Linear and Nonlinear Realizations of Supersymmetry

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Outline

- **Prelude**
  - Three Goldstino fields: linear/nonlinear/constrained
  - Two pion fields: linear/nonlinear
- **Superspace, linear/nonlinear realization of SUSY**
- **Reformulate linear SUSY into nonlinear ones**
- **Nonlinear Goldstino field out of linear superfield**
- **Constrained superfields**
- **Low energy effective theory**
- **Conclusions**

Prelude-1

- Spontaneous breaking of global symmetries $\longrightarrow$ massless Goldstone particles
- Properties of Goldstone particles $\leftrightarrow$ the nature of the broken and unbroken symmetries
- Strong interactions: pions $\leftrightarrow$ spontaneous breaking of chiral symmetry
- Low energy physics of pions: nonlinear realization of chiral symmetry
  - Expansion in terms of energy-momentum

Prelude-2

- Goldstino: spontaneous breaking of global SUSY
- Supergravity: Goldstino is part of the massive gravitino
- $M_{\mathrm{SUSY}} \ll M_{\mathrm{P}}$: lower energy physics dominated by Goldstino
- Goldstino physics could be of importance at the TeV scale and tested in LHC
- Low energy physics of Goldstinos: linear SUSY/nonlinear SUSY/constrained superfields

Goldstino Field in O'Raifeataigh-like models

- **Linear SUSY, chiral fields responsible for SSB**
- **Chiral super-multiplet $\Phi \sim (\phi, \psi, F)$**

$$\begin{aligned} \delta_\xi \phi &= \sqrt{2}\xi\psi, \\ \delta_\xi \psi_\alpha &= \sqrt{2}F\xi_\alpha + i\sqrt{2}(\sigma^\mu\bar{\xi})_\alpha \partial_\mu \phi \ , \\ \delta_\xi F &= i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu \partial_\mu \psi. \end{aligned}$$

- **The Goldstino field**
  - F-term in one Φ_0 develops a nonzero VEV $\langle F_0 \rangle$, SUSY is spontaneously broken
  - Goldstino field ψ_0: zero mass and changes as

$$\delta_\xi \psi_{0\alpha} = \sqrt{2}\langle F_0 \rangle \xi_\alpha + \cdots$$

  - If several F have nonzero VEVs, realignment can be made such that only Φ_0 has a nonzero VEV.

Goldstino Field in Nonlinear Realization of SUSY

- **Non-chiral version**

$$\delta_\xi \lambda^\alpha = \frac{1}{\kappa}\xi^\alpha - i\kappa(\lambda\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\lambda})\partial_\mu\lambda^\alpha$$

$$\delta_\xi \bar{\lambda}_{\dot{\alpha}} = \frac{1}{\kappa}\bar{\xi}_{\dot{\alpha}} - i\kappa(\lambda\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\lambda})\partial_\mu\bar{\lambda}_{\dot{\alpha}}$$

- **Chiral version**

$$\delta_\xi \tilde{\lambda}_\alpha = \frac{\xi_\alpha}{\kappa} - 2i\kappa\tilde{\lambda}\sigma^\mu\bar{\xi}\partial_\mu\tilde{\lambda}_\alpha$$

- **Conversion**

$$\tilde{\lambda}_\alpha(x) = \lambda_\alpha(z), \quad z = x - i\kappa^2\lambda(z)\sigma\bar{\lambda}(z)$$

Work of Komargodski/Seiberg-1

- Conservation equation: $\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$
- Supercurrent multiplet:

$$\begin{aligned}
\mathcal{J}_\mu &= j_\mu + \left[\theta^\alpha\left(S_{\mu\alpha} + \frac{1}{3}(\sigma_\mu\bar{\sigma}^\rho S_\rho)_\alpha\right) + h.c.\right] \\
&\quad +(\theta\sigma^\nu\bar{\theta})\left(2T_{\nu\mu} - \frac{2}{3}\eta_{\nu\mu}T - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j^{\sigma]}\right) \\
&\quad +\frac{i}{2}\theta^2\partial_\mu\bar{x} - \frac{i}{2}\bar{\theta}^2\partial_\mu x + \cdots \\
X &= x(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)
\end{aligned}$$

$$\partial^\mu T_{\mu\nu} = \partial^\mu S_{\mu\alpha} = 0\ , T_{\mu\nu} = T_{\nu\mu}\ .$$

$$\psi_\alpha = \frac{\sqrt{2}}{3}\sigma^\mu_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_\mu,\ F = \frac{2}{3}T + i\partial_\mu j^\mu,\ y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

Work of Komargodski/Seiberg-2

- SUSY spontaneously broken, the low-energy supercurrent in terms of the massless Goldstino G_α

$$S_{\mu\alpha} = \sqrt{2} f \sigma_{\mu\alpha\dot{\alpha}} \bar{G}^{\dot{\alpha}} + f'(\sigma_{\mu\nu})_\alpha^\beta \partial^\nu G_\beta \ + \cdots$$

- Low-energy Goldstino *not* accompanied by a massless scalar, the simplest bosonic state $\sim$ two Goldstinos
- SUSY partners:
  one Goldstino $Q^\dagger_{\dot{\alpha}}|0\rangle$ $\sim$ "two Goldstinos" $Q^\dagger_{\dot{1}} Q^\dagger_{\dot{2}}|0\rangle$
- ψ_ϕ creates a one Goldstino state
  $\phi \sim x$ creates a two Goldstino state

Work of Komargodski/Seiberg-3

- In combination with the SUSY algebra

$$X_{NL} = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F, \quad X_{NL}^2 = 0$$

- Integrate out heavy fields by constraints
- Coupling with superfields: spurion $Y \to \frac{m_{soft}}{f} X_{NL}$

$$\begin{aligned}\mathcal{L}_{soft} &= -\int d^4\theta \left|\frac{X_{NL}}{f}\right|^2 (m^2)_i^j (Qe^V\bar{Q})_j^i \\ &+ \int d^2\theta \frac{X_{NL}}{f}\left(-\frac{1}{2}B_{ij}Q^iQ^j + \frac{1}{6}A_{ijk}Q^iQ^jQ^k\right) + c.c.\end{aligned}$$

$$\mathcal{L}_{soft} = \int d^4\theta \left|\frac{X_{NL}}{f}\right|^2 \xi D^\alpha W_\alpha + \int d^2\theta \frac{X_{NL}}{f} m_\lambda W_\alpha W^\alpha + c.c. \ ,$$

The Web of Relations

- **Relation between $\tilde{\lambda}$ and ψ_0**
  - $\tilde{\lambda}$ is closely related, but not identical, to ψ_0
  - $\tilde{\lambda} \simeq \psi_0$ to the leading order, if $\kappa^{-1} = \sqrt{2}\langle F_0 \rangle$
- **Construct $\tilde{\lambda}$ out of $(\phi_0,\ \psi_0,\ F_0) \subset \Phi_0$**

$$\tilde{\lambda} = \frac{\psi_0}{\sqrt{2}\kappa F_0} - i\frac{\sigma^\mu \bar{\tilde{\lambda}}}{F_0}\left(\partial_\mu \phi_0 - \sqrt{2}\kappa\tilde{\lambda}\partial_\mu\psi_0 + \kappa^2\tilde{\lambda}^2\partial_\mu F_0\right)$$

- **Expressing X_{NL} in the language of nonlinear SUSY**
- **Comparison with the nonlinear/linear σ-model**
  - $\lambda \sim \vec{\pi}$
  - $(\phi_0, \psi_0, F_0) \sim \phi_a$
  - $X_{NL}^2 = 0 \sim \sum_a \phi_a^2 = \text{constant}$

Parallels: Pions as Goldstone Bosons-1

- **Linear σ-model**
  - SO(4)-invariant Lagrangian of the linear σ-model

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{2}\partial_\mu\phi_n\partial^\mu\phi_n - \frac{\mathcal{M}^2}{2}\phi_n\phi_n - \frac{\lambda}{4}(\phi_n\phi_n)^2 \\
&= -\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}\sigma^2\sum_{n=1}^{4}\partial^\mu R_{n4}\partial_\mu R_{n4} - \frac{1}{2}\mathcal{M}^2\sigma^2 - \frac{\lambda}{4}\sigma^4
\end{aligned}
$$

$$
\phi_n(x) = R_{n4}(x)\sigma(x), \quad R^T R = 1, \quad \sigma = \sqrt{\sum_n \phi_n^2}
$$

  - if $\mathcal{M}^2 < 0$, σ has a non-zero VEV

Parallels: Pions as Goldstone Bosons-2

- **Nonlinear σ-model**
  - Redefine the fields

$$\zeta_a \equiv \frac{\phi_a}{\phi_4+\sigma}, \;\; \phi_a/\sigma = R_{a4} = \frac{2\zeta_a}{1+\vec{\zeta}^2}, \;\; \phi_4/\sigma = R_{44} = \frac{1-\vec{\zeta}^2}{1+\vec{\zeta}^2}$$

$$R_{a4} = \frac{2\zeta_a}{1+\vec{\zeta}^2} = -R_{4a}, \;\; R_{44} = \frac{1-\vec{\zeta}^2}{1+\vec{\zeta}^2}, \;\; R_{ab} = \delta_{ab} - \frac{2\zeta_a\zeta_b}{1+\vec{\zeta}^2}$$

  - Nonlinear σ-Lagrangian

$$\mathcal{L} \;=\; -\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - 2\sigma^2\vec{D}_\mu\vec{D}^\mu - \frac{1}{2}\mathcal{M}^2\sigma^2 - \frac{\lambda}{4}\sigma^4$$

$$\vec{D}_\mu \;\equiv\; \frac{\partial_\mu\vec{\zeta}}{1+\vec{\zeta}^2}$$

Parallels: Pions as Goldstone Bosons-3

- **Transformation rules in nonlinear σ-model**
  - Isospin: $\delta\vec{\phi} = \vec{\theta} \times \vec{\phi}, \delta\phi_4 = 0$

$$\delta\sigma = 0,\ \delta\vec{\zeta} = \vec{\theta} \times \vec{\zeta},\ \delta\vec{D}_\mu = \vec{\theta} \times \vec{D}_\mu$$

  - Axial isospin: $\delta\vec{\phi} = 2\vec{\epsilon}\phi_4, \quad \delta\phi_4 = -2\vec{\epsilon} \cdot \vec{\phi}$

$$\delta\sigma = 0,\ \delta\vec{\zeta} = \epsilon(1 - \vec{\zeta}^2) + 2\vec{\zeta}(\vec{\epsilon} \cdot \vec{\zeta}),\ \delta\vec{D}_\mu = 2(\vec{\zeta} \times \vec{\epsilon}) \times \vec{D}_\mu$$

- Take $F = 2\langle\sigma\rangle$and $\vec{\pi} \equiv F\vec{\zeta}$,
  (Set σ to its VEV, constraint: $\sum_a \phi_a^2 = \langle\sigma\rangle^2$)

$$\mathcal{L} = -\frac{F^2}{2}\vec{D}_\mu\vec{D}^\mu = -\frac{1}{2}\frac{\partial_\mu\vec{\pi} \cdot \partial^\mu\vec{\pi}}{(1 + \vec{\pi}^2/F^2)^2}$$

Superspace, Translations, Induced Realization

- Superalgebra

$$\{\hat{Q}_\alpha, \bar{\hat{Q}}_{\dot\alpha}\} = -2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \quad \{\hat{Q}_\alpha, \hat{Q}_\beta\} = 0, \quad \{\bar{\hat{Q}}_{\dot\alpha}, \bar{\hat{Q}}_{\dot\beta}\} = 0$$

- Supergroup element in superspace $(x, \theta, \bar\theta)$
  - $G(x, \theta, \bar\theta) = \exp\left[i(-x^\mu P_\mu + \theta\hat{Q} + \bar\theta\bar{\hat{Q}})\right]$
- Multiplication
  - $G(0, \xi, \bar\xi)G(x, \theta, \bar\theta) = G(x + i(\theta\sigma\bar\xi - \xi\sigma\bar\theta), \theta + \xi, \bar\theta + \bar\xi)$
- Translation in superspace
  - $x' = x + i(\theta\sigma\bar\xi - \xi\sigma\bar\theta)$, $\theta' = \theta + \xi$, $\bar\theta' = \bar\theta + \bar\xi$

  generated by $\xi Q + \bar\xi\bar{Q}$

$$Q_\alpha = \partial_\alpha - i(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{Q}_{\dot\alpha} = -\partial_{\dot\alpha} + i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu$$

Linear Realization of SUSY

- Induced linear realization of the superalgebra

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu, \quad \{Q_\alpha, Q_\beta\} = 0, \quad \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

- Change in sign, the order of multiplication reversed
- Superfield:

$$\begin{aligned} F(x,\theta,\bar{\theta}) &= f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) \\ &+ \theta\sigma^\mu\bar{\theta}v_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta}d(x) \end{aligned}$$

- Linear transformation mixes different components

$$\begin{aligned} \delta_\xi F(x,\theta,\bar{\theta}) &= (\xi Q + \bar{\xi}\bar{Q})F(x,\theta,\bar{\theta}) \\ &= \delta_\xi f(x) + \theta\delta_\xi\phi(x) + \bar{\theta}\delta_\xi\bar{\chi}(x) + \theta\theta\delta_\xi m(x) + \bar{\theta}\bar{\theta}\delta_\xi n(x) \\ &+ \theta\sigma^\mu\bar{\theta}\delta_\xi v_\mu(x) + \theta\theta\bar{\theta}\bar{\delta}_\xi\lambda(x) + \bar{\theta}\bar{\theta}\theta\delta_\xi\psi(x) + \theta\theta\bar{\theta}\bar{\theta}\delta_\xi d(x) \end{aligned}$$

Nonlinear Realizations of SUSY-1

- Induced nonlinear realization: $\theta \to \kappa\lambda(x)$

$$\lambda'(x') = \lambda(x) + \frac{1}{\kappa}\xi, \quad \bar{\lambda}'(x') = \bar{\lambda}(x) + \frac{1}{\kappa}\bar{\xi}$$

- Infinitesimal changes $\left[v_\xi^\mu(x) = \kappa(\lambda\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\lambda})\right]$

$$\delta_\xi\lambda^\alpha = \frac{1}{\kappa}\xi^\alpha - i v_\xi^\mu(x)\partial_\mu\lambda^\alpha, \quad \delta_\xi\bar{\lambda}_{\dot{\alpha}} = \frac{1}{\kappa}\bar{\xi}_{\dot{\alpha}} - i v_\xi^\mu(x)\partial_\mu\bar{\lambda}_{\dot{\alpha}}$$

- The SUSY algebra is closed

$$(\delta_\eta\delta_\xi - \delta_\xi\delta_\eta)\lambda^\alpha = -2i(\eta\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\eta})\partial_\mu\lambda^\alpha$$

- Matter fields

$$\delta_\xi f(x) = -i v_\xi^\mu(x)\partial_\mu f(x)$$

$$(\delta_\eta\delta_\xi - \delta_\xi\delta_\eta)f = -2i(\eta\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\eta})\partial_\mu f$$

Nonlinear Realizations of SUSY-2

- Taking $(x',\ \theta',\ \bar{\theta}')$ as functions of $(x,\ \theta,\ \bar{\theta})$

$$dx'^{\mu} = dx^{\mu} + id\theta\sigma^{\mu}\bar{\xi} - i\xi\sigma^{\mu}d\bar{\theta}$$

$$d\theta'^{\alpha} = d\theta^{\alpha}, \qquad d\bar{\theta}'_{\dot{\alpha}} = d\bar{\theta}_{\dot{\alpha}}$$

- Define differentials

$$e^{\mu} = dx^{\mu} - id\theta\sigma^{\mu}\bar{\theta} + i\theta\sigma^{\mu}d\bar{\theta}$$

$$e^{\alpha} = d\theta^{\alpha}, \qquad e_{\dot{\alpha}} = d\bar{\theta}_{\dot{\alpha}}$$

- $(x,\theta,\bar{\theta}) \to (x',\theta',\bar{\theta}')$, $e^{\mu} \to e'^{\mu}$

$$e'^{\mu} = dx'^{\mu} - id\theta'\sigma^{\mu}\bar{\theta}' + i\theta'\sigma^{\mu}d\bar{\theta}' = e^{\mu}$$

  It is invariant.

Nonlinear Realizations of SUSY-3

- **Akulov-Volkov Lagrangian**
  - Substituting $\theta = \kappa\lambda$ and $d\theta = \kappa(\partial\lambda/\partial x^\mu)dx^\mu$

$$e^\mu \to dx^\nu \left[\delta^\mu_\nu - i\kappa^2 \partial_\nu \lambda \sigma^\mu \bar{\lambda} + i\kappa^2 \lambda \sigma^\mu \partial_\nu \bar{\lambda}\right] = dx^\nu T^\mu_\nu$$

  - A-V Lagrangian

$$\mathcal{L} = -\frac{1}{\kappa^2} det\, T$$

  - $\mathcal{L}$ changes by a total derivative

$$\delta_\xi det\, T = -i\kappa \partial_\mu \left[(\lambda\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\lambda}) det\, T\right]$$

- **Any SUSY non-invariant theory can be prompted to an (nonlinearly) invariant one**
  (Low energy effective theory, later)

From Linear to Nonlinear SUSY-1

- **General linear super-multiplet**

$$\Phi_k^\sigma\left(x,\theta,\bar\theta\right) = e^{-\kappa\lambda(x)Q-\kappa\bar\lambda(x)\bar Q}\Phi_k(x,\theta,\bar\theta) = \Phi_k(\tilde x,\tilde\theta,\bar{\tilde\theta})$$

$$\tilde x = x + i\kappa\lambda(x)\sigma\bar\theta - i\kappa\theta\sigma\bar\lambda(x)$$

$$\tilde\theta = \theta - \kappa\lambda(x), \quad \bar{\tilde\theta} = \bar\theta - \kappa\bar\lambda(x)$$

- **All components of Φ_k^σ change as matter fields**

$$\delta_\xi\Phi_k^\sigma(x,\theta,\bar\theta) = -iv_\xi^\nu(x)\frac{\partial}{\partial x^\nu}\Phi_k^\sigma(x,\theta,\bar\theta)$$

From Linear to Nonlinear SUSY-2

- Generic action, linear

$$S_{gen} = \int d^4x d^4\theta \mathcal{L}_{gen} \times \left(\Phi_t(x,\theta), \Phi_k(x,\theta,\bar{\theta}), D_\alpha \Phi_k, D_\alpha D_\beta \Phi_k, ...\right)$$

- Generic action, nonlinear

$$S_{gen} = \int d^4x d^4\theta Ber\left(x,\theta,\bar{\theta}\right) \times \mathcal{L}_{gen}\left(\Phi_t^\sigma(x,\theta,\bar{\theta}), \Phi_k^\sigma(x,\theta,\bar{\theta}), \triangle_\alpha \Phi_k^\sigma, \triangle_\alpha \triangle_\beta \Phi_k^\sigma, ...\right)$$

$$Ber\left(x,\theta,\bar{\theta}\right) = \det T(x) \det M(x,\theta,\bar{\theta})$$
$$M_\mu^\nu(x,\theta,\bar{\theta}) = \delta_\mu^\nu + i\kappa \nabla_\mu \lambda(x) \sigma^\nu \bar{\theta} - i\kappa \theta \sigma^\nu \nabla_\mu \bar{\lambda}(x)$$

- Covar. derivatives: $\nabla^\nu = (T^{-1})_\rho^\nu \partial^\rho$, $\triangle_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma_\mu \bar{\theta})_\alpha \triangle^\mu$

$$\triangle^\mu = (M^{-1})_\nu^\mu \left(\nabla^\nu + \nabla^\nu \lambda(x) \frac{\partial}{\partial\theta} + \nabla^\nu \bar{\lambda}(x) \frac{\partial}{\partial\bar{\theta}}\right)$$

From Linear to Nonlinear SUSY-3

- **Chiral superfield Φ_t**

$$\Phi_t(x,\theta,\bar\theta) = \exp\left(i\theta\sigma^\mu\bar\theta\partial_\mu\right) S_t(x,\theta)$$

$$\Phi_t^\sigma(x,\theta,\bar\theta) = L^+(\partial/\partial x, \partial/\partial\theta) S_t^\sigma(x,\theta)$$

$$S_t^\sigma(x,\theta) = S_t\left(\tilde{x}^+, \tilde\theta\right)$$

$$\tilde{x}^+ = x - 2i\kappa\theta\sigma\bar\lambda(x) + i\kappa^2\lambda(x)\sigma\bar\lambda(x)$$

$$L^+(\partial/\partial x, \partial/\partial\theta) = 1 + i\theta\sigma^\mu\bar\theta\triangle_\mu^+ + \frac{1}{4}\theta^2\bar\theta^2\triangle_\mu^+\triangle^{\mu+}$$

$$M^{+\nu}{}_\mu(x,\theta) = \delta_\mu^\nu - 2i\kappa\theta\sigma_\mu\nabla^\nu\bar\lambda, \quad \triangle_\alpha^+ = \frac{\partial}{\partial\theta^\alpha} + i(\bar\theta\sigma^\mu)_\alpha\triangle_\mu^+,$$

$$\triangle^{+\mu} = (M_+^{-1})_\nu^\mu\left(\nabla^\nu + \nabla^\nu\lambda(x)\frac{\partial}{\partial\theta}\right)$$

From Linear to Nonlinear SUSY-4

- Chiral part of the action, linear

$$S_{ch} = \int d^4x \left(d^2\theta \mathcal{L}_{ch}(S_t(x,\theta)) + C.C\right)$$

- Chiral part of the action, nonlinear

$$S_{ch} = \int d^4x \left(d^2\theta Ber^+(x,\theta)\,\mathcal{L}_{ch}(S_t^\sigma(x,\theta)) + C.C\right)$$

$$Ber^+(x,\theta) = \det T(x) \det M^+(x,\theta)$$

From Linear to Nonlinear SUSY: Recapture

- Linear superfields to nonlinear ones

$$\Omega^\sigma \;=\; \exp\left[-\kappa\left(\lambda Q + \bar{\lambda}\bar{Q}\right)\right]\Omega$$

- SUSY transformation rules for Ω^σ

$$\delta_\xi \Omega^\sigma = -i(\lambda\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\lambda})\partial_\mu\Omega^\sigma$$

- All component fields in Ω^σ transform into themselves
- Any of them can be integrated out without breaking SUSY, via e.o.m. (tree level) or matching (QM), producing high dimensional operators
- Extremely heavy ones: set to zero directly
- Whether and how to integrate out a field are dynamical questions

Construct the Nonlinear Goldstino Field λ

- Generic OR model: ($\Phi_0 \to S_0$ by ridding of $i\theta\sigma\bar{\theta}$)

$$S_0(x,\theta) = \phi_0(x) + \sqrt{2}\theta\psi_0(x) + \theta^2 F_0(x)$$

- Essential: $\langle F_0 \rangle \neq 0$
- The corresponding nonlinear super-multiplet

$$S_0^\sigma = S_0(x - 2i\kappa\theta\sigma\bar{\lambda}(x) + i\kappa^2\lambda(x)\sigma\bar{\lambda}(x), \theta - \kappa\lambda(x))$$

- Construct λ out of the components of S_0: demanding ψ_0^σ to vanish and re-express λ in terms of $\tilde{\lambda}$

$$\tilde{\lambda} = \frac{\psi_0}{\sqrt{2}\kappa F_0} - i\frac{\sigma^\mu \bar{\tilde{\lambda}}}{F_0}\left(\partial_\mu\phi_0 - \sqrt{2}\kappa\tilde{\lambda}\partial_\mu\psi_0 + \kappa^2\tilde{\lambda}^2\partial_\mu F_0\right)$$

- The analog of representing $\vec{\pi}$ in terms of ϕ_a in σ-models

Comments on the Construction

- Taking $\kappa^{-1} = \sqrt{2}\langle F_0\rangle$: $\tilde{\lambda} \simeq \psi_0$ to the leading order
- $\tilde{\lambda}$ (λ) transforms properly
- $\psi_0^g = 0$ in $\Phi_0^g = \exp\left[-\kappa\left(\lambda Q + \bar{\lambda}\bar{Q}\right)\right]\Phi_0$ when this λ is used, it is realized by the definition of λ
- ψ_0 cannot be dropped by the reasoning of dynamics for it's not heavy, it is actually massless
- Feasibility due to the SUSY algebras
- Can always construct a λ for any chiral super-multiplet, but cannot be used to separate the Goldstino field from the others

From Linear to Nonlinear Lagrangians

- Standard procedure, with *this* definition of λ
- No explicit form of λ is needed, the key element is $\psi_0^\sigma = 0$, which is all needed
- In the process, the Goldstino field disappears from the original Lagrangian, but reemerges in the Jacobian of the transformation and covariant derivatives
- Vertices with Goldstino fields carry at least one space-time derivative, as one would have expected
- All fields are kept, heavy ones can be integrated out, via e.o.m. or matching

Mass Spectrum in Nonlinear Lagrangians

- Space-time derivatives are not allowed in potential terms, Goldstino field is absent in the nonlinear version
- Potential terms in the nonlinear version

$$\int d^4x(d^2\theta W(S_t^\sigma, S_0^\sigma) + h.c.)$$

- The same structure as the linear version

$$\int d^4x(d^2\theta W(S_t, S_0) + h.c.)$$

ψ_0 is massless: no bilinear terms $\psi_0\psi_0$ or $\psi_0\psi_i$

- The mass spectrum is not changed by going from the linear version to nonlinear one by setting $\psi_0^\sigma = 0$

Construction of the Goldstino field in F-I models

- Abelian gauge field

$$V = D\theta^2\bar{\theta}^2 + \chi\theta\bar{\theta}^2 + \bar{\chi}\bar{\theta}\theta^2 + \cdots$$

- Non-zero VEV for $D \rightarrow$ SUSY spontaneously broken
- χ: massless, the Goldstino field
- Define a nonlinear Goldstino field λ by demanding $\chi^\sigma = 0$ in nonlinearly realized super-multiplet V^σ
- Problems about gauge and supergravity

Constrained Field for the Goldstino-1

- Goldstino field in a linearly chiral superfield X_{NL}
- $X_{NL}^2 = 0$, to rid of the scalar component
  - Supersymmetry structure and its breaking
  - $X_{NL} = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F$
- Define $\lambda^{NL} = G/\sqrt{2}\kappa F$

$$\delta_\xi \lambda_\alpha^{NL} = \frac{\xi_\alpha}{\kappa} - 2i\kappa\lambda^{NL}\sigma^\mu\bar{\xi}\partial_\mu\lambda_\alpha^{NL}$$

- λ^{NL} transforms in exactly the same way as $\tilde{\lambda}$
- $\lambda^{NL} = \tilde{\lambda}$ and $X_{NL} = F\Theta^2$, $\Theta = \theta + \kappa\tilde{\lambda}$

Constrained Field for the Goldstino-2

- Self consistent check:
  - $X^{\sigma}_{NL} = \theta^2 F^{\sigma}$
  - λ disappears in the nonlinearly realized super-multiplet
- Reverse the logic
  - For any chiral superfield $\Phi = \phi + \sqrt{2}\theta\psi + \theta^2 F$ define $\lambda^{\Phi} = \psi/\sqrt{2}\kappa F$

$$\delta_\xi \lambda^{\Phi}_\alpha = \frac{\xi_\alpha}{\kappa} - 2i\kappa\lambda^{\Phi}\sigma^\mu\bar{\xi}\partial_\mu\lambda^{\Phi}_\alpha + \frac{i}{\kappa F}(\sigma^\mu\bar{\xi})_\alpha\partial_\mu\left(\phi - \frac{\psi^2}{2F}\right)$$

  - Demanding λ^{Φ} to transform in the same way as that of $\tilde{\lambda}$, one obtains $\phi = \psi^2/2F$ and $\Phi^2 = 0$

Constrained Field for the Goldstino-3

- Prompt λ to a linear superfield

$$\Lambda(\lambda) = \exp(\theta Q + \bar{\theta}\bar{Q}) \times \lambda$$

- Construct two chiral fields out of Λ and $\bar{\Lambda}$

$$\Phi_2 = -\frac{1}{4}\bar{D}^2\Lambda\Lambda, \quad \Phi_4 = -\frac{1}{4}\bar{D}^2\Lambda\Lambda\bar{\Lambda}\bar{\Lambda}.$$

$$\Phi_2 = f_2(\lambda)\Theta^2, \quad \Phi_4 = f_4(\lambda)\Theta^2,$$

- $f_2,\ f_4$: definite functions of λ
- f_4: the AV Lagrangian up to an overall constant and possible total derivative terms
- f_4/f_2 and F/f_2 transform as matter fields

Constrained Field for the Goldstino-4

- Some history:

$$\Phi_4 \bar{D}^2 \bar{\Phi}_4 \sim \Phi_4$$

  while Φ_2 does not have such relation
- The rationale to choose Φ_4 instead of Φ_2 to be the superfield for Goldstino
- They differ only by a matter field in the standard realization
- Obvious in retrospect, since $\Phi_2^2 = \Phi_4^2 = 0$, the same form of factorization

Constrained Field for the Goldstino-4

- Real superfield $V_4 = \Lambda^2\bar{\Lambda}^2$
- $V_4^2 = 0$ and $V_4 = f_4(\lambda)\Theta^2\bar{\Theta}^2$, $\Theta = \theta + \kappa\lambda$
- f_4: the AV Lagrangian up to an overall constant and possible total derivative terms
- $V^2 = 0$ cannot be preserved under a general gauge transformation
- For any $V = D\theta^2\bar{\theta}^2 + \chi\theta\bar{\theta}^2 + \bar{\chi}\bar{\theta}\theta^2 + \cdots$
  - Define $\lambda^V = \chi/2\kappa D$

$$\delta_\xi \lambda_\alpha^V = \frac{\xi_\alpha}{\kappa} - i(\lambda^V\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\lambda}^V)\partial_\mu\lambda_\alpha^V + \frac{i}{D}(\text{total derivatives})$$

  - Demanding λ^V to transform in the same way as that of λ, one gets $V = D\Theta^2\bar{\Theta}^2$
  - D/f_4 transforms as a matter field

Constrained Field for the Goldstino-5

- The constraint to rid of the scalar component in a chiral superfield, $Q_{NL} = \phi_q + \sqrt{2}\theta\psi_q + \theta^2 F_q$

$$X_{NL} Q_{NL} = 0$$

  From which, one gets

$$\phi_q = \frac{\psi_q G}{F} - \frac{G^2}{2F^2} F_q$$

- Equivalent to the constraint $\phi^\sigma = 0$

Low Energy Effective Theory-1

- Prompt λ and matter fields to linear super-multiplets

$$\Lambda(\lambda) = \exp(\theta Q + \bar{\theta}\bar{Q}) \times \lambda$$

$$\Omega(\lambda) = \exp(\theta Q + \bar{\theta}\bar{Q}) \times \varphi$$

- SUSY non-invariant action $\longrightarrow$ SUSY invariant one

$$\int d^4x L(\partial_\mu \varphi, \varphi)$$

$$\Downarrow$$

$$\kappa^4 \int d^4x d^4\theta \Lambda\Lambda\bar{\Lambda}\bar{\Lambda} L(\partial_\mu \Omega, \Omega)$$

- $\Lambda(x) = \kappa^{-1}\theta' = \kappa^{-1}\theta + \lambda(z)$
  $\Omega(x) = \varphi(z)$, $z = x - i\kappa\lambda(z)\sigma\bar{\theta} + i\kappa\theta\sigma\bar{\lambda}(z)$

Low Energy Effective Theory-2

- Integrate out the Grassmann variables: $(x, \theta) \longrightarrow (z, \theta^{'})$

$$S = \int d^4 x (\det T) \; L(\nabla_\mu \varphi, \varphi)$$

- Same results by changing $\partial_\mu \longrightarrow \nabla_\mu$ and inserting $\det T$
- S is invariant under nonlinear SUSY transformations
- Integrations over the Grassmann variables can always be carried out in a similar manner for arbitrary functionals of Λ and Ω $\longrightarrow$ extra operators for effective theories

Low Energy Effective Theory-3

- Subtleties for gauge theories
- Wess-Zumino gauge, starting with the transformation

$$\begin{aligned} \delta_\xi A_\mu &= -i\chi\sigma_\mu\bar{\xi} + i\xi\sigma_\mu\bar{\chi} \\ \delta_\xi \chi_\alpha &= \sigma^{\mu\nu}\xi_\alpha F_{\mu\nu} + i\xi D \\ \delta_\xi D &= -D_\mu\chi\sigma^\mu\bar{\xi} - \xi\sigma^\mu D_\mu\bar{\chi} \end{aligned}$$

- $D_\mu = \partial_\mu - iA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - i\partial_\nu A_\mu - i[A_\mu, A_\nu]$
- Construct four superfields $V_\mu = \exp(\theta Q + \bar{\theta}\bar{Q}) \times A_\mu$
- Four nonlinearly realized superfields

  $\widehat{V}_\mu = \exp\left[-\kappa(\lambda Q + \bar{\lambda}\bar{Q})\right] \times V_\mu = \hat{A}_\mu + i\theta\sigma_\mu\bar{\hat{\chi}} - i\hat{\chi}\sigma_\mu\bar{\theta} + \cdots$

Low Energy Effective Theory-4

- Transformation rules of $\widehat{A}_\mu$

$$\delta_\xi \widehat{A}_\mu = -i\kappa v_\xi^\nu \widehat{F}_{\nu\mu}$$

$\widehat{F}_{\mu\nu} = \partial_\mu \widehat{A}_\nu - i\partial_\nu \widehat{A}_\mu - i[\widehat{A}_\mu, \widehat{A}_\nu]$

- This can be rewritten as

$$\delta_\xi \widehat{A}_\mu = -i\kappa v_\xi^\nu \partial_\nu \widehat{A}_\mu - i\kappa \partial_\mu v_\xi^\nu \widehat{A}_\nu + D_\mu(i\kappa v_\xi^\nu \widehat{A}_\nu)$$

- The last term can be compensated by a gauge transformation of the parameter $-i\kappa v_\xi^\nu \widehat{A}_\nu$
- Under this combination of SUSY and gauge transformations

$$\delta'_\xi \widehat{A}_\mu = -i\kappa v_\xi^\nu \partial_\nu \widehat{A}_\mu - i\kappa \partial_\mu v_\xi^\nu \widehat{A}_\nu$$

Low Energy Effective Theory-5

- Define

$$\mathcal{D}_\mu = (T^{-1})_\mu^\nu D_\nu = (T^{-1})_\mu^\nu(\partial_\nu - iA_\nu)$$

$$\mathcal{F}_{\mu\nu} = (T^{-1})_\mu^\rho (T^{-1})_\nu^\sigma(\partial_\rho A_\sigma - \partial_\sigma A_\rho - i[A_\rho, A_\sigma])$$

- $\mathcal{D}_\mu$ and $\mathcal{F}_{\mu\nu}$ transform covariantly under both SUSY and gauge rotation
- Substitute $D_\mu \to \mathcal{D}_\mu$ and $F_{\mu\nu} \to \mathcal{F}_{\mu\nu}$:
  non-SUSY Lagrangians $\to$ SUSY Lagrangians
  (with gauge invariance)

Conclusions

- Construct $\tilde{\lambda}$ out of $(\phi_0,\ \psi_0,\ F_0) \subset \Phi_0$

$$\tilde{\lambda} = \frac{\psi_0}{\sqrt{2}\kappa F_0} - i\frac{\sigma^\mu \bar{\tilde{\lambda}}}{F_0}\left(\partial_\mu \phi_0 - \sqrt{2}\kappa\tilde{\lambda}\partial_\mu \psi_0 + \kappa^2 \tilde{\lambda}^2 \partial_\mu F_0\right)$$

- Linear SUSY theories reformulated into non-linear ones
- Goldstino field disappears in the process, reemerges in the Jacobian and covariant derivatives
- Vertices with Goldstinos carry space-time derivatives
- Heavy ones can be integrated out, via e.o.m. or matching, without breaking SUSY
- Constrained superfield reformulated in terms of the standard realization: $X_{NL}^2 = 0 \to \tilde{\lambda} = \psi/\sqrt{2}\kappa F$
- SUSY non-invariant theories can be prompted to non-linearly invariant ones

Thank You!