Short Range Correlations and Spectral Functions in Exotic Nuclear Matter

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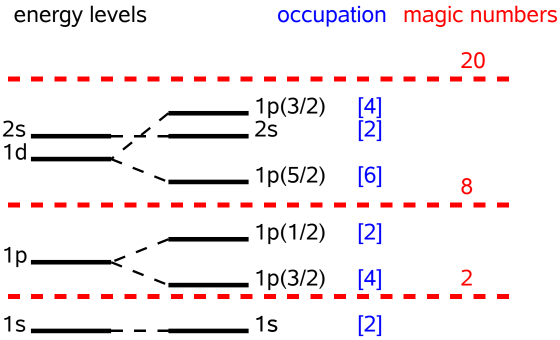
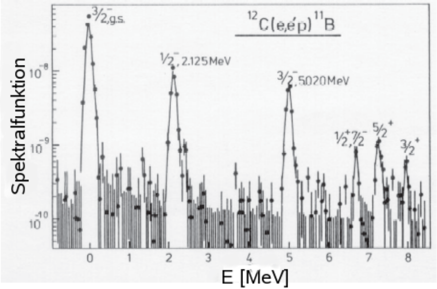
Shell-model

- nucleons move independently in static nuclear potential
- shell structures like in atoms



Experimental Evidences for Correlations

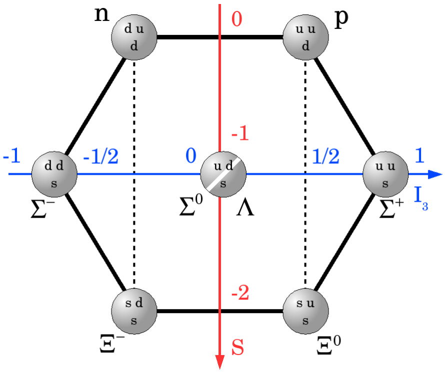
$^{12}C + e \longrightarrow {}^{11}B + p + e'$ (de Witt Huberts, J. Phys. G, 1990)



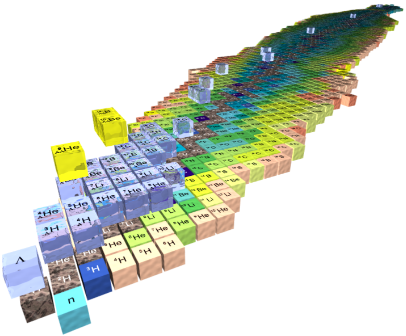
knock-out of protons

- $E < 6$ MeV: $1p$ shell (predictions of the shell-model)
- $E > 6$ MeV: $2s(1/2)$ and $1d(5/2, 3/2)$ shells (Correlations!!!)

Exotic Matter



Exotic Matter



(experiments for example @GSI)

Aim

investigation of correlations in neutron rich infinite nuclear matter with strangeness admixture

$\Rightarrow$ better understanding of neutron stars

One-Body Green's Function

Definition

$$G(\vec{r}_1, t_1, \vec{r}_2, t_2) = -\frac{i}{\hbar}\left\langle T\left\{\Psi_H(\vec{r}_1, t_1)\Psi_H^\dagger(\vec{r}_2, t_2)\right\}\right\rangle$$

propagation of an extra particle (hole) added to our system

Temperature $T = 0$

$$\langle A\rangle = \langle\Psi_0\,|A|\,\Psi_0\rangle$$

Temperature $T > 0$

thermal average over the grand canonical ensemble

$$\langle A\rangle = \frac{\mathrm{Tr}\left[Ae^{-\beta(H-\mu N)}\right]}{\mathrm{Tr}\left[e^{-\beta(H-\mu N)}\right]}$$

Time ordering operator

$$T\left\{\Psi(\vec{r}_1,t_1)\Psi^\dagger(\vec{r}_2,t_2)\right\} = \begin{cases}\Psi(\vec{r}_1,t_1)\Psi^\dagger(\vec{r}_2,t_2) & \text{for } t_1 > t_2\\ -\Psi^\dagger(\vec{r}_2,t_2)\Psi(\vec{r}_1,t_1) & \text{for } t_1 < t_2\end{cases}$$

$$G(\vec{r}_1,t_1,\vec{r}_2,t_2) = \theta(t_1-t_2)\, G^>(\vec{r}_1,t_1,\vec{r}_2,t_2) + \theta(t_2-t_1)\, G^<(\vec{r}_1,t_1,\vec{r}_2,t_2)$$

Particle Correlation Function

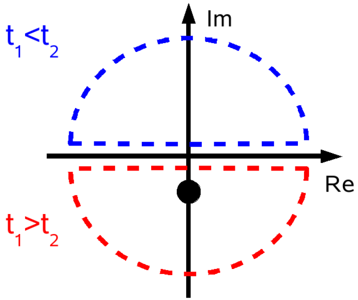
$$G^>(\vec{r}_1,t_1,\vec{r}_2,t_2) = -\frac{i}{\hbar}\langle\Psi_H(\vec{r}_1,t_1)\Psi_H^\dagger(\vec{r}_2,t_2)\rangle$$

Hole Correlation Function

$$G^<(\vec{r}_1,t_1,\vec{r}_2,t_2) = \frac{i}{\hbar}\langle\Psi_H^\dagger(\vec{r}_2,t_2)\Psi_H(\vec{r}_1,t_1)\rangle$$

Spectral Representation of the Step Function

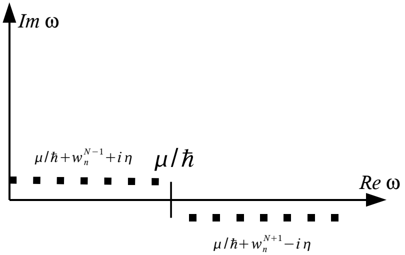
$$\theta(t_1 - t_2) = -\int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi i} \frac{e^{-i\omega(t_1-t_2)}}{\omega + i\eta}$$



Lehmann Representation

$$G(\vec{k},\omega) = \frac{1}{\hbar}\sum_n \frac{\langle\Psi_0|\Psi(\vec{k})|\Psi_n^{N+1}\rangle\langle\Psi_n^{N+1}|\Psi^\dagger(\vec{k})|\Psi_0\rangle}{\omega - \mu/\hbar - w_n^{N+1} + i\eta} + \frac{1}{\hbar}\sum_m \frac{\langle\Psi_0|\Psi^\dagger(\vec{k})|\Psi_m^{N-1}\rangle\langle\Psi_m^{N-1}|\Psi(\vec{k})|\Psi_0\rangle}{\omega - \mu/\hbar - w_m^{N-1} - i\eta}$$

$$w_n^{N\pm1} = \pm\frac{1}{\hbar}(E_n^{N\pm1} - E)$$ excitation energy



Thermodynamic Limit

$$G(\vec{k},\omega) = \int_0^\infty \frac{\mathrm{d}\omega'}{2\pi}\left[\frac{\mathcal{A}_p(\vec{k},\omega')}{\omega-\mu/\hbar-\omega'+i\eta} + \frac{\mathcal{A}_h(\vec{k},\omega')}{\omega-\mu/\hbar+\omega'-i\eta}\right]$$

for sharp eigen-states:

Hole Spectral Function $\mathcal{A}_h(\vec{k},\omega) = 0$ for $\omega > 0$

$$\mathcal{A}_h(\vec{k},\omega) = 2\pi\sum_n \left|\langle\Psi_n^{N-1}|\Psi(\vec{k})|\Psi_0\rangle\right|^2 \delta(\omega - w_n^{N-1})$$

Particle Spectral Function $\mathcal{A}_p(\vec{k},\omega) = 0$ for $\omega < 0$

$$\mathcal{A}_p(\vec{k},\omega) = 2\pi\sum_n \left|\langle\Psi_n^{N+1}|\Psi^\dagger(\vec{k})|\Psi_0\rangle\right|^2 \delta(\omega - w_n^{N+1})$$

Retarded Green's Function

$$G^{(r)}(\vec{r}_1, t_1, \vec{r}_2, t_2) = -\frac{i}{\hbar}\langle\Psi_0|[\Psi_H(\vec{r}_1, t_1)\Psi_H^\dagger(\vec{r}_2, t_2)]_+|\Psi_0\rangle\theta(t_1 - t_2)$$

analytic in the upper complex plane

Advanced Green's Function

$$G^{(a)}(\vec{r}_1, t_1, \vec{r}_2, t_2) = \frac{i}{\hbar}\langle\Psi_0|[\Psi_H(\vec{r}_1, t_1)\Psi_H^\dagger(\vec{r}_2, t_2)]_+|\Psi_0\rangle\theta(t_2 - t_1)$$

analytic in the lower complex plane

Lehmann Representation

$$G^{(r,a)}(\vec{k}, \omega) = \int_0^\infty \frac{\mathrm{d}\omega'}{2\pi}\left[\frac{\mathcal{A}_p(\vec{k}, \omega')}{\omega - \mu/\hbar - \omega' \pm i\eta} + \frac{\mathcal{A}_h(\vec{k}, \omega')}{\omega - \mu/\hbar + \omega' \pm i\eta}\right]$$

Dirac Formula

$$\frac{1}{\omega \pm i\eta} = \mathcal{P}\frac{1}{\omega} \mp i\pi\delta(\omega)$$

$$G^{(r,a)}(\vec{k},\omega) = \mathcal{P}\int_0^\infty \frac{\mathrm{d}\omega'}{2\pi}\frac{\mathcal{A}_p(\vec{k},\omega')}{\omega - \mu/\hbar - \omega'} + \mathcal{P}\int_0^\infty \frac{\mathrm{d}\omega'}{2\pi}\frac{\mathcal{A}_h(\vec{k},\omega')}{\omega - \mu/\hbar + \omega'}$$
$$\mp \frac{i}{2}\mathcal{A}_p(\vec{k},\omega - \mu/\hbar) \mp \frac{i}{2}\mathcal{A}_h(\vec{k},\mu/\hbar - \omega)$$

Imaginary Part of the Retarded Green's Function

$$\mathrm{Im}\left[G^{(r)}(\vec{k},\omega)\right] = \frac{1}{2}[\mathcal{A}_p(\vec{k},\omega - \mu/\hbar) + \mathcal{A}_h(\vec{k},\mu/\hbar - \omega)]$$
$$\equiv \frac{1}{2}\mathcal{A}(\vec{k},\omega)$$

Usefull Relations

Correlation Functions

$$G^{>}(\vec{k},\omega) = -e^{\beta(\hbar\omega-\mu)} G^{<}(\vec{k},\omega)$$

$$\begin{aligned}\mathcal{A}(\vec{k},\omega) &= -2\mathrm{Im}\left[G^{(r)}(\vec{k},\omega)\right] = i\left[G^{(r)}(\vec{k},\omega) - G^{(a)}(\vec{k},\omega)\right] \\ &= i\left[G^{>}(\vec{k},\omega) - G^{<}(\vec{k},\omega)\right]\end{aligned}$$

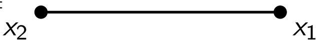
Spectral Function is a generalised density of states!!!

$$G^{<}(\vec{k},\omega) = i\mathcal{A}(\vec{k},\omega)f(\omega) \qquad G^{>}(\vec{k},\omega) = -i\mathcal{A}(\vec{k},\omega)[1-f(\omega)]$$

Perturbation Expansion in the Interaction

$$G(x_1, x_2) = G_0(x_1, x_2) + G_1(x_1, x_2) + O(n = 2)$$

$=$



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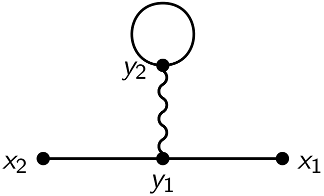

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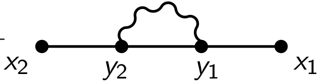
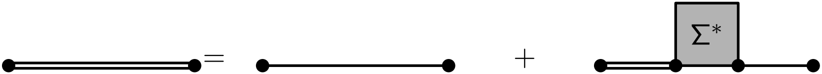


$+ \quad O(n = 2)$

Dyson Equation

$$G = G_0 + G_0 \Sigma^* G$$



Formal Solution

$$G^{(r)}(\vec{k}, \omega) = \frac{1}{\hbar\omega - \epsilon_0(\vec{k}) - \Sigma^{*(r)}(\vec{k}, \omega)}$$

Spectral Function

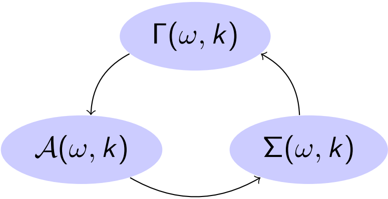
$$\mathcal{A}(k,\omega) = \frac{\Gamma(k,\omega)}{\left(\hbar\omega - \frac{\hbar^2 k^2}{2m} - \mathrm{Re}\Sigma^{(r)}(k,\omega)\right)^2 + \left(\frac{\Gamma(k,\omega)}{2}\right)^2}$$

Width $\Gamma > 0$

$$\Gamma(k,\omega) = -2\mathrm{Im}\Sigma^{(r)}(k,\omega)$$

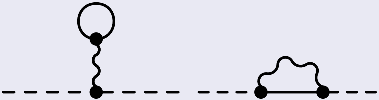
The Model

Self-Consistent Calculation

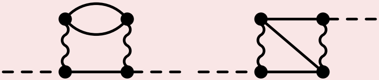


Self-Energy: $\Sigma(k,\omega) = \boxed{\Sigma_{MF}(k)} + \Sigma_C(k,\omega)$

Mean-Field Contribution



Collisional Contribution



Mean-Field Contribution

Energy Density Functional

Hartree-Fock Method

$$E = \langle\phi|(T + \frac{1}{2}V)|\phi\rangle = \int \mathcal{E}(\vec{r})\mathrm{d}^3 r.$$

Single Particle Energy

expansion with respect to p up to the second order

$$\epsilon_q(p) = \frac{\delta\mathcal{E}}{\delta n_q(p)} = \frac{p^2}{2m_q} + \Sigma_q^{MF}(k,\rho)$$
$$\approx \frac{p^2}{2m_q^*(\rho)} + U_q^{eff}(\rho)$$

Binding Energy

$$\frac{E}{A}(\rho) = \frac{V}{A}\rho = \frac{\mathcal{E}(\rho)}{\rho}$$

Mean-Field Contribution

Skyrme Parameterization

- parameterization of the N-N, N-Λ, Λ-Λ interaction
- local interaction with 2- and 3-body-forces
- simple analytic structure
- determination of the parameters using experimental data (i.e. binding-energies)

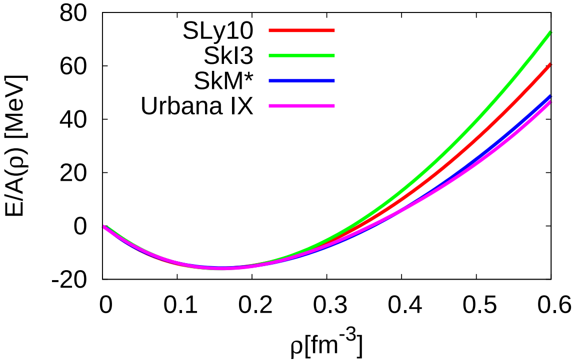
$$
\begin{aligned}
v_{ij}^{(2)}(r_1 - r_2) = {} & t_0(1 + x_0 P_\sigma)\delta(r_1 - r_2) \\
& + \frac{1}{2} t_1(1 + x_1 P_\sigma)\left[k'^2 \delta(r_1 - r_2) + \delta(r_1 - r_2) k^2\right] \\
& + t_2(1 + x_2 P_\sigma)\vec{k}' \delta(r_1 - r_2)\vec{k} \\
& + i W_0 \vec{\sigma} \cdot [\vec{k}' \times \delta(r_1 - r_2)\vec{k}]
\end{aligned}
$$

Equation of State

Symmetric Nuclear Matter

- $Y = \rho_p/(\rho_n + \rho_p) = 0.5$
- saturation density $\rho_0 \approx 0.16\ \mathrm{fm}^{-3}$

Urbana IX: A. Akmal et al., Phys. Rev. C58, 1804-1828

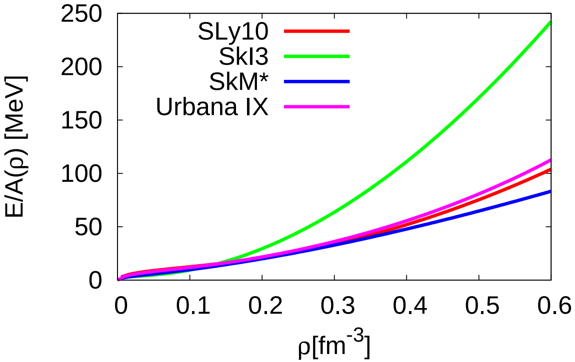


Equation of State

Pure Neutron Matter

- $Y = \rho_p/(\rho_n + \rho_p) = 0$
- no binding

Urbana IX: A. Akmal et al., Phys. Rev. C58, 1804-1828

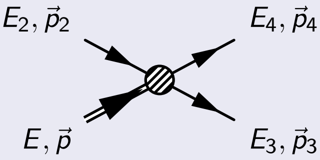


Collisional Contribution

$$\Sigma_C(p, E) = \Sigma^{>}(p, E) + \Sigma^{<}(p, E)$$

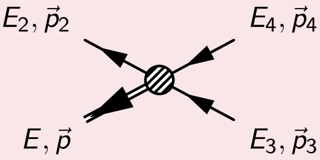
1-Particle-2-Hole Self-Energy

"scattering-out rate"



2-Particle-1-Hole Self-Energy

"scattering-in rate"



Collisional Contribution

$$\Sigma^{\gtrless}(p,E) \sim \int \mathrm{d}\xi \ldots \boxed{\mid \mathcal{M} \mid^2} \ldots \boxed{G^{\lessgtr}(p_2,E_2)G^{\gtrless}(p_3,E_3)G^{\gtrless}(p_4,E_4)}$$

$$\mathrm{d}\xi = \prod_{i=2}^{4} \mathrm{d}p_i \mathrm{d}E_i$$

Local Interaction

$$\mid \mathcal{M}(p,E,\rho) \mid^2 \approx \mid \mathcal{M}(\rho) \mid^2$$

Density of States

- particle: $G^{<}(p,E) = i\mathcal{A}(p,E)f(E)$
- hole: $G^{>}(p,E) = i\mathcal{A}(p,E)[1-f(E)]$
- Fermi-distribution: $f(E)$

$$\Rightarrow \Sigma^{\gtrless}(p,E) \sim \boxed{\mid \mathcal{M} \mid^2} \int \mathrm{d}\xi \ldots \boxed{G^{\lessgtr}(p_2,E_2)G^{\gtrless}(p_3,E_3)G^{\gtrless}(p_4,E_4)}$$

The Spectral Function

$$\mathcal{A}(p,E) = \frac{\boxed{\Gamma(p,E)}}{\left(E - \frac{p^2}{2m} - \boxed{\mathrm{Re}\Sigma(p,E)}\right)^2 + \left(\frac{\Gamma(p,E)}{2}\right)^2}$$

Width

broadening of the spectral function

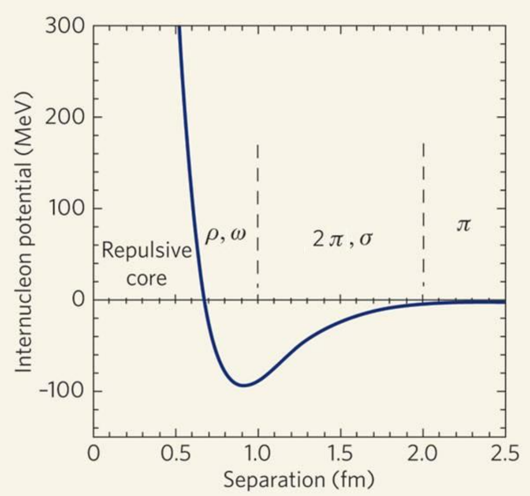
$$\Gamma(p,E) = i[\Sigma^>(p,E) - \Sigma^<(p,E)]$$

Real Part

shifting of the peak

$$\mathrm{Re}\Sigma(p,E) = \Sigma_{MF}(p) + \mathcal{P}\int \frac{\mathrm{d}E'}{2\pi}\frac{\Gamma(p,E')}{E-E'}$$

The Nucleon-Nucleon Potential



Landau-Migdal-Parameters

Energy Density Functional:

$$\mathcal{E}(\rho) = \sum_{qs} \int \mathrm{d}^3 p \epsilon^0_{qs}(p) n_{qs}(p) + \frac{1}{2} \sum_{qq'ss'} \int \mathrm{d}^3 p \mathrm{d}^3 p' \boxed{f^{qsq's'}(p,p')} n_{qs}(p) n_{q's'}(p')$$

Residual Interaction

$$f^{qsq's'}(p,p') = \frac{\delta^2 E}{\delta n_{qs}(p) \delta n_{q's'}(p')}$$

Landau-Migdal-Parameter

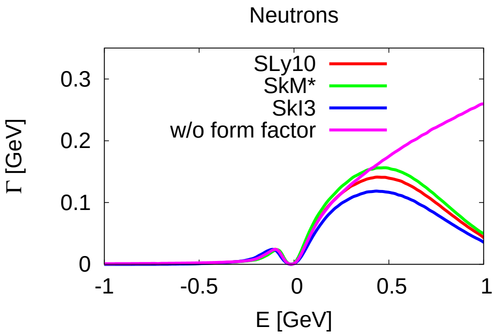
$$f_{qsq's'} = f^{qsq's'}(p_f, p'_{f'})$$

$p_f \approx$ Fermi-Momentum

The Nucleon Width

Symmetric Nuclear Matter

- $\Gamma \sim \tau^{-1}$, direct measure for correlations
- momentum $p = 0.100$ GeV/c
- form factor $F_q(E_2, E_3) = e^{(E_2 - E_3 - \epsilon_{F_q})^4/\Lambda^4}$

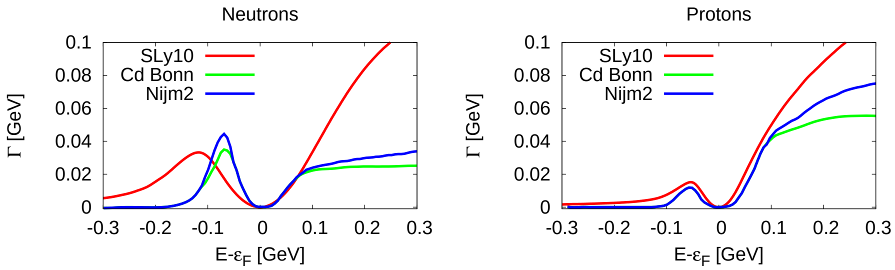


The Nucleon Width

Asymmetric Nuclear Matter

- asymmetry Y=0.25
- density $\rho = 0.17\ \text{fm}^{-3}$
- momentum $p = 0.25 p_{F_q}$

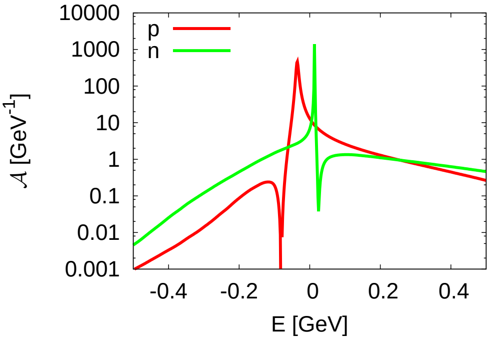
Kh.S.A.Hassaneen and H.Müther, Phys. Rev. C70 (2004)



The Nucleon Spectral Function

Asymmetric Nuclear Matter

- neutron rich nuclear matter $Y = \rho_p/(\rho_n + \rho_p) = 0.1$
- $p = 0.300$ GeV/c
- quasi-particle peak is on-shell $\epsilon^q_{on} = \frac{p^2}{2m_q} + \Sigma^q_{MF}$



Momentum Distribution and Spectroscopic Factor

Momentum Distribution

$$n_q(p) = \int_{-\infty}^{\epsilon_{F_q}} \frac{\mathrm{d}E}{2\pi} \mathcal{A}_q(E,p)$$

expansion of the real part at the on-shell point ϵ_{on}:

$$(E - \epsilon_{on} - \mathrm{Re}\Sigma(p,E)) \simeq (E - \epsilon_{on}) \left(1 - \left.\frac{\partial \mathrm{Re}\Sigma(p,E)}{\partial E}\right|_{E=\epsilon_{on}}\right)$$

Spectroscopic Factor

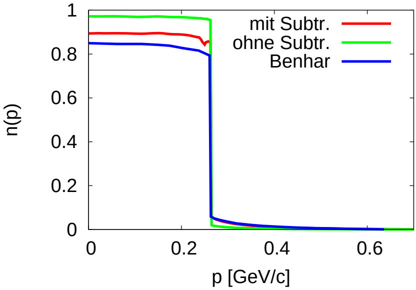
$$Z_F(p) = \left(1 - \left.\frac{\partial \mathrm{Re}\Sigma(p,E)}{\partial E}\right|_{E=\epsilon_{on}}\right)^{-1}$$

measure of how many particles remain in the "Quasi-particle" state

Momentum Distribution

Symmetric Nuclear Matter

- strength shifted above the Fermi-edge
- substraction of longe-range part important

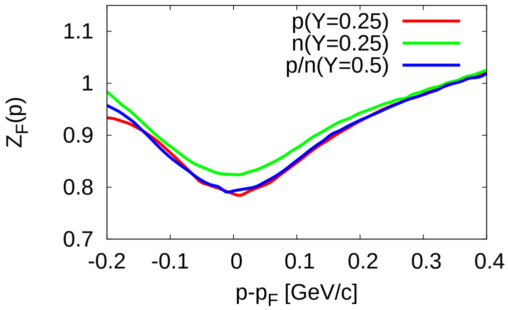


O. Benhar, A. Fabrocini, Phys. Rev. C62 (2000)

Spectroscopic Factor

Asymmetric Nuclear Matter

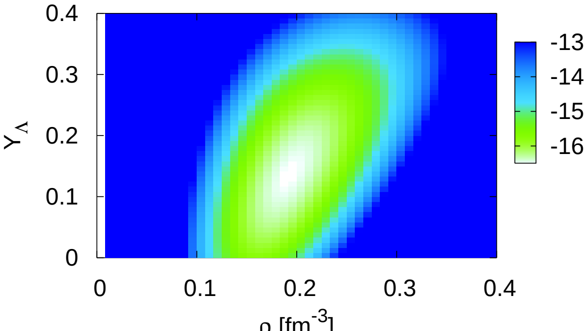
- most correlations take place close to the Fermi-edge
- less correlations for higher particle number (Pauli-blocking)



Equation of State

Hypernuclear Matter

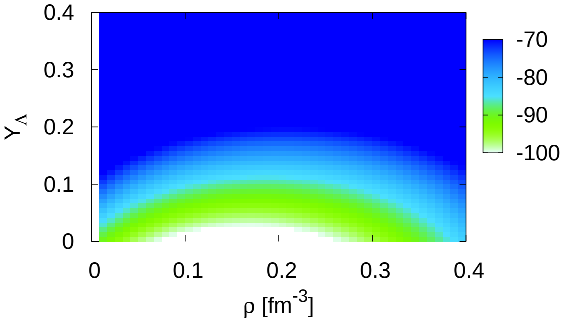
- here: equal number of proton and neutrons
- maximal binding at $Y_\Lambda = \rho_\Lambda / \rho \approx 0.15$
- $E/A_{min} \approx -21$ MeV



Equation of State

Hypernuclear Matter

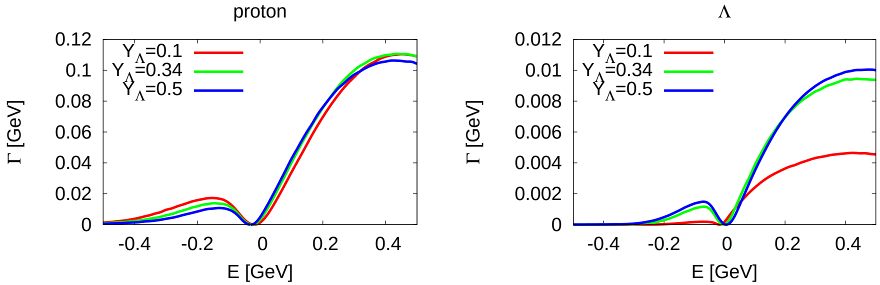
- consideration of masses: $E/A + Y_\Lambda m_\Lambda + Y_N m_N$
- substraction of the average mass $m_{av} = (m_\Lambda + m_N)/2$
- maximal binding at $Y_\Lambda = \rho_\Lambda/\rho = 0$



Width

Hypernuclear Matter

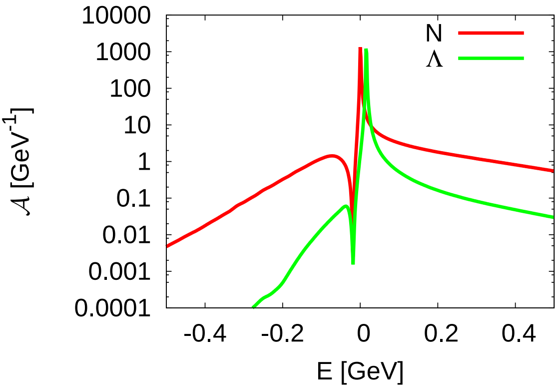
- strangeness fraction $Y_\Lambda = 0.1$
- smaller width for Λ-hyperons than for nucleons



Spectral Function

Hypernuclear Matter

- strangeness fraction $Y_\Lambda = 0.1$
- sharp spectral function for Λ-hyperons



ΛN-Matter im β-Equilibrium

Condition for β-Equilibrium

all processes are allowed, which conserve charge and Baryon number:

$$p + e^- \longleftrightarrow n + \nu_e$$
$$p + e^- \longleftrightarrow \Lambda + \nu_e$$

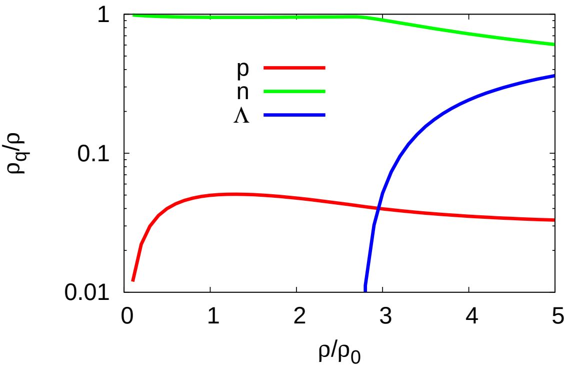
Chemical Potentials

$$(\mu_e + \mu_n) - (\mu_p + m_p) = \mu_e - \mu_\nu$$
$$(\mu_n + m_n) = (\mu_\Lambda + m_\Lambda)$$
$$\text{neutrality:} \quad n_e = n_p$$
$$\text{free Neutrinos:} \quad \mu_\nu = 0$$

Particle Fraction in β-Equilibrium

β-equilibrium

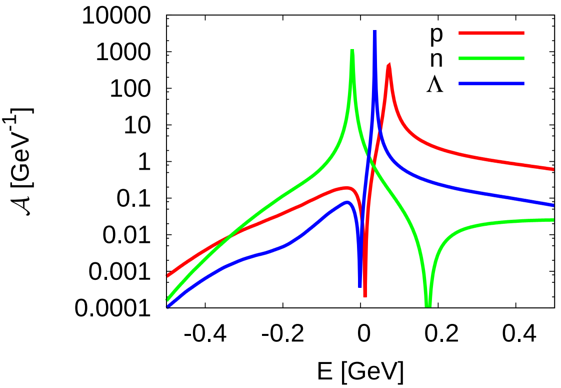
- at low densities consists ΛN-matter in β-Equilibrium nearly out of neutrons
- Λ-formation around $2.6\rho_0$



Spectral Function

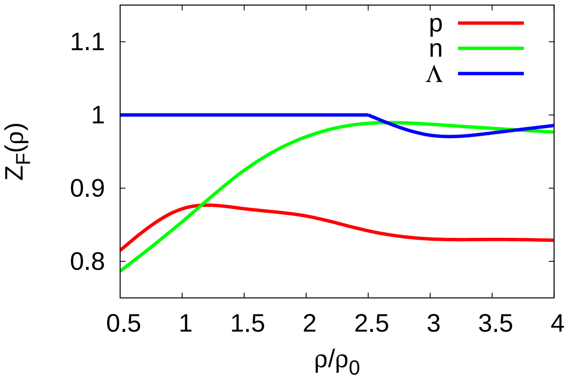
β-equilibrium

- same structure as in hypernuclear matter
- sharp spectral functions for Λ-hyperons and neutrons



Spectroscopic Factor in ΛN-Matter

- suppression of Λ-correlations bescause of $\mathcal{M}_\Lambda < \mathcal{M}_N$
- suppression of n-correlations by Pauli-blocking



Summary

- calculation of the spectral function in exotic nuclear matter using a self-consistent method
- energy and momentum independent interaction matrix element
- subtraction of the longe-range part
  - good agreement with other many-body calculations
  - correlations do not depend on the details of the interaction
  - correlations are suppressed for high number densities

Outlook

- calculation including Σ-hyperons
- finite temperature calculations