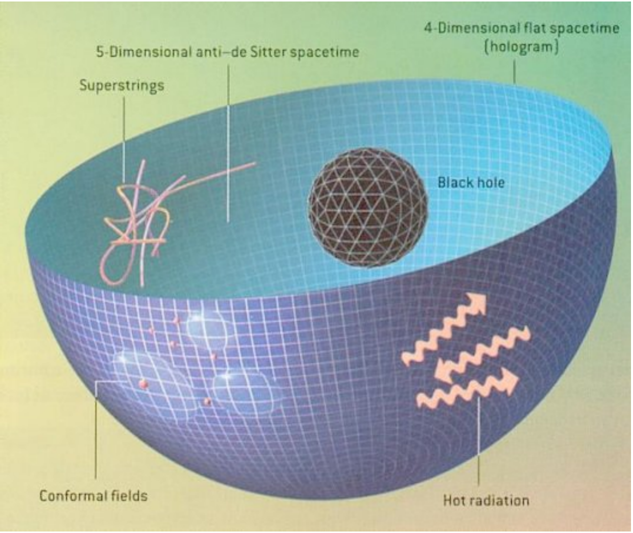
- QCD in the infrared is strongly coupled (Conformal window?)

  [Deur, Korsch et. al: hep-ph/0509113]



- QGP produced at strong coupling ($T >\approx \Lambda_{QCD}$)

Motivation: AdS/CFT Holography



Motivation: AdS/CFT Holography

The Original Correspondence (weakest form)

IIB Supergravity on $AdS_5 \times S^5$ with $(R/\ell_s)^4 = 2\lambda \gg 1$

$$\mathrm{d}s^2_{AdS_5 \times S^5} = R^2 \left(\frac{\mathrm{d}x^{\mu 2} + \mathrm{d}u^2}{u^2} + \mathrm{d}\Omega_5^2 \right)$$

$$\Leftrightarrow$$

large N_c limit of $\mathcal{N} = 4$ $SU(N_c)$ Super Yang-Mills with $\lambda = g_{YM}^2 N_c$

1. Strong-Weak Coupling Duality: $G_N \propto g_s^2 = g_{YM}^4$
2. Gubser-Klebanov-Polyakov-Witten relation:

$$\left\langle e^{-\int \mathrm{d}^4 x J_{\mathcal{O}} \mathcal{O}} \right\rangle = e^{-S_{IIB,\text{onshell}}[J_{\mathcal{O}}]}$$

3. Operator-Field Dictionary:

$$\phi_{m^2 = \Delta(\Delta-4)} \simeq u^{4-\Delta} J_{\mathcal{O}} + u^{\Delta} \langle \mathcal{O} \rangle$$

Motivation: Holographic “Quarks” in AdS5/CFT4

- AdS_5/CFT_4 : Strongly Coupled $\mathcal{N} = 4$ SYM $\rightarrow$ All Adjoint Fields
- What about Quarks ? $\rightarrow$ Fundamental Fields
- Quenched Approximation: Probe Branes, e.g. D3/D7 [Karch/Katz] & D4/D8/$\bar{D}$8 [Sakai/Sugimoto]

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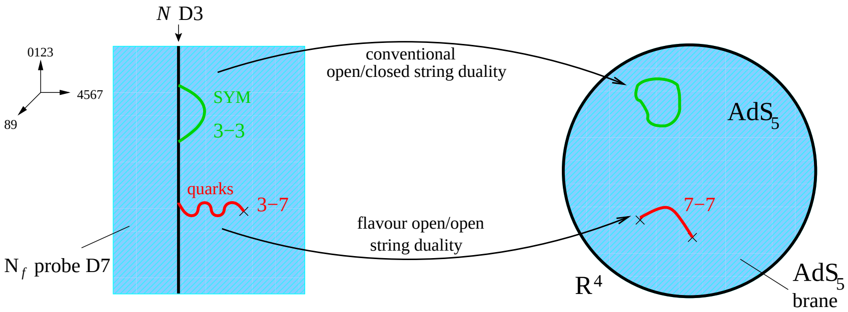

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"Now, what about 2+1 dimensions and duals to CS theories?"

Aharony, Bergman, Jafferis & Maldacena [hep-th/0806.1218]

Duality between
$\mathcal{N} = 6$ $U(N_c)_k \times U(N_c)_{-k}$ Chern-Simons-Matter Theory
and
IIA Supergravity on $AdS_4 \times \mathbb{CP}_3$

- "Matter ": (Anti)-bifundamentals $\rightarrow$ How to add "Quarks"?
- Goal : Classification of Supersymmetric Probe Branes in ABJM and Identification of their CFT Duals
- Possible Applications : Quantum Critical Points? FQHE?

Outline

ABJM Review: Field Theory [hep-th/0806.1218]

- Gauge Group $U(N_c)_k \times U(N_c)_{-k}$
- Gauge Fields : (V_1, Φ_1), $(V_2, \Phi_2) \to \mathcal{N} = 4$ Vector Multiplets
- Bifundamentals : $(A_{a=1,2}, B_{\dot{a}=1,2}) \in \{(N_c, \bar{N}_c), (\bar{N}_c, N_c)\}$
  $\to \mathcal{N} = 4$ Hyper Multiplets: $(A_a, B^\dagger_{\dot{a}})$

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$$
\begin{aligned}
S_{ABJM} &= S_{CS} + S_{kin} + S_{pot} \\
S_{CS} &= -i\,\frac{k}{4\pi} \int d^3x\, d^4\theta \int_0^1 dt\, \mathrm{Tr}\left(V_1 \bar{D}^\alpha \left(e^{tV_1} D_\alpha e^{-tV_1}\right) - (1 \leftrightarrow 2)\right) \\
S_{kin} &= -\int d^3x\, d^4\theta\, \mathrm{Tr}\left(\bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1}\right) \\
S_{pot} &= \int d^3x\, d^2\theta\, W + c.c. \\
W &= -\frac{k}{8\pi} \mathrm{Tr}\left(\Phi_1^2 - \Phi_2^2\right) + \mathrm{Tr}\left(B_a \Phi_1 A_a\right) + \mathrm{Tr}\left(A_a \Phi_2 B_a\right)
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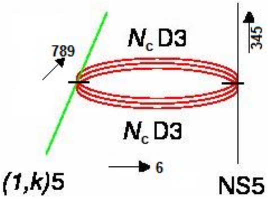
S_{pot} &= \int d^3x\, d^2\theta\, W + c.c. \\
W &= \frac{2\pi}{k} \epsilon^{ab} \epsilon^{\dot{a}\dot{b}} \mathrm{Tr} \left(A_a B_{\dot{a}} A_b B_{\dot{B}} \right)
\end{aligned}
$$

- Symmetries : $\mathcal{N} = 6$ SUSY, $SO(6)_\mathcal{R} \simeq SU(4)_\mathcal{R}$, $U(1)_b : Q_A = -Q_B$

ABJM Review: Brane Construction [hep-th/0806.1218]

- S_{ABJM} is the IR Fixed Point Theory of a IIB Brane Construction:



	0	1	2	3	4	5	6	7	8	9
N_c D3	•	•	•	–	–	–	•	–	–	–
NS5	•	•	•	•	•	•	–	–	–	–
$(1,k)5$	•	•	•	37_θ	48_θ	59_θ	–	–	–	–

$$\theta = \arg(\tau) - \arg(k+\tau)\,, \quad \tau = \frac{i}{g_s} + \chi$$

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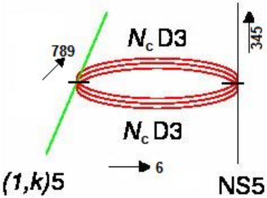

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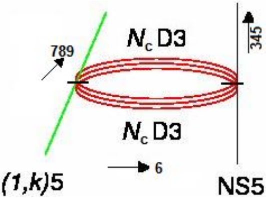

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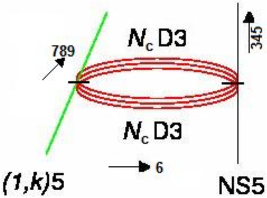

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NS5	•	•	•	•	•	•	–	–	–	–
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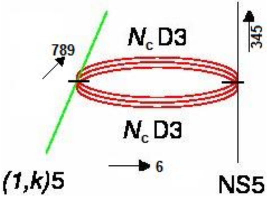

	0	1	2	3	4	5	6	7	8	9
N_c D3	•	•	•	–	–	–	•	–	–	–
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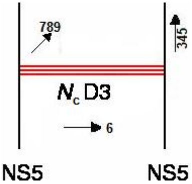
- Field Theory :
  - Both D3 Stacks: $U(N_c) \times U(N_c)$ $\mathcal{N} = 4$ SYM
  - Bifundamentals: Strings between D3 Stacks
  - Effect of NS5/(1,k)5 Boundary Conditions :
    4D $\mathcal{N} = 4$ SYM $\rightarrow$ 3D $\mathcal{N} = 3$ SYM+CS+Matter

Reminder: Hanany-Witten 3D Gauge Theories [hep-th/9611230]

- IIB Brane Construction of a 3D $\mathcal{N}=4$ SYM Theory :



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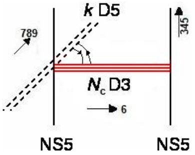
N_c D3	•	•	•	–	–	–	•	–	–	–
NS5	•	•	•	•	•	•	–	–	–	–
NS5	•	•	•	•	•	•	–	–	–	–

- 4D $\mathcal{N}=4$ SYM: $(V, \Phi_1, \Phi_2, \Phi_3)$ w. bosonic field content $(A_\mu, X^{3,4,5,7,8,9})$
- 3D Point of View: $\mathcal{N}=4$ Vector w. $(A_i, X^{3,4,5})$ + $\mathcal{N}=4$ Hyper w. $(A_6, X^{7,8,9})$
- Effect of NS5 Branes: Project out $\mathcal{N}=4$ Hyper Multiplet by Dirichlet Boundary Conditions: ~~$(A_6, X^{7,8,9})$~~
- Low Energy Effective Theory: 3D $\mathcal{N}=4$ $U(N_c)$ SYM Theory (w. gauge coupling $g^2_{YM} \propto g_s/\Delta x^6$)
- Symmetries: $SU(2)_V \times SU(2)_H \simeq SO(3)_{345} \times SO(3)_{789}$

BHKK+ABJM: 3D CS Gauge Theories [hep-th/9908075,ABJM]

- IIB Brane Construction of a 3D SYM-CS-Matter Theory :



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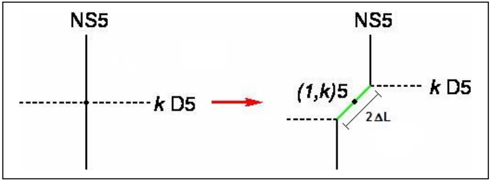
N_c D3	•	•	•	–	–	–	•	–	–	–
NS5	•	•	•	•	•	•	–	–	–	–
NS5	•	•	•	•	•	•	–	–	–	–
k D5	•	•	•	•	•	–	–	–	–	•

- SUSY: $\frac{16}{2\times 2} = 4_{\mathbb{R}}$ supercharges $\Rightarrow \mathcal{N} = 2$ in 3D
- Field Content: $\mathcal{N} = 4$ Vector $(A_i, X^{3,4,5})$ + $\mathcal{N} = 2$ Chirals $q^I, \tilde{q}_I$
- Web Deformation: $q^I, \tilde{q}_I$ acquire real mass $m \propto \Delta L$



$$\int d^4\theta \left[Q_I^\dagger e^{m\bar{\theta}\theta} Q^I + \tilde{Q}^{I\dagger} e^{m\bar{\theta}\theta} \tilde{Q}_I \right]$$

$\mathcal{N} = 2$ SUSY: Tilt by $\theta = \arg\tau - \arg(k+\tau)$ in [59] plane

- Decoupling of $q^I, \tilde{q}_I$: $m \propto \Delta L \to \infty \Rightarrow$ Parity Anomaly!

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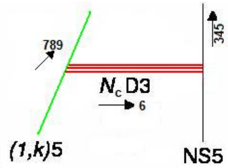

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$(1,k)5$	•	•	•	37_θ	48_θ	59_θ	–	–	–	–

$\theta = \arg(\tau) - \arg(k+\tau)\,, \quad \tau = \frac{i}{g_s} + \chi$

- SUSY Enhancement: Tilt in [37], [48], [59] by angle $\theta \Rightarrow \mathcal{N} = 3$
- Parity Anomaly: Generates Level k Chern-Simons term

$$S_{CS} = \frac{k}{4\pi}\int \mathrm{d}^3x \mathrm{Tr}\left(A \wedge \mathrm{d}A + \frac{2}{3}A^3 - \bar{\chi}\chi + 2D\sigma\right)$$

- By Simple Guessing: [hep-th/9908075] BCs on ONE (p,q)5 Brane

$$0 = \partial_i A_6 - \partial_6 A_i - \frac{g^2_{YM,4}}{4\pi}\frac{q}{p}\epsilon_{ijk}\partial^j A^k$$

$\Rightarrow$ Boundary Term $\frac{q}{4\pi p}\int A \wedge dA$ cancels the last contribution

$\Rightarrow$ Guess: NS5/(1,k)5 BCs are equivalent to NS5/NS5 BCs (forbidding the $\mathcal{N} = 4$ Hyper multiplet) + Generation of the CS term

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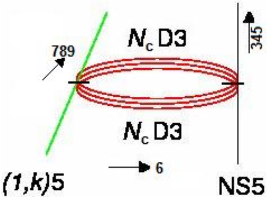

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- SUSY : $6_{\mathbb{R}}$ supercharges $\Rightarrow \mathcal{N} = 3$ in 3D
- T_6 & M-Theory Uplift : N_c M2 Branes at KK/KK+Flux Intersection X_8
- Infrared Limit : Near-Singularity Limit of X_8 $\rightarrow$ N_c M2s at $\mathbb{C}^4/\mathbb{Z}_k$ $(12_{\mathbb{R}})$
- Field Theory :
  - Effect of NS5/(1,k)5 Boundary Conditions :
    4D $\mathcal{N} = 4$ SYM $\rightarrow$ 3D $\mathcal{N} = 3$ SYM+CS+Matter
  - Topological Mass : $m^2_{YM} \propto g^2_{YM} k$
  - IR : 3D $\mathcal{N} = 6$ ABJM Theory

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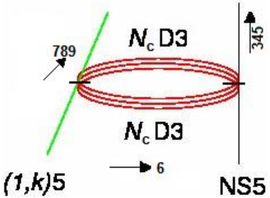

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- ABJM Duality :

  3D $\mathcal{N} = 6$ $U(N_c)_k \times U(N_c)_{-k}$ ABJM Theory

  $\Updownarrow$

  Low Energy Limit of N_c M2s at $\mathbb{C}^4/\mathbb{Z}_k$

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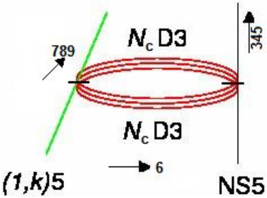

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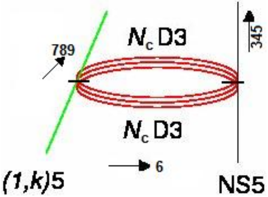
- Two Regimes :
  1. Large $N_c \gg k^5$: 11D SUGRA on $AdS_4 \times S^7/\mathbb{Z}_k$
  2. Large k : $(k^5 \gg N_c \gg k)$ S^7 : $S^1_{\mathbb{Z}_k} \rightarrow \mathbb{CP}_3$

IIA SUGRA on $AdS_4 \times \mathbb{CP}_3$

ABJM Review: Brane Construction [hep-th/0806.1218]

- S_{ABJM} is the IR Fixed Point Theory of a IIB Brane Construction:



	0	1	2	3	4	5	6	7	8	9
N_c D3	•	•	•	–	–	–	•	–	–	–
NS5	•	•	•	•	•	•	–	–	–	–
$(1,k)5$	•	•	•	37_θ	48_θ	59_θ	–	–	–	–

$$\theta = \arg(\tau) - \arg(k+\tau)\,, \quad \tau = \frac{i}{g_s} + \chi$$

- SUSY : $6_{\mathbb{R}}$ supercharges $\Rightarrow \mathcal{N} = 3$ in 3D
- T_6 & M-Theory Uplift : N_c M2 Branes at KK/KK+Flux Intersection X_8
- Infrared Limit : Near-Singularity Limit of X_8 $\to$ N_c M2s at $\mathbb{C}^4/\mathbb{Z}_k$ $(12_{\mathbb{R}})$
- Symmetries :
  1. $SU(4)_R$: $(A_1, B_1^\dagger, B_2^\dagger, A_2) \simeq (z^1, z^2, z^3, z^4) \in \mathbb{C}^4$ transform in **4** of $SU(4)_R$
  2. $U(1)_b$: $z^i \mapsto e^{i\alpha} z^i$ (shifts of M-theory circle)

Outline

SUSY Branes in ABJM & $SU(4)$ Equivalence

- Classification of SUSY Branes in ABJM (along coordinate axes):

#	Type IIB	Type IIA	M theory	codim	wrapping	SUSY	SUSY (anti)
1	D1	D2	M2	2	0 7	2	2
2	D3	D2	M2	0	012 6	6	0
3	D3	D4	M5	1	01 3 7	3	3
4	D3	D4	M5	1	01 3 8	2	2
5	D3	D2	M2	2	0 34 6	2	2
6	D3	D2	M2	2	0 6 78	2	2
7	D5	D6	KK	0	012 34 7	2	2
8	D5	D6	KK	0	012 34 9	4	2
9	D5	D6	KK	0	012 789	6	0
10	D5	D4	M5	1	01 345 6	3	3
11	D5	D4	M5	1	01 3 6 78	2	2
12	D5	D4	M5	1	01 3 6 89	3	3
13	D5	D6	KK	2	0 34 789	2	2
14	D7	D6	KK	0	012 34 6 78	2	4
15	D7	D6	KK	0	012 34 6 79	2	2
16	D7	D8	M9	1	01 345 789	3	3

SUSY Branes in ABJM & $SU(4)$ Equivalence

- Classification of SUSY Branes in ABJM (along coordinate axes):

#	Type IIB	Type IIA	M theory	codim	wrapping	SUSY	SUSY (anti)
1	D1	D2	M2	2	0 7	2	2
2	D3	D2	M2	0	012 6	6	0
3	D3	D4	M5	1	01 3 7	3	3
4	D3	D4	M5	1	01 3 8	2	2
5	D3	D2	M2	2	0 34 6	2	2
6	D3	D2	M2	2	0 6 78	2	2
7	D5	D6	KK	0	012 34 7	2	2
8	D5	D6	KK	0	012 34 9	4	2
9	D5	D6	KK	0	012 789	6	0
10	D5	D4	M5	1	01 345 6	3	3
11	D5	D4	M5	1	01 3 6 78	2	2
12	D5	D4	M5	1	01 3 6 89	3	3
13	D5	D6	KK	2	0 34 789	2	2
14	D7	D6	KK	0	012 34 6 78	2	4
15	D7	D6	KK	0	012 34 6 79	2	2
16	D7	D8	M9	1	01 345 789	3	3

- IIB: $SO(3)_R$ Equivalence - Simultaneous Rotations of (345) and (789)

e.g. D1 along 07, 08 and 09

SUSY Branes in ABJM & $SU(4)$ Equivalence

- Classification of SUSY Branes in ABJM (along coordinate axes):

#	Type IIB	Type IIA	M theory	codim	wrapping	SUSY	SUSY (anti)
1	D1	D2	M2	2	0 7	2	2
2	D3	D2	M2	0	012 6	6	0
3	D3	D4	M5	1	01 3 7	3	3
4	D3	D4	M5	1	01 3 8	2	2
5	D3	D2	M2	2	0 34 6	2	2
6	D3	D2	M2	2	0 6 78	2	2
7	D5	D6	KK	0	012 34 7	2	2
8	D5	D6	KK	0	012 34 9	4	2
9	D5	D6	KK	0	012 789	6	0
10	D5	D4	M5	1	01 345 6	3	3
11	D5	D4	M5	1	01 3 6 78	2	2
12	D5	D4	M5	1	01 3 6 89	3	3
13	D5	D6	KK	2	0 34 789	2	2
14	D7	D6	KK	0	012 34 6 78	2	4
15	D7	D6	KK	0	012 34 6 79	2	2
16	D7	D8	M9	1	01 345 789	3	3

- IIB: $SO(3)_R$ Equivalence - Simultaneous Rotations of (345) and (789)

  e.g. D1 along 07, 08 and 09

- On $\mathbb{C}^4/\mathbb{Z}_k$: $SU(4)_R$ Equivalence - e.g. #9 and #14

  Field Theory: Both IIB constructions flow to same IR fixed point

SUSY Branes in ABJM & $SU(4)$ Equivalence

- Classification of SUSY Branes in ABJM (along coordinate axes):

#	Type IIB	Type IIA	M theory	codim	wrapping	SUSY	SUSY (anti)
1	D1	D2	M2	2	0 7	2	2
2	D3	D2	M2	0	012 6	6	0
3	D3	D4	M5	1	01 3 7	3	3
4	D3	D4	M5	1	01 3 8	2	2
5	D3	D2	M2	2	0 34 6	2	2
6	D3	D2	M2	2	0 6 78	2	2
7	D5	D6	KK	0	012 34 7	2	2
8	D5	D6	KK	0	012 34 9	4	2
9	D5	D6	KK	0	012 789	6	0
10	D5	D4	M5	1	01 345 6	3	3
11	D5	D4	M5	1	01 3 6 78	2	2
12	D5	D4	M5	1	01 3 6 89	3	3
13	D5	D6	KK	2	0 34 789	2	2
14	D7	D6	KK	0	012 34 6 78	2	4
15	D7	D6	KK	0	012 34 6 79	2	2
16	D7	D8	M9	1	01 345 789	3	3

- Interesting Cases :

#2	ABJM Colour Branes	
#9 & 14	Cod. Zero $\mathcal{N} = 3$ Flavour [0901.0924,0903.1730,0903.2175]	#9 : D6 on $AdS_4 \times \mathbb{RP}_3$
#16	Cod. One $\mathcal{N} = (0, 6)$ Flavour	D8 on $AdS_3 \times \mathbb{CP}_3$
#3 & 10	Cod. One $\mathcal{N} = (3, 3)$ Flavour	#3 : D4 on $AdS_3 \times \mathbb{CP}_1$
#5 & 6	Cod. Two $\mathcal{N} = 4$ Flavour	#5 : D2 on $AdS_2 \times$ (1-cycle)

Outline

Cod. Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$ ($6_{\mathbb{R}}$ supercharges)
  $\stackrel{k\rightarrow\infty}{\rightarrow}$ IIA : D6 on $AdS_4 \times \mathbb{RP}^3$ ($12_{\mathbb{R}}$ supercharges)
- IIB : Anti-$D7$ wrapping 012 34 6 78 ($4_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $\Im z^i = 0\,\forall i \stackrel{SU(4)_R}{\rightarrow} z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$

Cod. Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$ ($6_{\mathbb{R}}$ supercharges)
  $\overset{k\to\infty}{\rightarrow}$ IIA : D6 on $AdS_4 \times \mathbb{RP}^3$ ($12_{\mathbb{R}}$ supercharges)
- IIB : Anti-$D7$ wrapping 012 34 6 78 ($4_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $\Im z^i = 0\,\forall i \overset{SU(4)_R}{\rightarrow} z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$
- D5 Theory : fixed by $\mathcal{N} = 3$ superconformal symmetry [0903.1730,0903.2175]
  Field Content : $\mathcal{N} = 4$ Hyper $(Q_i, \tilde{Q}_i^\dagger)\,,\ i = 1, 2$ (parity!)

Cod. Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1 = \bar{z}^3$, $z^2 = \bar{z}^4$ ($6_{\mathbb{R}}$ supercharges)
  $\overset{k\to\infty}{\rightarrow}$ IIA : D6 on $AdS_4 \times \mathbb{RP}^3$ ($12_{\mathbb{R}}$ supercharges)
- IIB : Anti-$D7$ wrapping 012 34 6 78 ($4_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $\Im z^i = 0 \, \forall i \overset{SU(4)_R}{\rightarrow} z^1 = \bar{z}^3$, $z^2 = \bar{z}^4$
- D5 Theory : fixed by $\mathcal{N} = 3$ superconformal symmetry [0903.1730,0903.2175]
  Field Content : $\mathcal{N} = 4$ Hyper $(Q_i, \tilde{Q}_i^\dagger)$, $i = 1, 2$ (parity!)

$$
\begin{aligned}
S_{D5} &= S_{CS} + S_{kin,A,B} + S_{flavour} + \int \mathrm{d}^3x\mathrm{d}^2\theta W + c.c. \\
S_{flavour} &= -\sum_{i=1,2} \int \mathrm{d}^x\mathrm{d}^4\theta \left(Q_i^\dagger e^{-V_i} Q_i + \tilde{Q}_i e^{V_i} \tilde{Q}_i^\dagger \right) \\
W &= -\frac{k}{8\pi} \mathrm{Tr}\left(\Phi_1^2 - \Phi_2^2\right) + \mathrm{Tr}\left(B_a \Phi_1 A_a\right) + \mathrm{Tr}\left(A_a \Phi_2 B_a\right) + \\
&\quad + \tilde{Q}_1 \Phi_1 Q_1 - \tilde{Q}_2 \Phi_2 Q_2
\end{aligned}
$$

Cod. Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$ ($6_{\mathbb{R}}$ supercharges)
  $\overset{k\to\infty}{\rightarrow}$ IIA : D6 on $AdS_4 \times \mathbb{RP}^3$ ($12_{\mathbb{R}}$ supercharges)
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- D5 Theory : fixed by $\mathcal{N} = 3$ superconformal symmetry [0903.1730,0903.2175]
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S_{D5} &= S_{CS} + S_{kin,A,B} + S_{flavour} + \int \mathrm{d}^3x\mathrm{d}^2\theta W + c.c. \\
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W &= \frac{2\pi}{k}\mathrm{Tr}\left[(A_aB_a + Q_1\tilde{Q}_1)^2 - (B_aA_a - Q_2\tilde{Q}_2)^2 \right]
\end{aligned}
$$

Symm. : $SU(2)_R \times \mathrm{diag}(SU(2)_A \times SU(2)_B) \times U(1)_b \times U(N_f)_1 \times U(N_f)_2$

Cod. Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$ ($6_{\mathbb{R}}$ supercharges)
  $\overset{k\to\infty}{\rightarrow}$ IIA : D6 on $AdS_4 \times \mathbb{RP}^3$ ($12_{\mathbb{R}}$ supercharges)
- IIB : Anti-$D7$ wrapping 012 34 6 78 ($4_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $\Im z^i = 0\,\forall i \overset{SU(4)_R}{\rightarrow} z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$
- Anti-D7 Theory :
  1. 4ND D3/D7: $\mathcal{N} = 2$ Hypers $(Q^I, \tilde{Q}^{I\dagger})$ coupled to $\mathcal{N} = 4$ SYM via $(A_{0,1,2,6}, X^5, X^9)$

Cod. Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_{\mathbb{R}}$ supercharges)
  $\to \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1 = \bar{z}^3$, $z^2 = \bar{z}^4$ ($6_{\mathbb{R}}$ supercharges)
  $\overset{k\to\infty}{\to}$ IIA : D6 on $AdS_4 \times \mathbb{RP}^3$ ($12_{\mathbb{R}}$ supercharges)
- IIB : Anti-$D7$ wrapping 012 34 6 78 ($4_{\mathbb{R}}$ supercharges)
  $\to \mathbb{C}^4/\mathbb{Z}_k$: KK w. $\Im z^i = 0 \, \forall i \overset{SU(4)_R}{\to} z^1 = \bar{z}^3 , \; z^2 = \bar{z}^4$
- Anti-D7 Theory :
  1. 4ND D3/D7: $\mathcal{N} = 2$ Hypers $(Q^I, \tilde{Q}^{I\dagger})$ coupled to $\mathcal{N} = 4$ SYM via $(A_{0,1,2,6}, X^5, X^9)$

$$\mathcal{L} = \Im\Big[\tau \int d^2\theta d^2\bar{\theta} \left(\mathrm{tr}(\bar{\Phi}_i e^V \Phi_i e^{-V}) + Q_I^\dagger e^V Q^I + \tilde{Q}^{I\dagger} e^{-V} \tilde{Q}_I\right) + \\ + \tau \int d^2\theta \left(\mathrm{tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) + \mathrm{tr}(\epsilon_{ijk}\Phi_i\Phi_j\Phi_k) + \tilde{Q}_I \Phi_3 Q^I\right)\Big]$$

Cod. Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_{\mathbb{R}}$ supercharges)
  $\rightarrow \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$ ($6_{\mathbb{R}}$ supercharges)
  $\stackrel{k\to\infty}{\rightarrow}$ IIA : D6 on $AdS_4 \times \mathbb{RP}^3$ ($12_{\mathbb{R}}$ supercharges)
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- Anti-D7 Theory :
  1. 4ND D3/D7: $\mathcal{N} = 2$ Hypers $(Q^I, \tilde{Q}^{I\dagger})$ coupled to $\mathcal{N} = 4$ SYM via $(A_{0,1,2,6}, X^5, X^9)$
  2. NS5/(1,k)5 Effects:
     [Hanany/Witten] NS5-D3s-NS5 $\Rightarrow$ 3D $\mathcal{N} = 4$ Vector w. $(A_{0,1,2}, X^{3,4,5})$ survives, 3D $\mathcal{N} = 4$ Hyper w. $(A_6, X^{7,8,9})$ is projected out
     [Bergman/Hanany/Karch/Kol] NS5-D3s-(1,k)5 $\Rightarrow$ Hanany-Witten-BCs & $\mathcal{N} = 3$ CS-Term
  3. D3/Anti-D7/NS5/(1,k)5: Preserves $\mathcal{N} = 2$ SUSY $\Rightarrow$ Flavours couple to a $\mathcal{N} = 2$ Vector Multiplet with $(A_{0,1,2}, X^5) \Rightarrow W_{Flavour} = 0?$
  4. IR Limit: [Gaiotto/Yin] Flow to $\mathcal{N} = 3$ Theory with $W_{Flavour}$ generated

Cod. Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_{\mathbb{R}}$ supercharges)
  $\to \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1 = \bar{z}^3$, $z^2 = \bar{z}^4$ ($6_{\mathbb{R}}$ supercharges)
  $\stackrel{k\to\infty}{\to}$ IIA : D6 on $AdS_4 \times \mathbb{RP}^3$ ($12_{\mathbb{R}}$ supercharges)
- IIB : Anti-$D7$ wrapping 012 34 6 78 ($4_{\mathbb{R}}$ supercharges)
  $\to \mathbb{C}^4/\mathbb{Z}_k$: KK w. $\Im z^i = 0 \, \forall i \stackrel{SU(4)_R}{\to} z^1 = \bar{z}^3$, $z^2 = \bar{z}^4$
- Anti-D7 Theory :
  1. 4ND D3/D7: $\mathcal{N} = 2$ Hypers $(Q^I, \tilde{Q}^{I\dagger})$ coupled to $\mathcal{N} = 4$ SYM via $(A_{0,1,2,6}, X^5, X^9)$
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Cod. Zero: $\mathcal{N}=3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_\mathbb{R}$ supercharges)
  $\to \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1=\bar{z}^3,\ z^2=\bar{z}^4$ ($6_\mathbb{R}$ supercharges)
  $\overset{k\to\infty}{\to}$ IIA : D6 on $AdS_4\times\mathbb{RP}^3$ ($12_\mathbb{R}$ supercharges)
- IIB : Anti-$D7$ wrapping 012 34 6 78 ($4_\mathbb{R}$ supercharges)
  $\to \mathbb{C}^4/\mathbb{Z}_k$: KK w. $\Im z^i=0\,\forall i \overset{SU(4)_R}{\to} z^1=\bar{z}^3,\ z^2=\bar{z}^4$
- Anti-D7 Theory :
  1. 4ND D3/D7: $\mathcal{N}=2$ Hypers $(Q^I,\tilde{Q}^{I\dagger})$ coupled to $\mathcal{N}=4$ SYM via $(A_{0,1,2,6},X^5,X^9)$
  2. NS5/(1,k)5 Effects:
     [Hanany/Witten] NS5-D3s-NS5 $\Rightarrow$ 3D $\mathcal{N}=4$ Vector w. $(A_{0,1,2},X^{3,4,5})$ survives, 3D $\mathcal{N}=4$ Hyper w. $(A_6,X^{7,8,9})$ is projected out
     [Bergman/Hanany/Karch/Kol] NS5-D3s-(1,k)5 $\Rightarrow$ Hanany-Witten-BCs & $\mathcal{N}=3$ CS-Term
  3. D3/Anti-D7/NS5/(1,k)5: Preserves $\mathcal{N}=2$ SUSY $\Rightarrow$ Flavours couple to a $\mathcal{N}=2$ Vector Multiplet with $(A_{0,1,2},X^5)\Rightarrow W_{Flavour}=0?$
  4. IR Limit: [Gaiotto/Yin] Flow to $\mathcal{N}=3$ Theory with $W_{Flavour}$ generated

Cod. Zero: $\mathcal{N} = 3$ Flavour & SU(4) Equivalence

- IIB : D5 wrapping 012 789 ($6_{\mathbb{R}}$ supercharges)
  $\to \mathbb{C}^4/\mathbb{Z}_k$: KK w. $z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$ ($6_{\mathbb{R}}$ supercharges)
  $\overset{k\to\infty}{\to}$ IIA : D6 on $AdS_4 \times \mathbb{RP}^3$ ($12_{\mathbb{R}}$ supercharges)
- IIB : Anti-$D7$ wrapping 012 34 6 78 ($4_{\mathbb{R}}$ supercharges)
  $\to \mathbb{C}^4/\mathbb{Z}_k$: KK w. $\Im z^i = 0\,\forall i \overset{SU(4)_R}{\to} z^1 = \bar{z}^3\,,\ z^2 = \bar{z}^4$

Codimension Zero & SU(4) Equivalence – Conclusion

At low energies, the D3/Anti-D7/NS5/(1,k)5 and the D3/D5/NS5/(1,k)5 system of [Kirsch/Hohenegger,Gaiotto/Jafferis] both flow to the $\mathcal{N} = 3$ superconformal CS-Bifundamental-Flavour theory

$$S_{D5} = S_{CS} + S_{kin,A,B} + S_{kin,Flavour} + \int \mathrm{d}^3x\mathrm{d}^2\theta W + c.c.$$

$$W = \frac{2\pi}{k}\mathrm{Tr}\left[(A_aB_a + Q_1\tilde{Q}_1)^2 - (B_aA_a - Q_2\tilde{Q}_2)^2\right]$$

In 11D, this is reflected by the SU(4) equivalence of both flavour M2 branes on $\mathbb{R}^{1,2} \times \mathbb{C}^4/\mathbb{Z}_k$.

Outline

Cod. One: $\mathcal{N} = (0, 6)$ Chiral Flavour

- IIB : D7 wrapping 01 345 789 ($3_{\mathbb{R}}$) $\rightarrow$ M9 wrapping $\mathbb{R}^{1,1} \times \mathbb{C}^4/\mathbb{Z}_k$ ($6_{\mathbb{R}}$) $\stackrel{k\to\infty}{\rightarrow}$ D8 on $AdS_3 \times \mathbb{CP}_3$ ($12_{\mathbb{R}}$)
- Field Theory :
  1. 8ND D3/D7: [Buchbinder/Gomis/Passerini,Harvey/Royston]
     3-7 Open String ZPEs: $E_R = 0\,,\ E_{NS} = \frac{\nu}{8} - \frac{1}{2} = \frac{1}{2}$
     $\rightarrow$ GSO: (R) Ground State is a single left-handed (e.g. left-moving) Weyl spinor $\psi_q \in (N_c, \bar{N}_f)$, coupling only to the gauge field via

     $$S_{(0,8)} = \int \mathrm{d}x^+ \mathrm{d}x^- \psi_q^\dagger (i\partial_- - A_-)\psi_q$$

  2. NS5/(1,k)5 Effects: Project out $(A_6, X^{7,8,9})$ + Generate CS-Term
  3. IR Flow: Marginality ($[\psi_q^\dagger \psi_q] = M$) and (1+1)-dimensional Lorentz invariance ($\psi_q \mapsto e^{\gamma}\psi_q$) forbid any additional terms
- SUSY : $S_{ABJM} + S_{(0,6)}$ preserves $\mathcal{N} = (0, 6)$ superconformal symmetry

  In particular: $\delta_\varepsilon A_- = 0$
- Symmetries : $SU(4) \times U(1)_b \times U(1)_q \times$ Flavour Symmetries

Cod. One: $\mathcal{N} = (0,6)$ Chiral Flavour

- IIB : D7 wrapping 01 345 789 ($3_{\mathbb{R}}$) $\to$ M9 wrapping $\mathbb{R}^{1,1} \times \mathbb{C}^4/\mathbb{Z}_k$ ($6_{\mathbb{R}}$) $\stackrel{k\to\infty}{\to}$ D8 on $AdS_3 \times \mathbb{CP}_3$ ($12_{\mathbb{R}}$)
- Field Theory :
  1. 8ND D3/D7: [Buchbinder/Gomis/Passerini,Harvey/Royston]
     3-7 Open String ZPEs: $E_R = 0\,,\ E_{NS} = \frac{\nu}{8} - \frac{1}{2} = \frac{1}{2}$
     $\to$ GSO: (R) Ground State is a single left-handed (e.g. left-moving) Weyl spinor $\psi_q \in (N_c, \bar{N}_f)$, coupling only to the gauge field via

     $$S_{(0,8)} = \int \mathrm{d}x^+ \mathrm{d}x^- \psi_q^\dagger (i\partial_- - A_-) \psi_q$$

  2. NS5/(1,k)5 Effects: Project out $(A_6, X^{7,8,9})$ + Generate CS-Term
  3. IR Flow: Marginality ($[\psi_q^\dagger \psi_q] = M$) and (1+1)-dimensional Lorentz invariance ($\psi_q \mapsto e^\gamma \psi_q$) forbid any additional terms
- SUSY : $S_{ABJM} + S_{(0,6)}$ preserves $\mathcal{N} = (0,6)$ superconformal symmetry

  In particular: $\delta_\varepsilon A_- = 0$
- Symmetries : $SU(4) \times U(1)_b \times U(1)_q \times$ Flavour Symmetries

Cod. One: $\mathcal{N} = (0,6)$ Chiral Flavour

- IIB : D7 wrapping 01 345 789 ($3_{\mathbb{R}}$) $\rightarrow$ M9 wrapping $\mathbb{R}^{1,1} \times \mathbb{C}^4/\mathbb{Z}_k$ ($6_{\mathbb{R}}$) $\stackrel{k\rightarrow\infty}{\rightarrow}$ D8 on $AdS_3 \times \mathbb{CP}_3$ ($12_{\mathbb{R}}$)
- Field Theory :
  1. 8ND D3/D7: [Buchbinder/Gomis/Passerini,Harvey/Royston]
     3-7 Open String ZPEs: $E_R = 0\,,\ E_{NS} = \frac{\nu}{8} - \frac{1}{2} = \frac{1}{2}$
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Cod. One: $\mathcal{N} = (0, 6)$ Chiral Flavour

- IIB : D7 wrapping 01 345 789 ($3_{\mathbb{R}}$) $\rightarrow$ M9 wrapping $\mathbb{R}^{1,1} \times \mathbb{C}^4/\mathbb{Z}_k$ ($6_{\mathbb{R}}$) $\overset{k\to\infty}{\rightarrow}$ D8 on $AdS_3 \times \mathbb{CP}_3$ ($12_{\mathbb{R}}$)
- Field Theory :
  1. 8ND D3/D7: [Buchbinder/Gomis/Passerini,Harvey/Royston]
     3-7 Open String ZPEs: $E_R = 0\,,\ E_{NS} = \frac{\nu}{8} - \frac{1}{2} = \frac{1}{2}$
     $\rightarrow$ GSO: (R) Ground State is a single left-handed (e.g. left-moving) Weyl spinor $\psi_q \in (N_c, \bar{N}_f)$, coupling only to the gauge field via

     $$S_{(0,8)} = \int \mathrm{d}x^+ \mathrm{d}x^- \psi_q^\dagger \,(i\partial_- - A_-)\, \psi_q$$

  2. NS5/(1,k)5 Effects: Project out $(A_6, X^{7,8,9})$ + Generate CS-Term
  3. IR Flow: Marginality ($[\psi_q^\dagger \psi_q] = M$) and (1+1)-dimensional Lorentz invariance ($\psi_q \mapsto e^\gamma \psi_q$) forbid any additional terms
- SUSY : $S_{ABJM} + S_{(0,6)}$ preserves $\mathcal{N} = (0, 6)$ superconformal symmetry

  In particular: $\delta_\varepsilon A_- = 0$
- Symmetries : $SU(4) \times U(1)_b \times U(1)_q \times$ Flavour Symmetries

Cod. One: $\mathcal{N} = (0,6)$ Chiral Flavour

- IIB : D7 wrapping 01 345 789 ($3_{\mathbb{R}}$) $\to$ M9 wrapping $\mathbb{R}^{1,1} \times \mathbb{C}^4/\mathbb{Z}_k$ ($6_{\mathbb{R}}$) $\overset{k\to\infty}{\to}$ D8 on $AdS_3 \times \mathbb{CP}_3$ ($12_{\mathbb{R}}$)
- Field Theory :
  1. 8ND D3/D7: [Buchbinder/Gomis/Passerini,Harvey/Royston]
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  In particular: $\delta_\varepsilon A_- = 0$
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Cod. One: $\mathcal{N} = (0,6)$ Chiral Flavour

- IIB : D7 wrapping 01 345 789 ($3_\mathbb{R}$) $\to$ M9 wrapping $\mathbb{R}^{1,1} \times \mathbb{C}^4/\mathbb{Z}_k$ ($6_\mathbb{R}$) $\stackrel{k\to\infty}{\to}$ D8 on $AdS_3 \times \mathbb{CP}_3$ ($12_\mathbb{R}$)
- Field Theory :
  1. 8ND D3/D7: [Buchbinder/Gomis/Passerini,Harvey/Royston]
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Outline

Cod. One: $\mathcal{N} = (3,3)$ Nonchiral Flavour

- IIB : D3 (01 3 7) $\rightarrow$ M5 at $z^1 = z^2$, $z^3 = z^4$ $\rightarrow$ IIA : D4 on $AdS_3 \times \mathbb{CP}_1$
- SU(4) Equivalence : D5 (01 345 6) $\rightarrow$ M5 at $z^1 = z^2 = 0$
- SUSY : IIB: $3_{\mathbb{R}} \rightarrow \mathbb{C}^4/\mathbb{Z}_k$: $6_{\mathbb{R}} \rightarrow AdS_4 \times S^7/\mathbb{Z}_k$: $12_{\mathbb{R}} \Rightarrow \mathcal{N} = (3,3)$ superconformal ($k \geq 2$)
- Symmetries : $SU(2) \times SU(2) \times U(1) \times U(1)_b \times$ Flavour Symmetries
- D3 Field Theory : What we know and what we don't.
  1. 4ND D3/D3: [Constable/Erdmenger/Kirsch/Guralnik hep-th/0211222]
     Fields: $\mathcal{N} = (4,4)$ Hyper $(Q, \tilde{Q}^\dagger)$
     Couplings: Only to $\mathcal{N} = (4,4)$ Vector $(A_{0,1}; X^{4,5,8,9}; D_{(2)}, F_{(2)})$
  2. NS5/(1,k)5 Effect: Flavours couple to $\mathcal{N} = (2,2)$ Vector $(A_{0,1}; X^{4,5}, D_{(2)})$ only & CS-Term is generated
  3. IR Flow: Elusive $\rightarrow$ Work in Progress

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  2. NS5/(1,k)5 Effect: Flavours couple to $\mathcal{N} = (2,2)$ Vector $(A_{0,1}; X^{4,5}, D_{(2)})$ only & CS-Term is generated

     Question 1: $D_{(2)} = \frac{1}{\sqrt{2}}(D_{(4)} + F_{26}) \overset{NS5/(1,k)5}{=} 0$? Meaning?
  3. IR Flow: Elusive $\rightarrow$ Work in Progress

Cod. One: $\mathcal{N} = (3,3)$ Nonchiral Flavour

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- D3 Field Theory : What we know and what we don't.
  1. 4ND D3/D3: [Constable/Erdmenger/Kirsch/Guralnik hep-th/0211222]
     Fields: $\mathcal{N} = (4,4)$ Hyper $(Q, \tilde{Q}^\dagger)$
     Couplings: Only to $\mathcal{N} = (4,4)$ Vector $(A_{0,1}; X^{4,5,8,9}; D_{(2)}, F_{(2)})$
  2. NS5/(1,k)5 Effect: Flavours couple to $\mathcal{N} = (2,2)$ Vector $(A_{0,1}; X^{4,5}, D_{(2)})$ only & CS-Term is generated

     Question 1: $D_{(2)} = \frac{1}{\sqrt{2}}(D_{(4)} + F_{26}) \overset{NS5/(1,k)5}{=} 0$? Meaning?

     Question 2: Identification of $SU(2) \times SU(2)$ in IR?
  3. IR Flow: Elusive $\rightarrow$ Work in Progress

Cod. One: $\mathcal{N} = (3,3)$ Nonchiral Flavour

- IIB : D3 (01 3 7) $\rightarrow$ M5 at $z^1 = z^2$, $z^3 = z^4$ $\rightarrow$ IIA : D4 on $AdS_3 \times \mathbb{CP}_1$
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- D3 Field Theory : What we know and what we don't.
  1. 4ND D3/D3: [Constable/Erdmenger/Kirsch/Guralnik hep-th/0211222]
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     Couplings: Only to $\mathcal{N} = (4,4)$ Vector $(A_{0,1}; X^{4,5,8,9}; D_{(2)}, F_{(2)})$
  2. NS5/(1,k)5 Effect: Flavours couple to $\mathcal{N} = (2,2)$ Vector $(A_{0,1}; X^{4,5}, D_{(2)})$ only & CS-Term is generated

     Question 1: $D_{(2)} = \frac{1}{\sqrt{2}}(D_{(4)} + F_{26}) \overset{NS5/(1,k)5}{=} 0$? Meaning?

     Question 2: Identification of $SU(2) \times SU(2)$ in IR?
  3. IR Flow: Elusive $\rightarrow$ Work in Progress

Outline

Cod. Two: $\mathcal{N} = 4$ Localised Flavour

- IIB : D3 (0 34 6) $\rightarrow$ M2 at $z^1 = z^2 = 0\,, z^3 = \bar{z}^4$ $\rightarrow$ IIA : D2 on $AdS_2 \times$ (1-cycle)
- SU(4) Equivalence : D3 (0 6 78)
- SUSY : IIB: $2_{\mathbb{R}} \rightarrow \mathbb{C}^4/\mathbb{Z}_k$: $4_{\mathbb{R}} \rightarrow AdS_4 \times S^7/\mathbb{Z}_k$: $8_{\mathbb{R}} \Rightarrow \mathcal{N} = 4$ superconformal symmetry
- Flavour Action : What we know and what needs to be done.

Cod. Two: $\mathcal{N} = 4$ Localised Flavour

- IIB : D3 (0 34 6) $\rightarrow$ M2 at $z^1 = z^2 = 0\,, z^3 = \bar{z}^4$ $\rightarrow$ IIA : D2 on $AdS_2 \times$ (1-cycle)
- SU(4) Equivalence : D3 (0 6 78)
- SUSY : IIB: $2_{\mathbb{R}} \rightarrow \mathbb{C}^4/\mathbb{Z}_k$: $4_{\mathbb{R}} \rightarrow AdS_4 \times S^7/\mathbb{Z}_k$: $8_{\mathbb{R}} \Rightarrow \mathcal{N} = 4$ superconformal symmetry
- Flavour Action : What we know and what needs to be done.

Cod. Two: $\mathcal{N} = 4$ Localised Flavour

- IIB : D3 (0 34 6) $\rightarrow$ M2 at $z^1 = z^2 = 0\,, z^3 = \bar{z}^4$ $\rightarrow$ IIA : D2 on $AdS_2 \times$ (1-cycle)
- SU(4) Equivalence : D3 (0 6 78)
- SUSY : IIB: $2_{\mathbb{R}} \rightarrow \mathbb{C}^4/\mathbb{Z}_k$: $4_{\mathbb{R}} \rightarrow AdS_4 \times S^7/\mathbb{Z}_k$: $8_{\mathbb{R}} \Rightarrow \mathcal{N} = 4$ superconformal symmetry
- Flavour Action : What we know and what needs to be done.

Cod. Two: $\mathcal{N} = 4$ Localised Flavour

- IIB : D3 (0 34 6) $\rightarrow$ M2 at $z^1 = z^2 = 0\,, z^3 = \bar{z}^4$ $\rightarrow$ IIA : D2 on $AdS_2 \times$ (1-cycle)
- SU(4) Equivalence : D3 (0 6 78)
- SUSY : IIB: $2_{\mathbb{R}} \rightarrow \mathbb{C}^4/\mathbb{Z}_k$: $4_{\mathbb{R}} \rightarrow AdS_4 \times S^7/\mathbb{Z}_k$: $8_{\mathbb{R}} \Rightarrow \mathcal{N} = 4$ superconformal symmetry
- Symmetries : $SU(2) \times U(1)_1 \times U(1)_2 \times$ ~~$U(1)_b$~~ $\times$ Flavour Symmetries

  $SU(2)$: (z^1, z^2), $U(1)_1$: $(z^1, z^2) \mapsto e^{i\alpha}(z^1, z^2)$, $U(1)_2$: $(z^3, z^4) \mapsto (e^{i\alpha} z^3, e^{-i\alpha} z^4)$
- Flavour Action : What we know and what needs to be done.

Cod. Two: $\mathcal{N} = 4$ Localised Flavour

- IIB : D3 (0 34 6) $\rightarrow$ M2 at $z^1 = z^2 = 0\,, z^3 = \bar{z}^4$ $\rightarrow$ IIA : D2 on $AdS_2 \times$ (1-cycle)
- SU(4) Equivalence : D3 (0 6 78)
- SUSY : IIB: $2_{\mathbb{R}} \rightarrow \mathbb{C}^4/\mathbb{Z}_k$: $4_{\mathbb{R}} \rightarrow AdS_4 \times S^7/\mathbb{Z}_k$: $8_{\mathbb{R}} \Rightarrow \mathcal{N} = 4$ superconformal symmetry
- Symmetries : $SU(2)_R \times U(1)_1 \times U(1)_2 \times$ ~~$U(1)_b$~~ $\times$ Flavour Symmetries

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- R Symmetry? [Britto/Michelson/Strominger/Volovich hep-th/9911066] $8_{\mathbb{R}}$ SUSYs + One $SU(2) \Rightarrow SU(1,1|2) \Rightarrow SU(2)_R$
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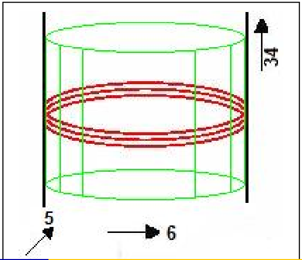

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Outline

Conclusions & Outlook

- Large Variety of SUSY Probe Branes in ABJM IIB Construction
- $SU(4)_R$ Equivalence : Flow to same IR Fixpoint
- Cod. Zero $\mathcal{N} = 3$:
  – D5 along (012 789) $\equiv$ Anti-D7 along (012 34 6 78)
  – Field Theory
- Cod. One $\mathcal{N} = (0, 6)$:
  – D7 along (01 345 789) $\rightarrow$ (1+1)-dim. Gauged Chiral Fermion
- Cod. One $\mathcal{N} = (3, 3)$:
  – D3 along (01 3 7) $\equiv$ D5 along (01 345 6)
  – $\mathcal{N} = (4, 4)$ Hypers $(Q, \tilde{Q}^\dagger)$ coupled to $\mathcal{N} = (2, 2)$ Vector $(A_{0,1}, X^{4,5})$
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