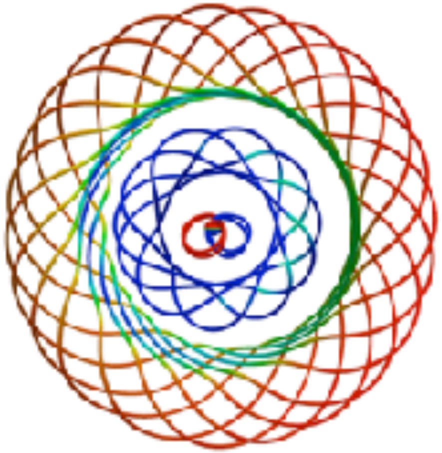
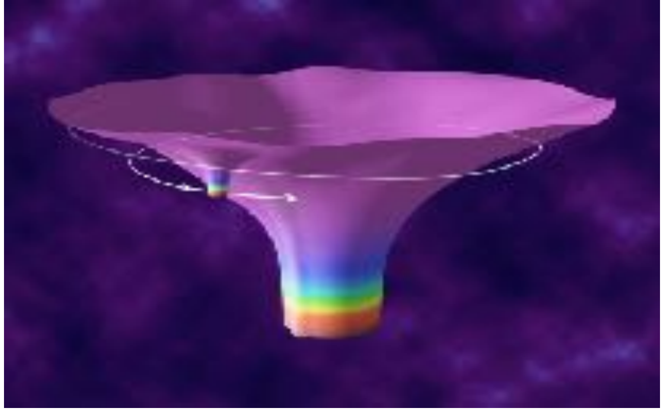
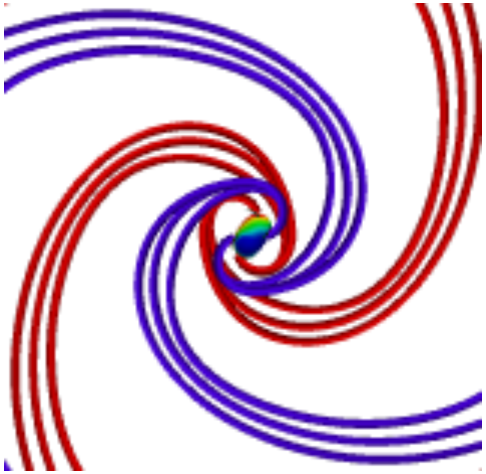
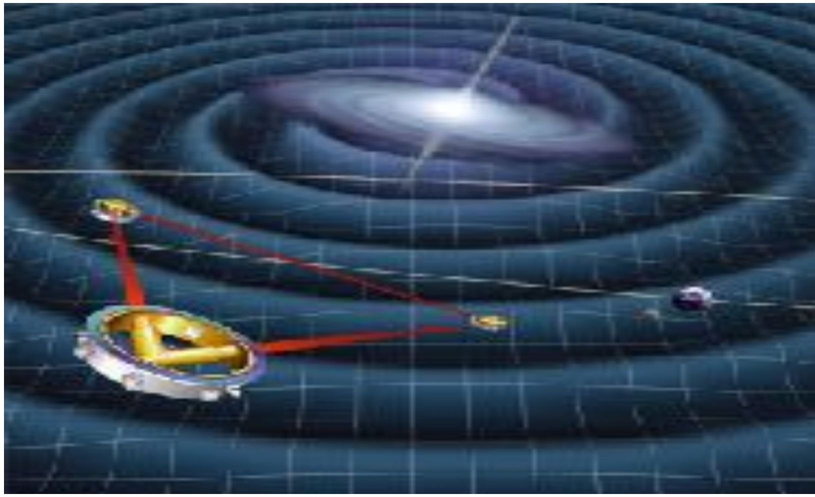




Geometrodynamics

Exploring the nonlinear dynamics of curved spacetime via computer simulations and gravitational wave observations

Kip Thorne

ITP Physics Colloquium, Beijing, 20 December 2017

John Wheeler: Geometrodynamics

The Nonlinear Dynamics of Curved Spacetime

Nonlinear Dynamics Elsewhere in Physics

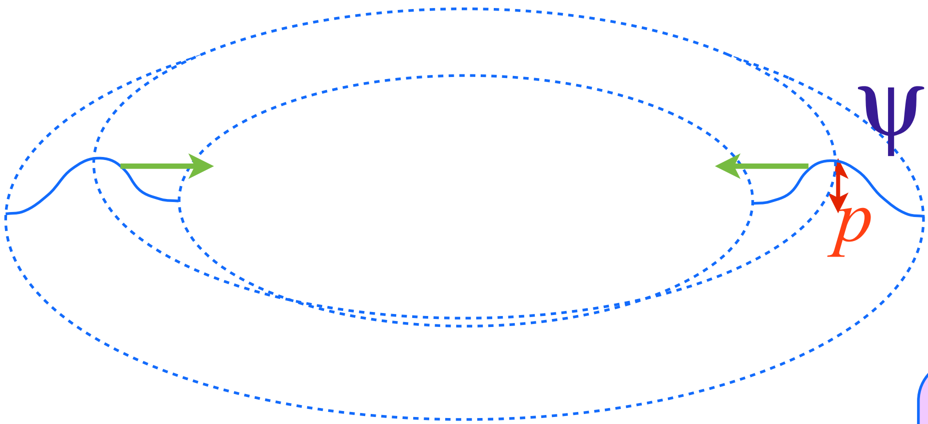
- fluid turbulence, tornados, …
- phase transitions in condensed matter
- nonlinear optics (modern optical technology)
- colliding solitons in fluids, plasmas,
  nonlinear crystals, optical fibers, …
- chaotic maps, strange attractors, …

Four Arenas for Geometrodynamics probed by numerical & analytical relativity

- Gravitational waves: nonlinear self coupling in ***critical gravitational collapse***
- Spacetime dynamics near ***singularities***
  - cosmological singularities
  - singularities inside black holes
- ***Binary black hole*** mergers
- Gravitational-wave ***observations***

Gravitational Waves: Nonlinear Self-coupling

- Motivation: Choptuik's analysis of spherical scalar-wave implosion (1993 -)



scalar wave's energy generates spacetime curvature, then wave interacts with the curvature

$p > p_*$: Black hole forms

$$M_{BH} \propto (p - p_*)^\beta \ , \quad \beta \simeq 0.374$$

$p < p_*$: Wave disperses

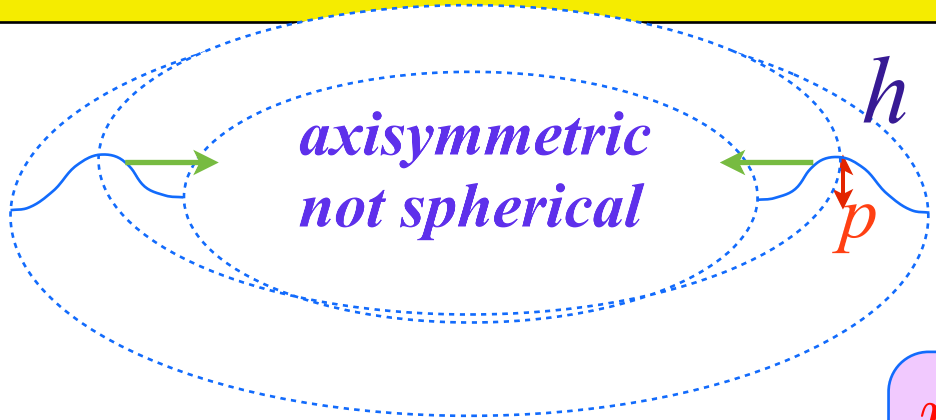
$$(R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta})^{-1/4}_{\max} \propto (p_* - p)^\beta$$

$p = p_*$:Discretely self-similar

Gravitational Waves: Nonlinear Self-coupling

- ***Imploding Gravitational Wave***

Abrahams & Evans (1993)
Evgeny Sorkin (2011)



wave self-coupling via
nonlinear Einstein equations

$p > p_*$: Black hole forms

$$M_{BH} \propto (p - p_*)^{\beta}\ , \quad \beta \ \simeq 0.38$$

$p < p_*$: Wave disperses

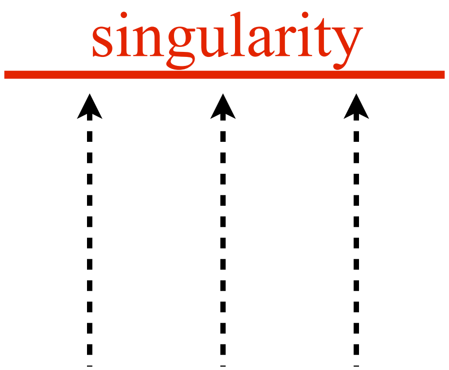
$$(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})^{-1/4}_{\max} \propto (p_* - p)^{\beta}$$

$p = p_*$: Moderately strong evidence for discrete self similarity

Numerical studies are in their infancy.
Great richness remains to be uncovered!

Geometrodynamics Near Singularities

- Some ancient history:
  - » 1960s: ***Singularity theorems*** - Penrose, Hawking, ...
  - » 1969 - 71: ***BKL*** approximate analysis (Belinsky, Khalatnikov, Lifshitz): ***geometrodynamics near generic spacelike singularity:***
    - – spatial decoupling; PDEs -> ODEs in time
    - – temporal dynamics is ***Mixmaster*** (Misner; Belinsky & Khalatnikov)
    - – matter has negligible influence



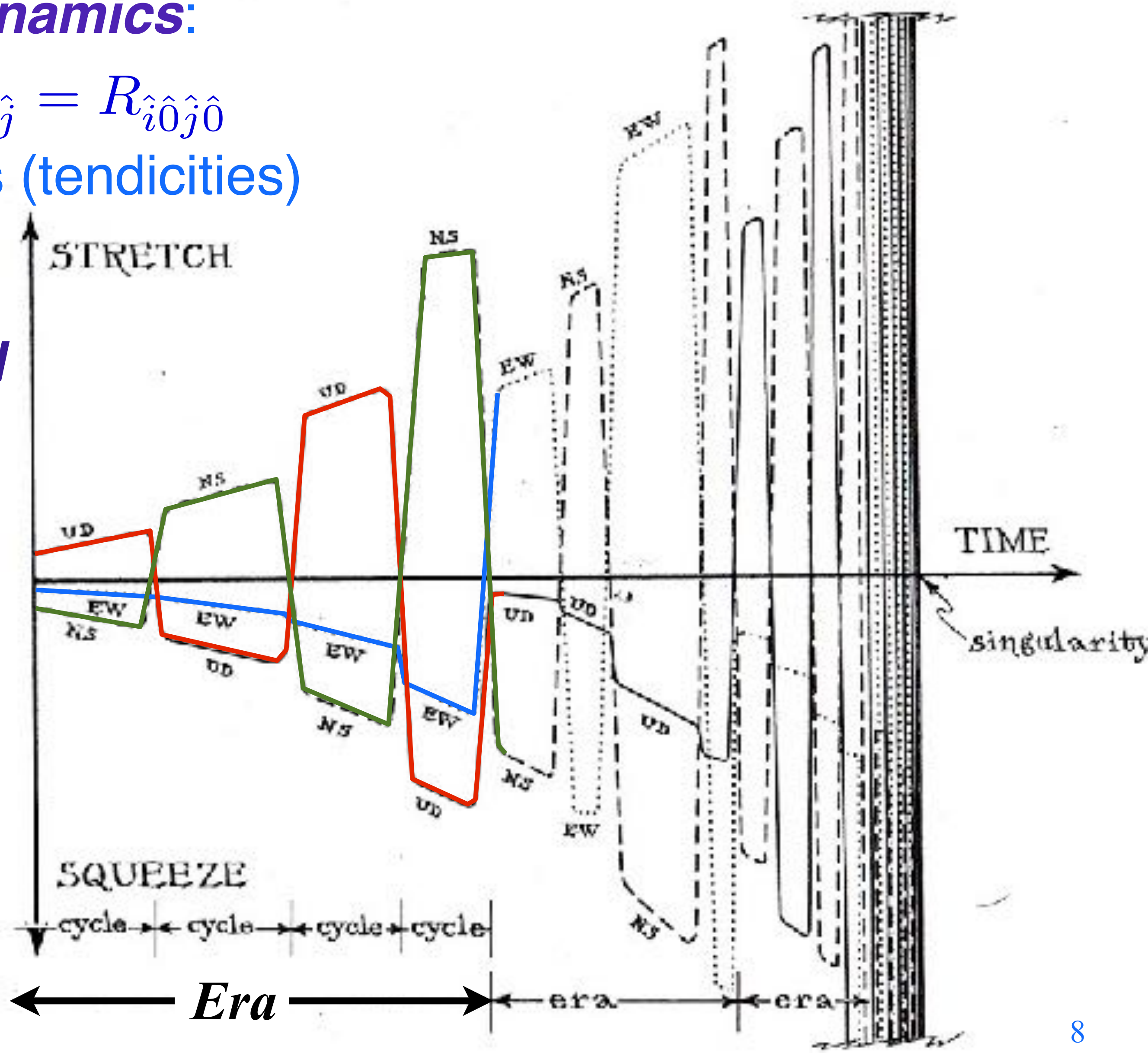
  - » ***Skepticism in the West:***
    - – BKL “conjecture” and “heuristic arguments”

Geometrodynamics Near Singularities

- ***Mixmaster dynamics***:
  - » tidal field $\mathcal{E}_{\hat{i}\hat{j}} = R_{\hat{i}\hat{0}\hat{j}\hat{0}}$
  - » eigenvalues (tendicities)

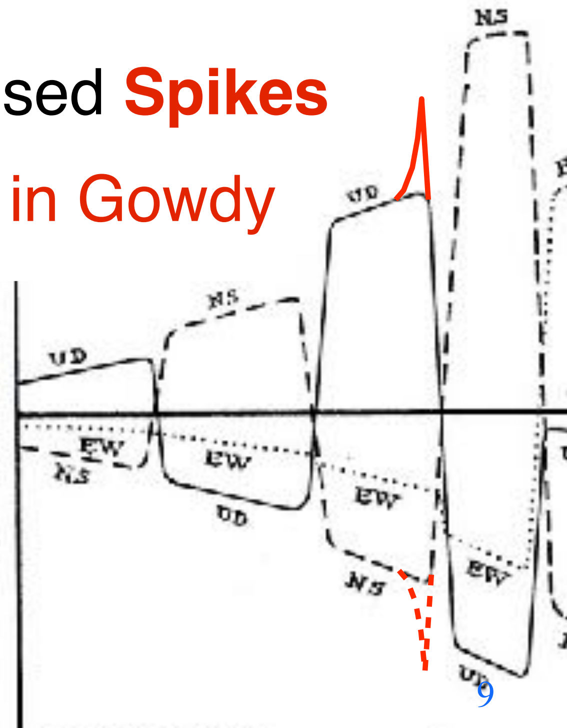
- ***Era transitions driven by spatial curvature***

- Chaotic map



Geometrodynamics Near Singularities

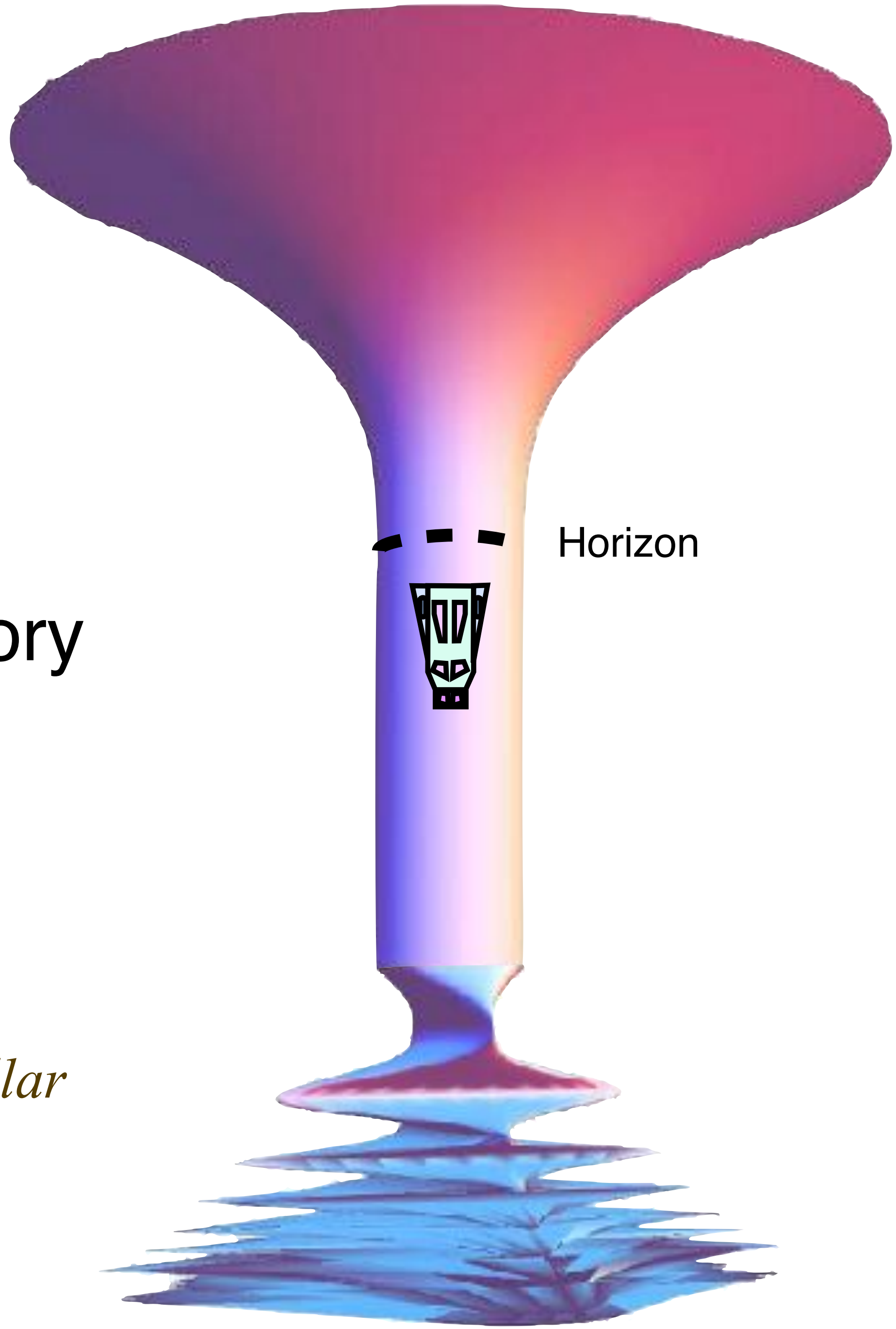
- Program to test BKL via numerical simulations:
  - » Formulated by Beverly Berger and Vince Moncrief (1994)
  - » Carried out by Berger, Moncrief, **Garfinkle**, Isenberg, Weaver: 1994 - ....
  - » Analytic studies motivated by simulations: Rendall, Weaver, ... 2001 - ....
- ***BKL largely confirmed, Except:*** BKL missed **Spikes**
  - » Discovered by Berger & Moncrief (1994) in Gowdy
  - » ***Triggered by spatial inhomogeneity***
  - » Recur; sharper at later times
  - » Modify the chaotic map (Lim, Andersson, Garfinkle, Pretorius, 2009)

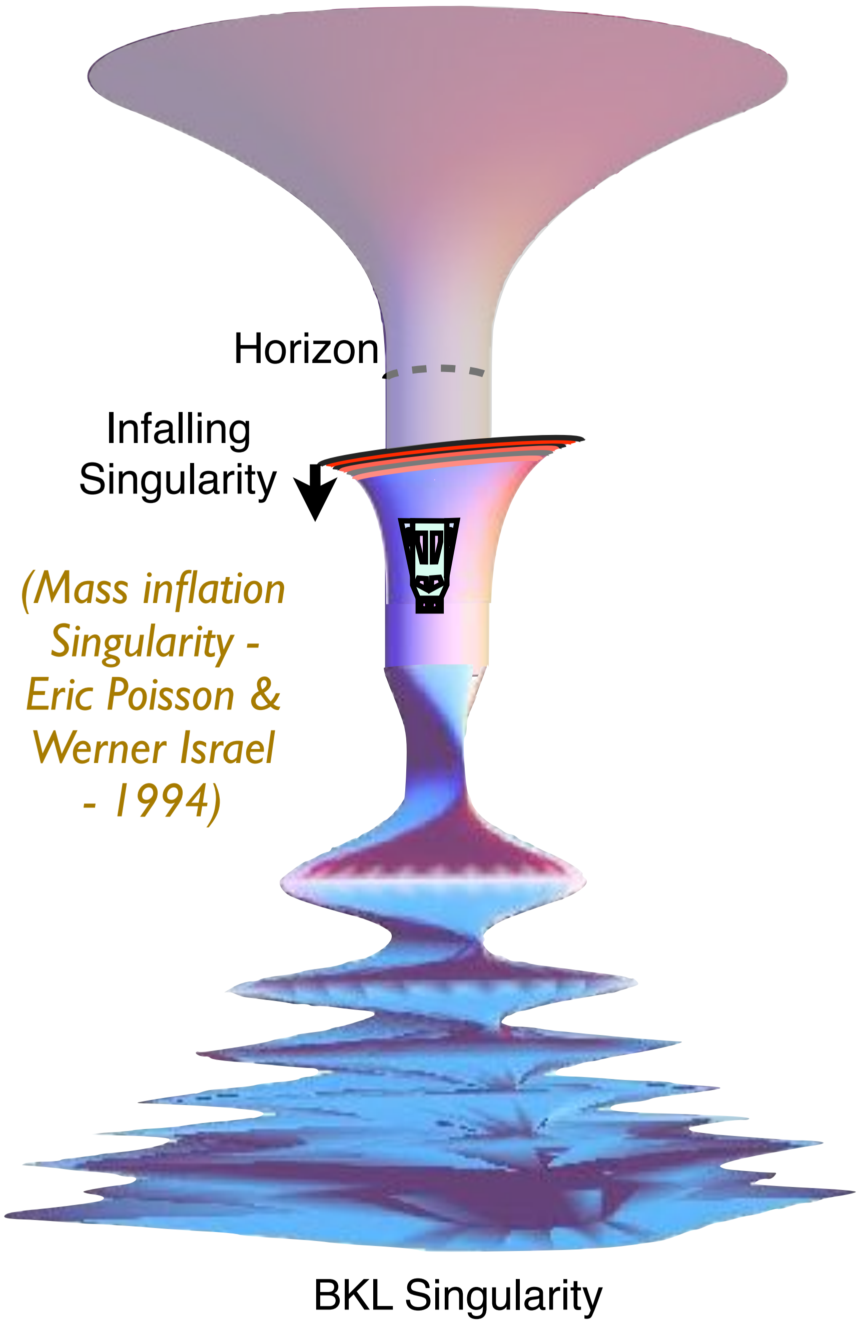


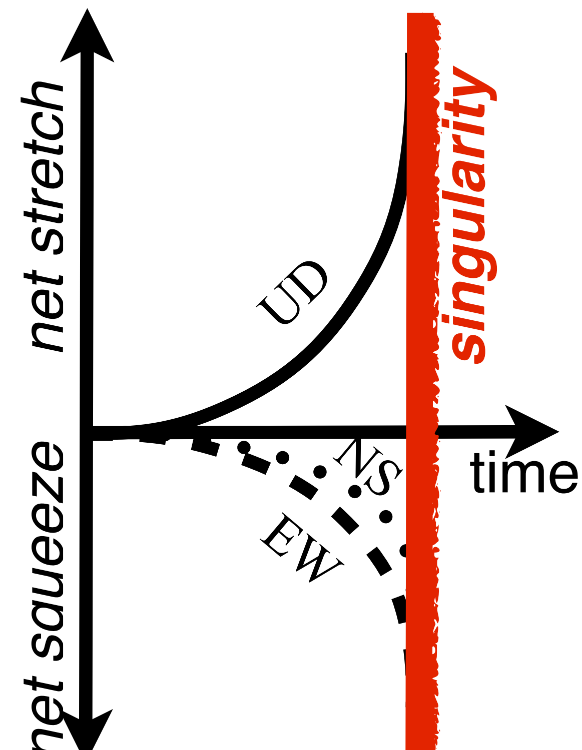
Generic Singularity Inside a Black Hole

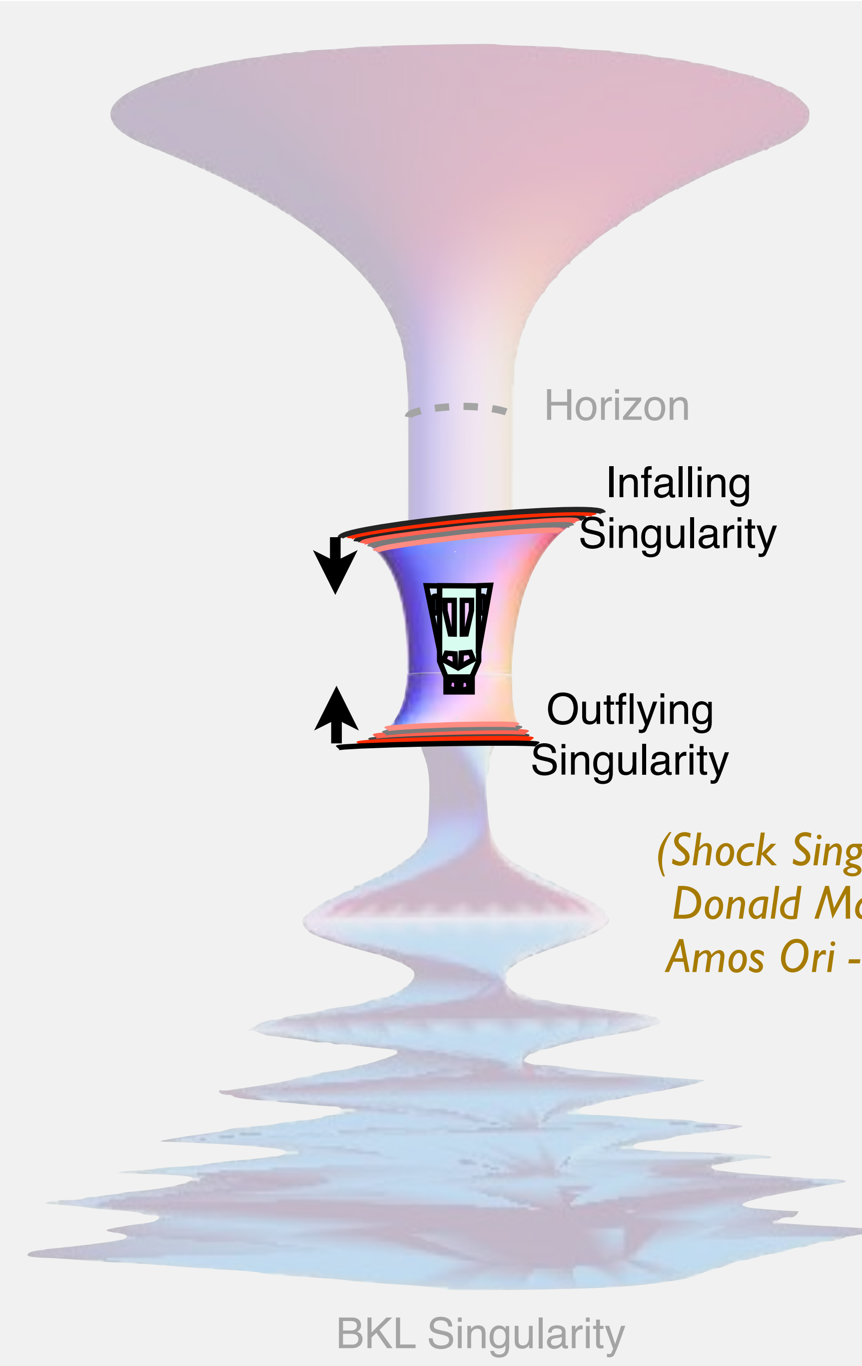
Educated guess based largely on perturbation theory

from my book *The Science of Interstellar*



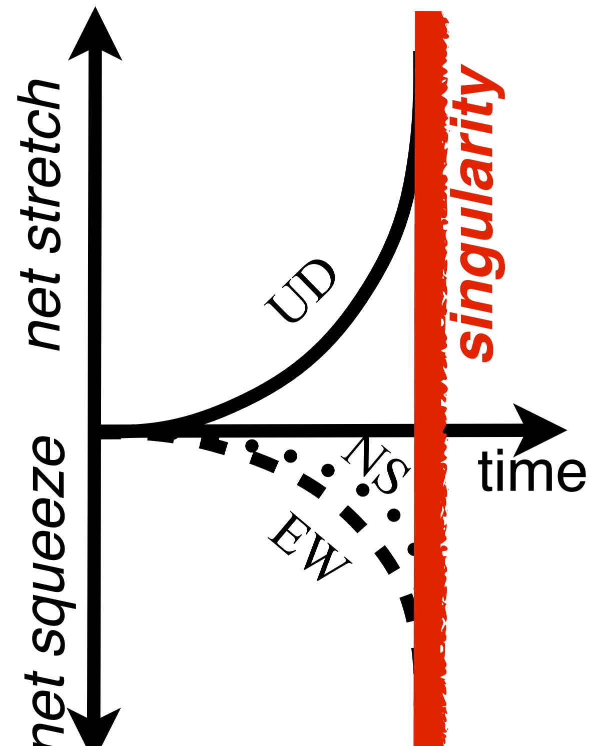

Horizon
Infalling
Singularity
(Mass inflation
Singularity -
Eric Poisson &
Werner Israel
- 1994)
BKL Singularity


net stretch
net squeeze
time
singularity
UD
NS
EW



(Shock Singularity - Donald Marolf & Amos Ori - 2012)

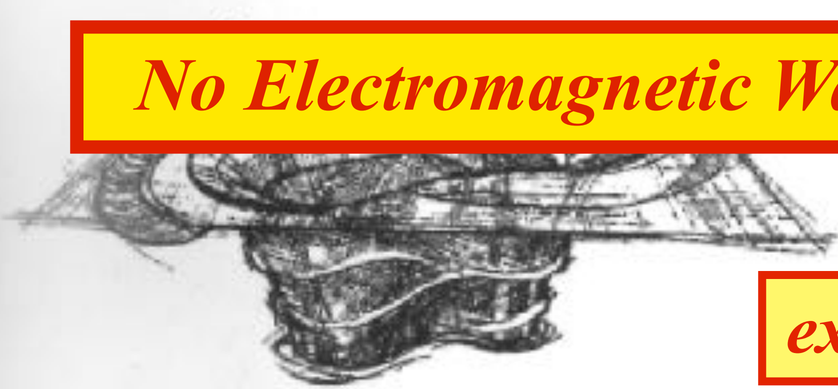
Simulations Greatly Needed



Geometrodynamics in Binary Black Holes

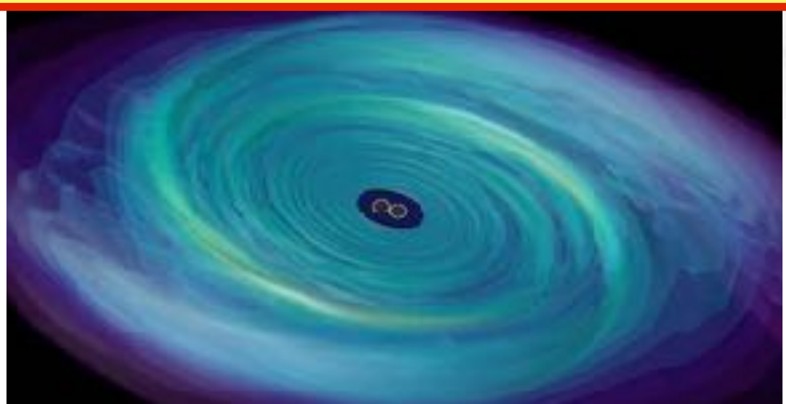
Collisions of Black Holes: The most violent events in the Universe

Power output in gravitational waves: ~ 50 times luminosity of all stars in Universe

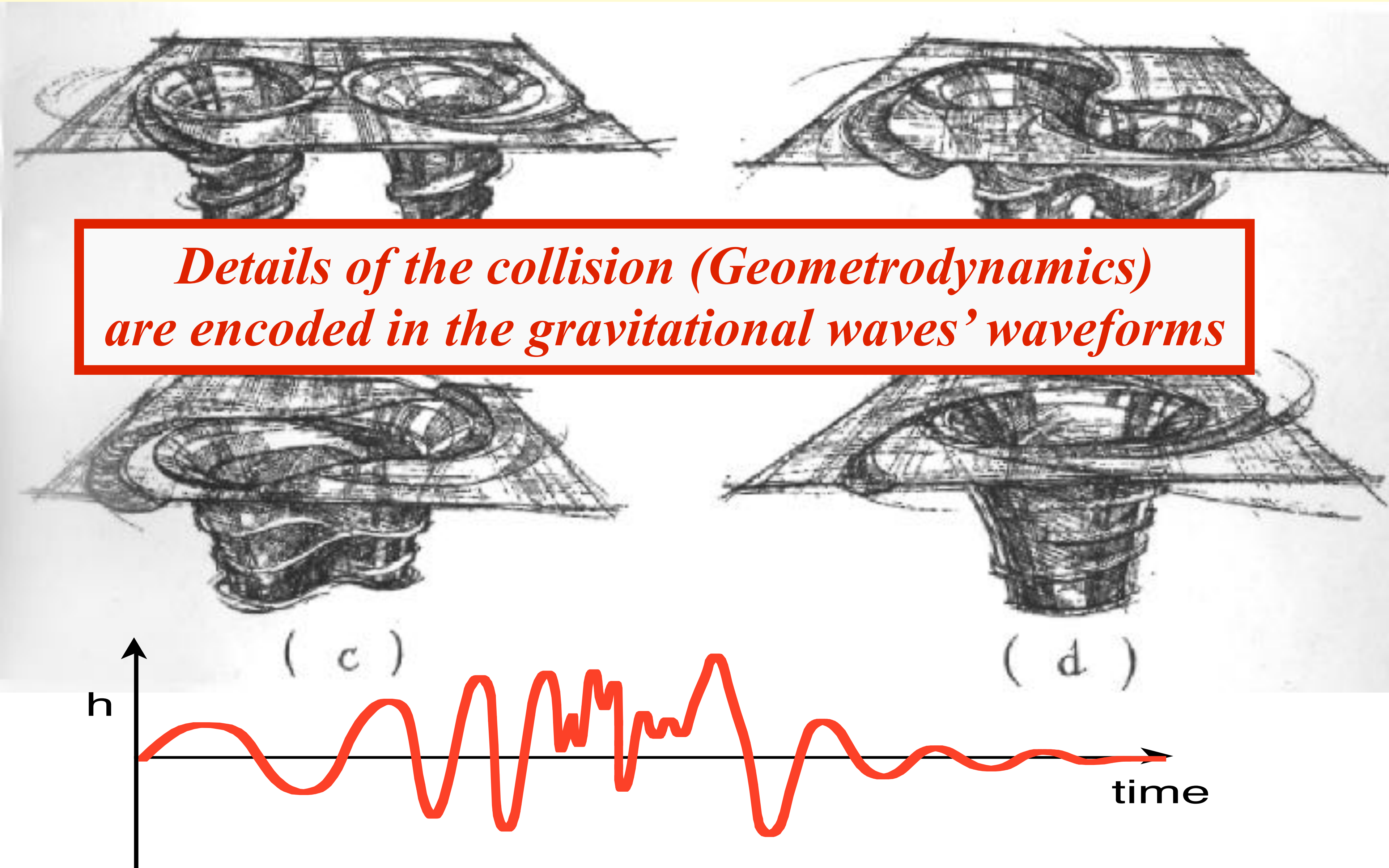


No Electromagnetic Waves emitted whatsoever

except from disturbed accretion disks



Collisions of Black Holes:
The most violent events in the Universe



Details of the collision (Geometrodynamics) are encoded in the gravitational waves' waveforms

Example of Numerical Simulation

GW150914

SXS* Collaboration:

[Project to Simulate eXtreme Spacetimes]

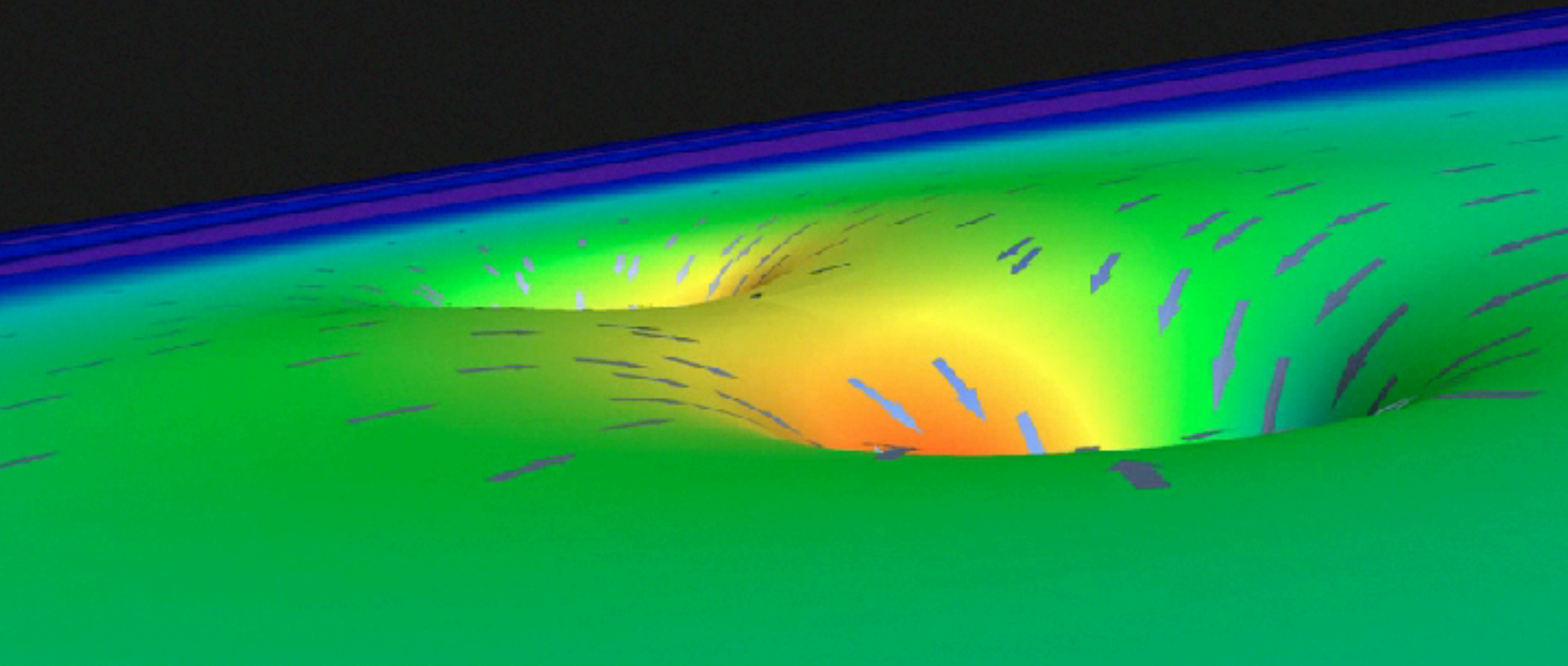
Cornell/Caltech/CITA/CalState Fullerton/Oberlin/WSU

[Kidder, Pfeiffer, Scheel, Teukolsky,…]

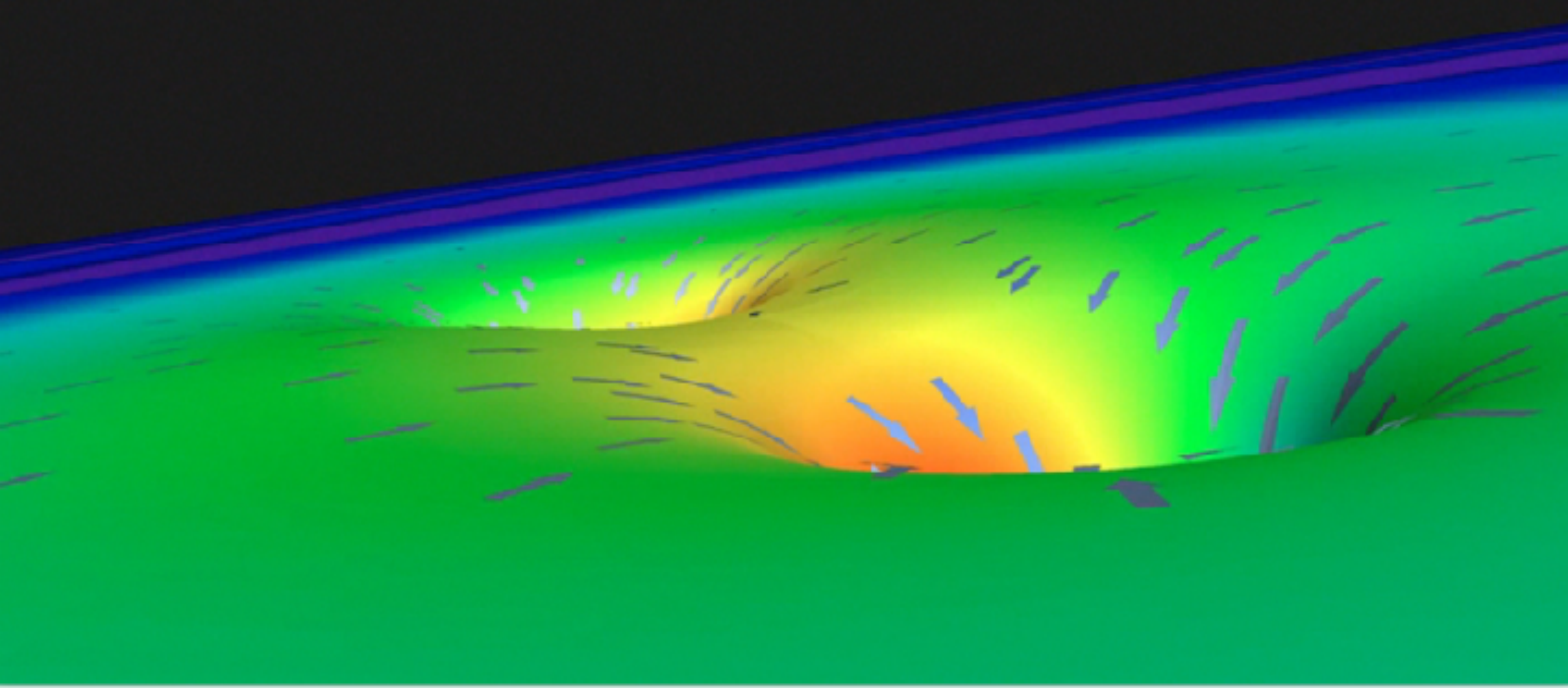
Depiction of spacetime metric (geometry) in orbital plane

Pseudo Embedding Diagram
Video by Harald Pfeiffer
from SXS Simulation

$$\frac{-\mathrm{sign}(R)}{|R|^{1/2}}$$



PROBLEM:
Too little of the spacetime geometry is depicted this way!



Visualizing the Vacuum Riemann Curvature Tensor

Rob Owen, Jeandrew Brink, Yanbei Chen, Jeff Kaplan, Geoffrey Lovelace, Keith Matthews, David Nichols, Mark Scheel, Fan Zhang, Aaron Zimmerman, and Kip Thorne

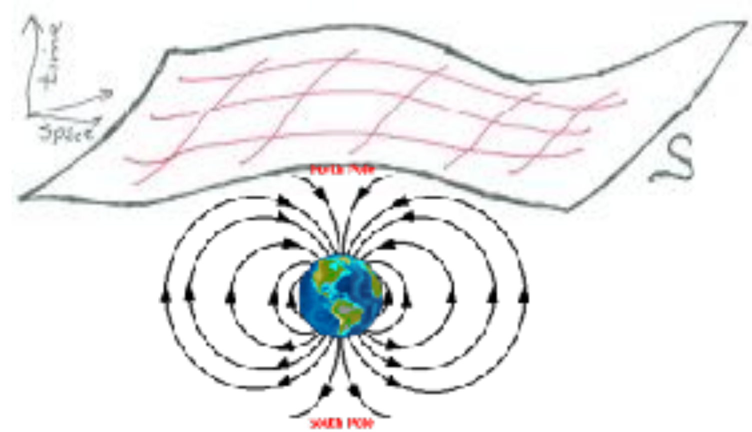
Caltech, Cornell, and NiTheP (South Africa)

Physical Review Letters , **106,** 151101 (2011)

[arXiv:1012.4869]

Tidal Field & *Frame-Drag Field*

- Slice spacetime into space plus time
- EM field tensor ***F*** ➔ Electric field and magnetic field; visualize with field lines

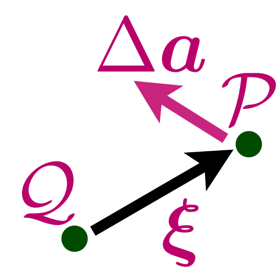


- Weyl curvature tensor ➔ **"electric" part** $\mathcal{E}_{jk}$ and **"magnetic" part** $\mathcal{B}_{jk}$

$$\mathcal{E}_{jk} = C_{0j0k} \qquad \mathcal{B}_{jk} = \frac{1}{2}\epsilon_{jpq}C^{pq}{}_{k0}$$

Symmetric, Trace-Free (STF) tensors

- $\mathcal{E}_{jk}$ describes **tidal accelerations** $\Delta a_j = -\mathcal{E}_{jk}\,\xi^k$



We call $\mathcal{E}_{jk}$ the ***tidal field***

- $\mathcal{B}_{jk}$ describes **differential frame dragging**: Gyroscope at P precesses relative to inertial frames at Q with angular velocity

$$\Delta\Omega_j = \mathcal{B}_{jk}\xi^k$$

We call $\mathcal{B}_{jk}$ the ***frame-drag field***

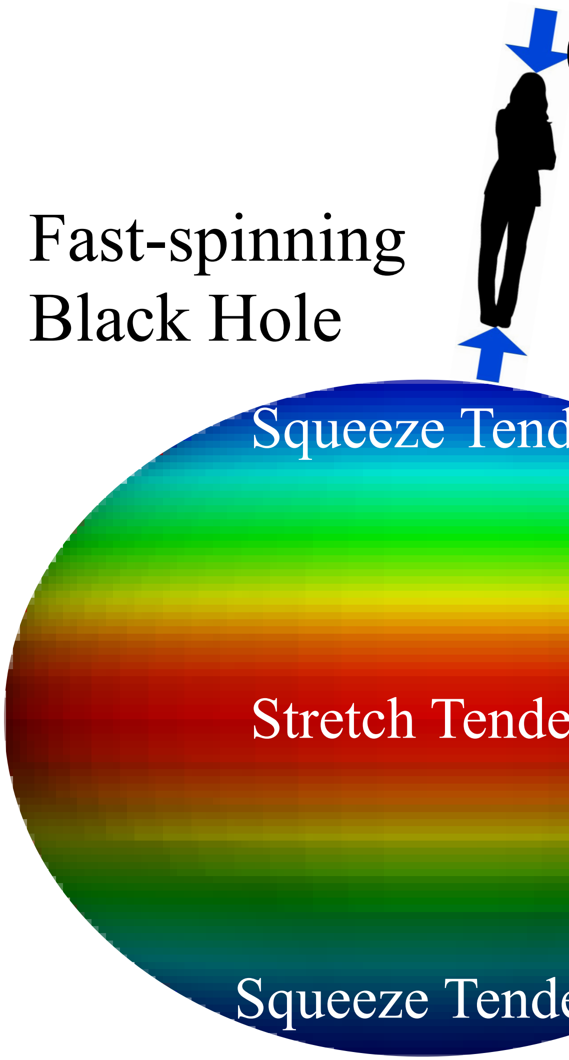
Horizon Tendicity

$\text{Tendicity} = \mathcal{E}_{nn} = (\text{RelativeAcceleration})/\text{height}$

Blue: Squeeze; $\mathcal{E}_{nn} > 0$

Red: Stretch; $\mathcal{E}_{nn} < 0$

Green: small tendicity; $\mathcal{E}_{nn} \simeq 0$



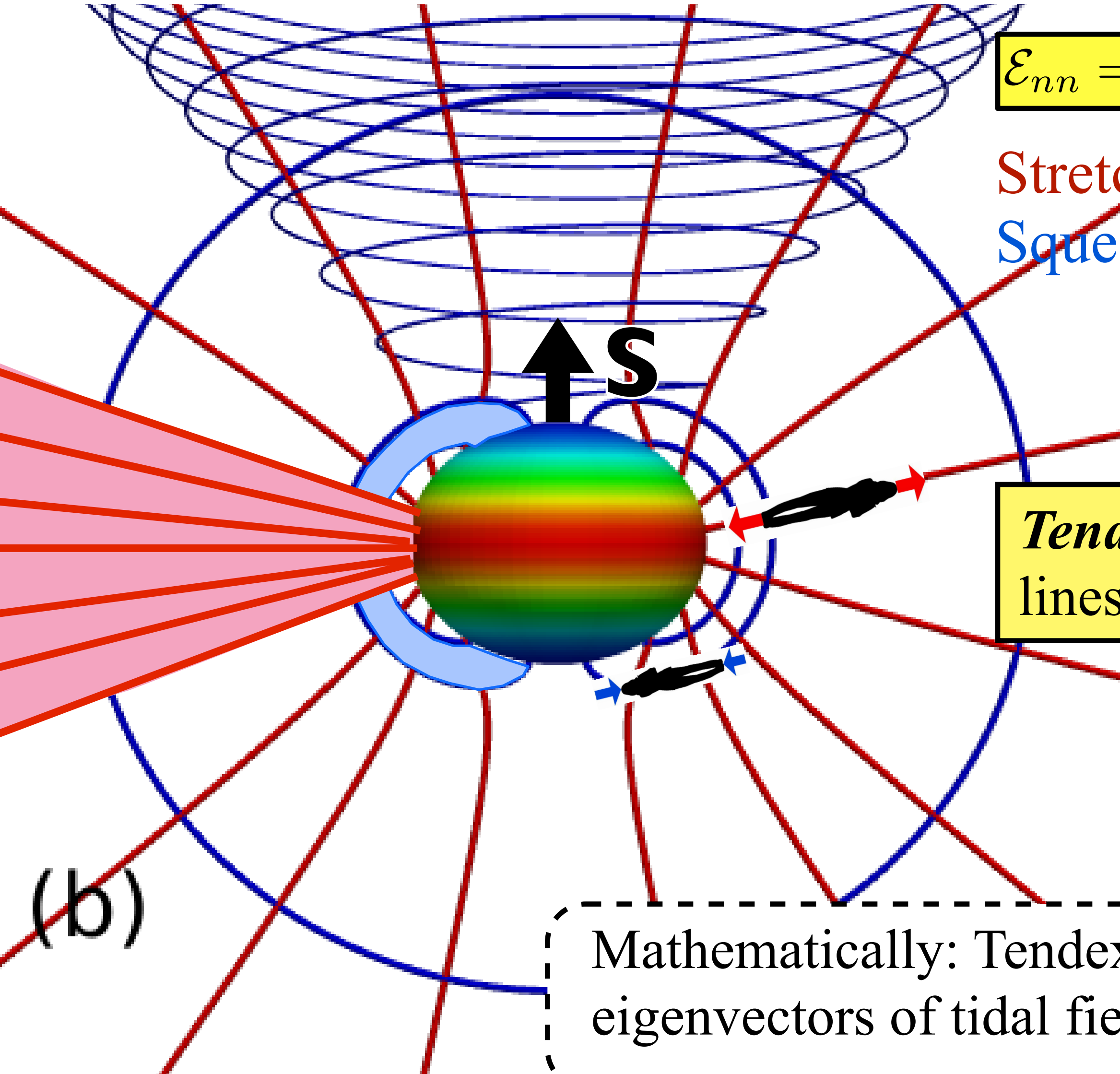
Horizon Tendex: Region with large tendicity

Mathematically: $\mathcal{E}_{nn}$ is normal-normal component of tidal field

Tendex Lines and their Tendicities

Fast Spinning Black Hole

Each tendex line has a tendicity $\mathcal{E}_{nn}$



$\mathcal{E}_{nn} = (\text{RelativeAcceleration})/\text{height}$

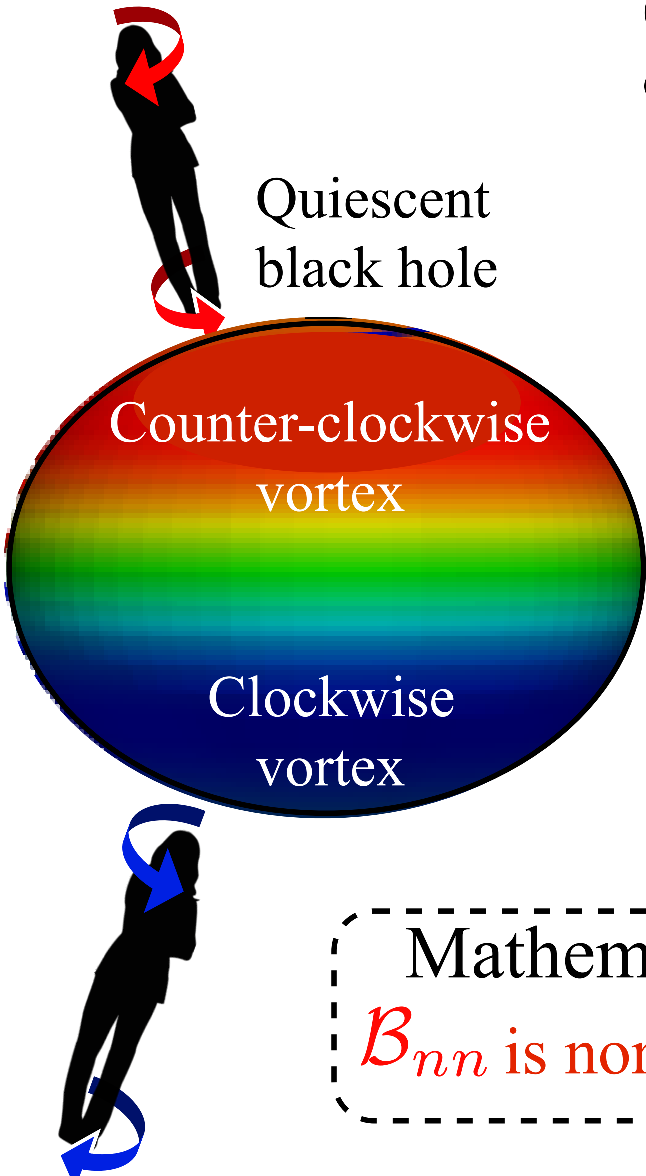
Stretch along red lines $\mathcal{E}_{nn} < 0$

Squeeze along blue lines $\mathcal{E}_{nn} > 0$

Tendex: a collection of tendex lines with large tendicity

Mathematically: Tendex lines are Integral curves of eigenvectors of tidal field $\mathcal{E}_{ij}$; tendicity is eigenvalue

Horizon Vorticity



(Angular velocity of feet as seen by head, or head as seen by feet) = Ω

$$\boxed{\text{Horizon Vorticity} = \mathcal{B}_{nn} = \Omega/\text{height}}$$

Blue: Clockwise vorticity; $\mathcal{B}_{nn} > 0$

Red: Counter-clockwise vorticity; $\mathcal{B}_{nn} < 0$

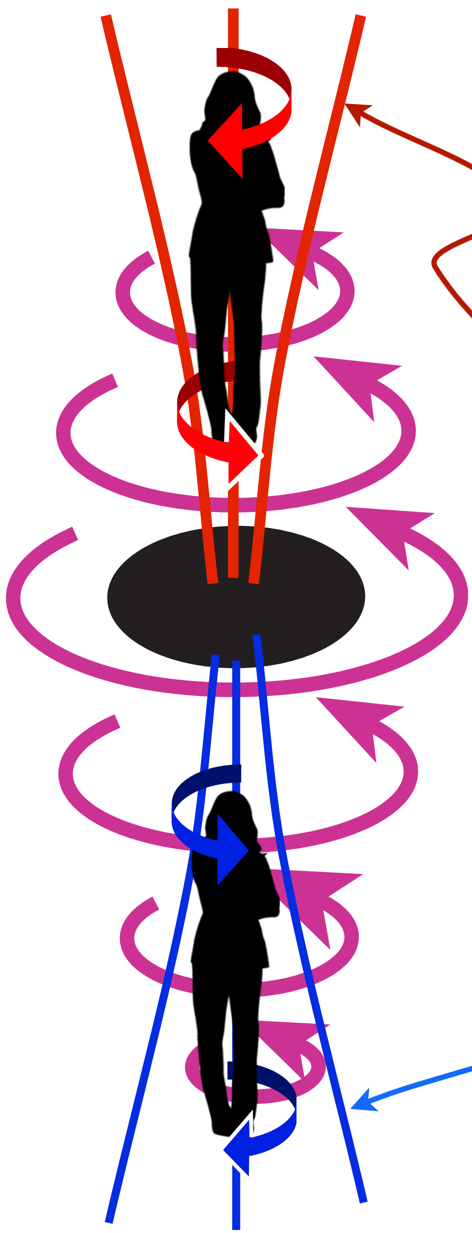
Green: small vorticity; $\mathcal{B}_{nn} \simeq 0$

Horizon Vortex: Region with high vorticity

Mathematically:

$\mathcal{B}_{nn}$ is normal-normal component of frame-drag field

Vortex Lines Outside Black Hole



Each vortex line has a vorticity

Vorticity $= \mathcal{B}_{nn} = \Omega/\text{height}$

Blue: Clockwise vorticity; $\mathcal{B}_{nn} > 0$

Red: Counter-clockwise vorticity; $\mathcal{B}_{nn} < 0$

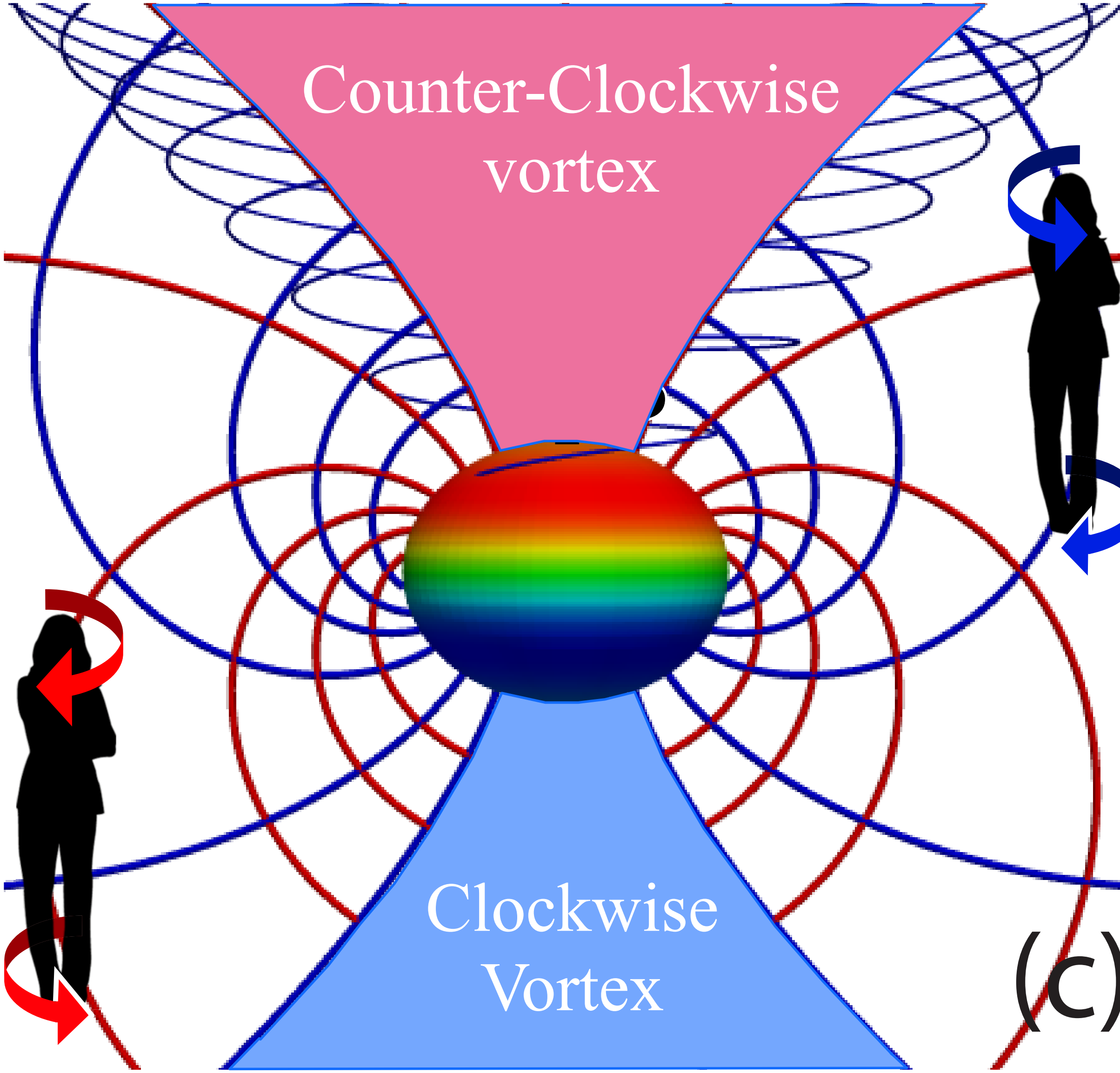
Vortex lines guide the whirling vortex

Mathematically: ***Vortex line*** is Integral curve of an eigenvector ***n*** of frame-drag field $\mathcal{B}_{ij}$

Vorticity $\mathcal{B}_{nn}$ is eigenvalue of $\mathcal{B}_{ij}$

Vortex lines & their vorticities completely characterize the frame-drag field $\mathcal{B}_{ij}$

Kerr Black Hole

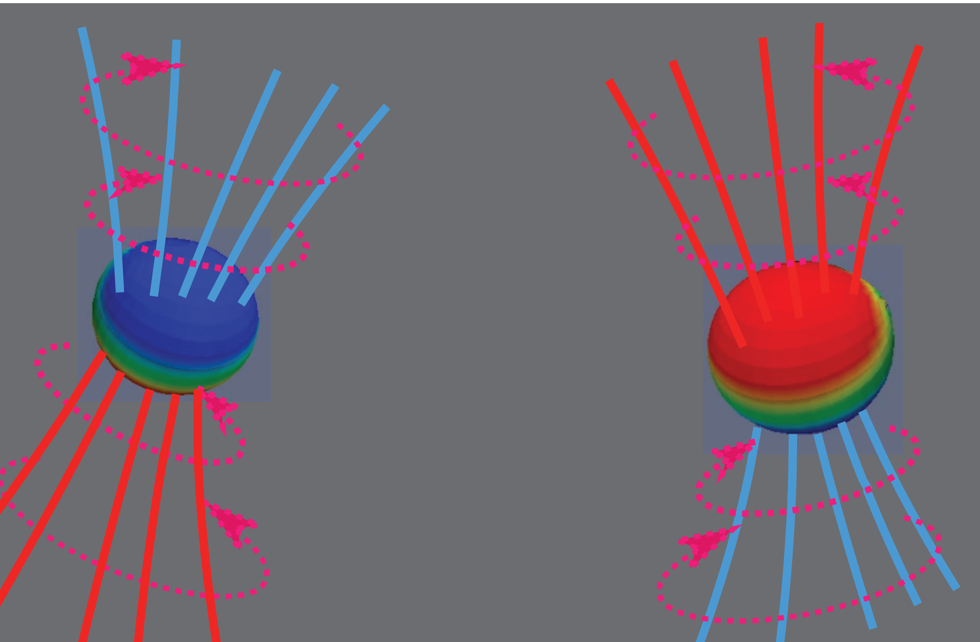


Vortex
A collection of vortex lines with large vorticity

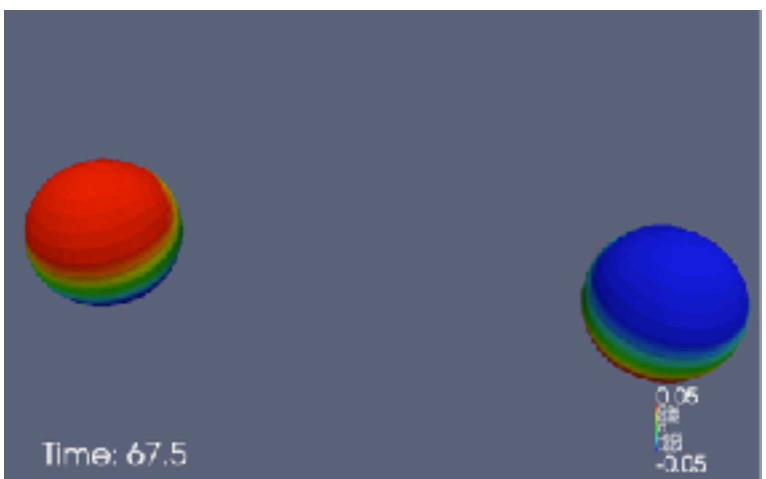
Physically:
A strong "tornado" of twisting space

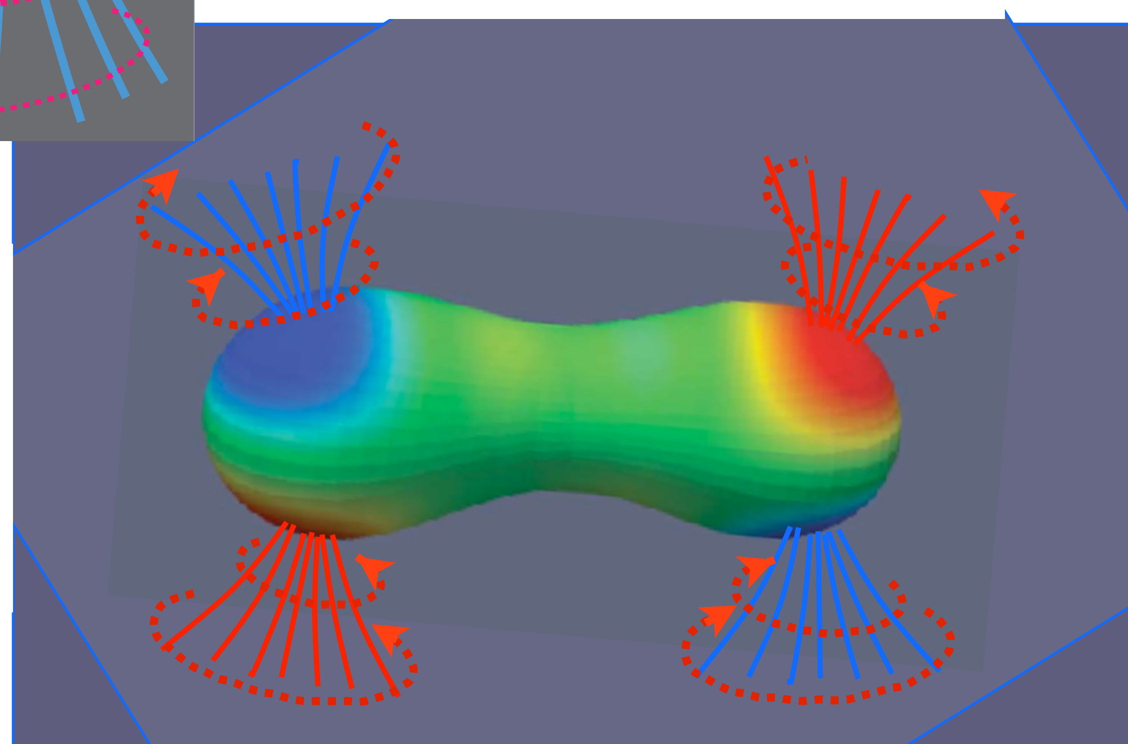
Head-On Collision with Transverse Spin

Keith Matthews, Geoffrey Lovelace, Mark Scheel



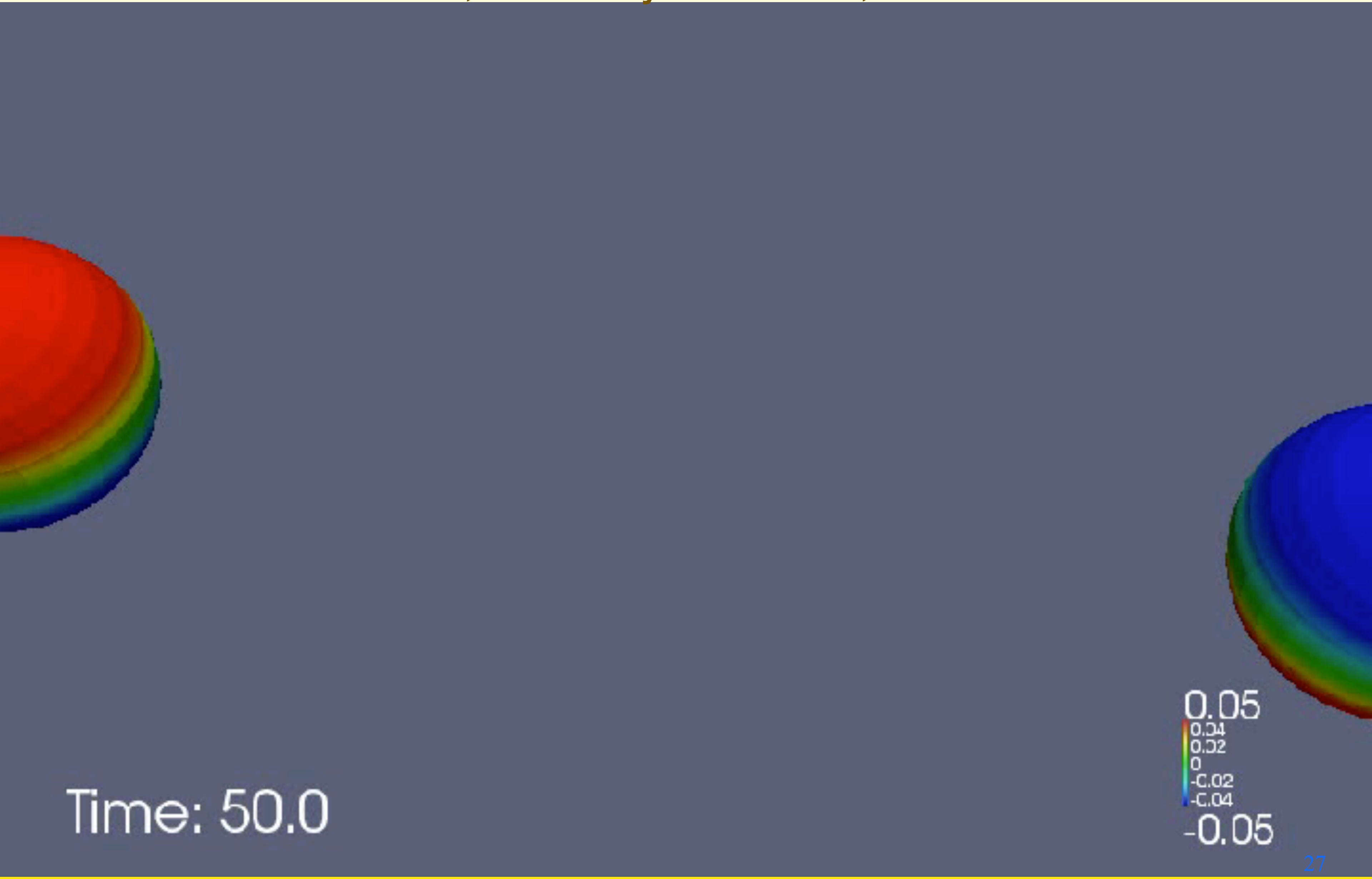
Vortices robustly retain their individuality



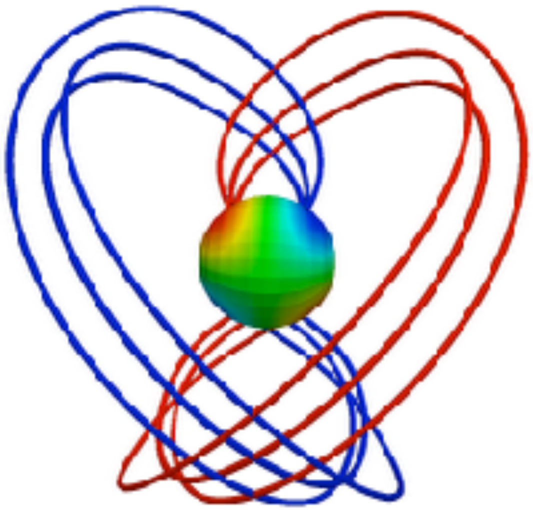


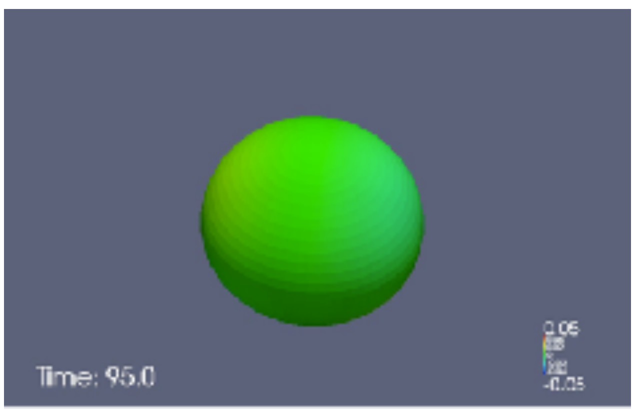
Head-On Collision with Transverse Spin

Keith Matthews, Geoffrey Lovelace, Mark Scheel

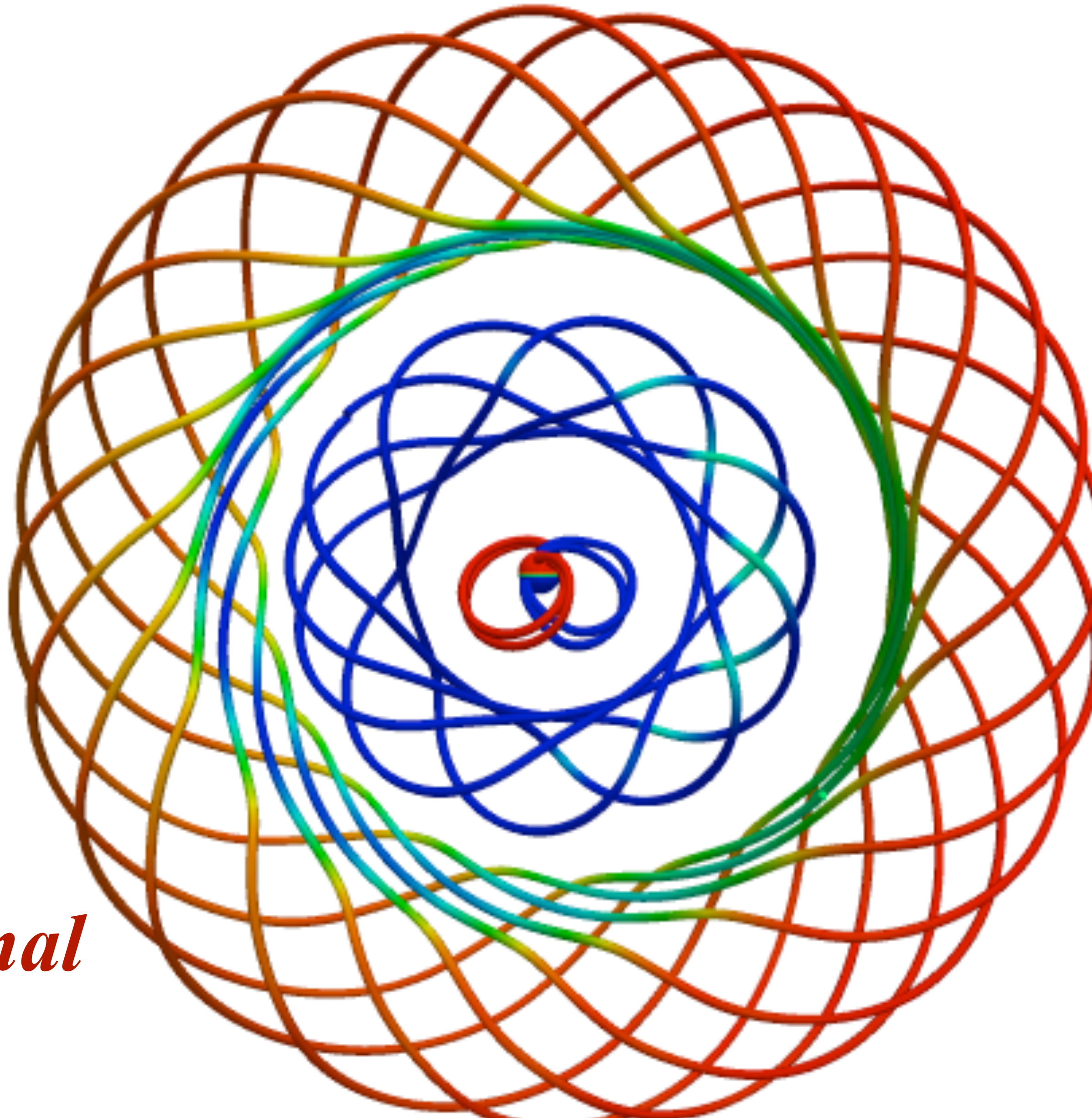


Sloshing Ejects Vortices





gravitational waves

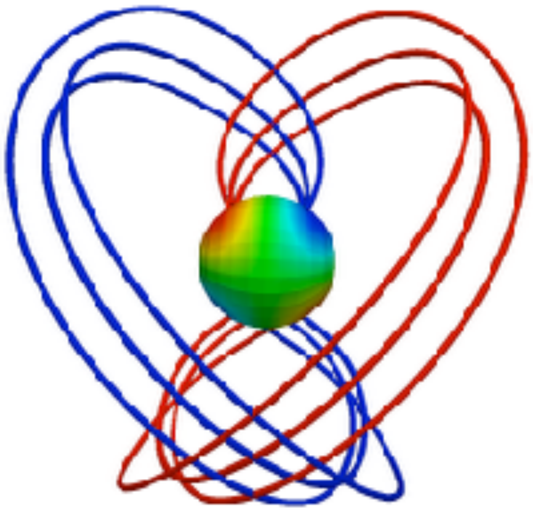


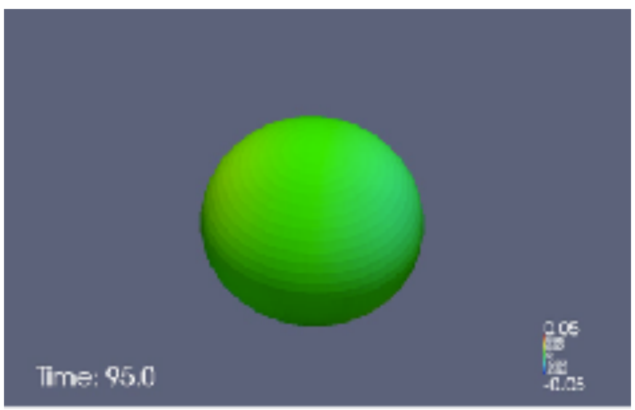
Sloshing Ejects Vortices

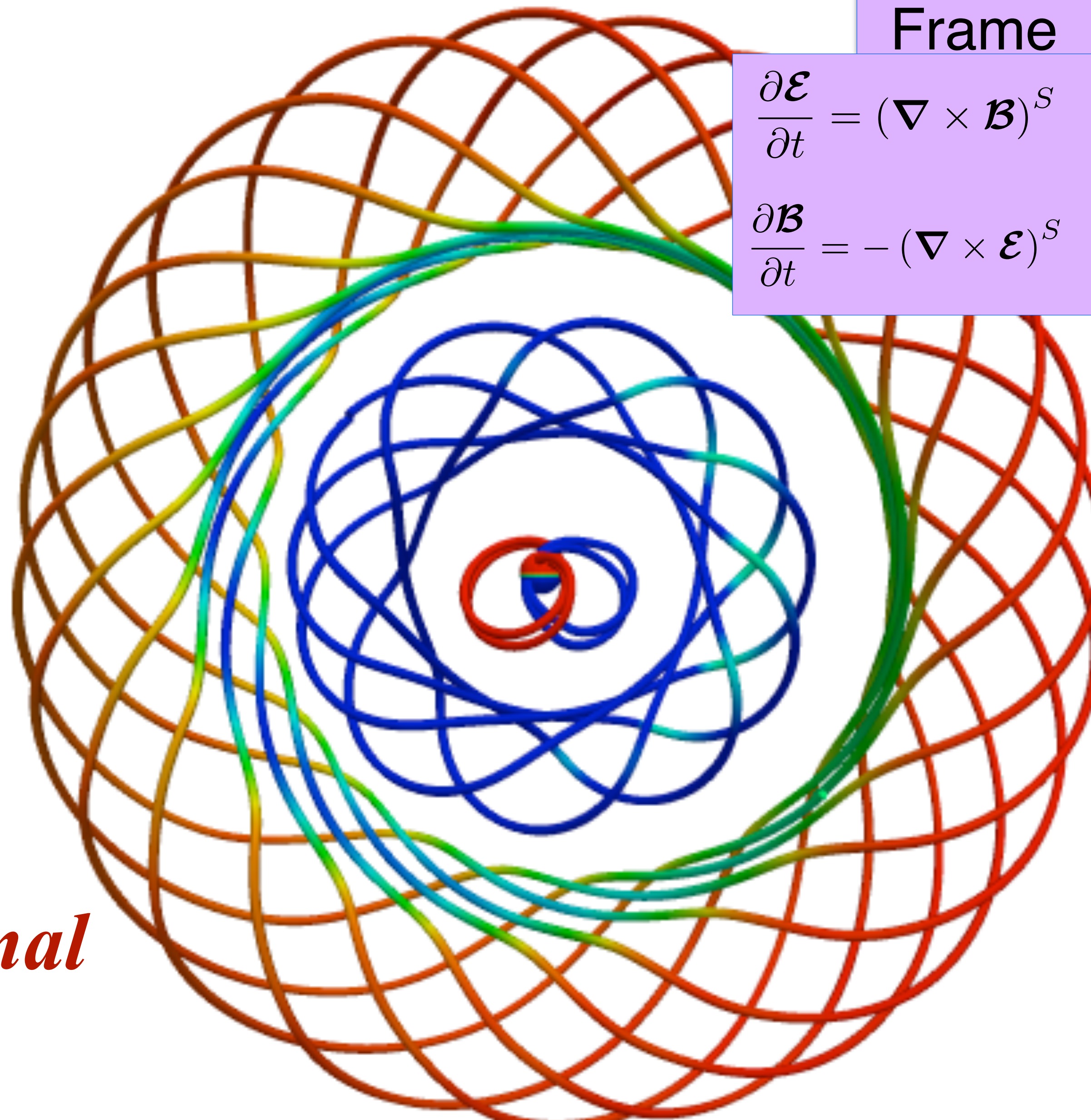
In Local Lorentz Frame

$$\frac{\partial \boldsymbol{\mathcal{E}}}{\partial t} = (\boldsymbol{\nabla} \times \boldsymbol{\mathcal{B}})^S$$

$$\frac{\partial \boldsymbol{\mathcal{B}}}{\partial t} = -(\boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}})^S$$

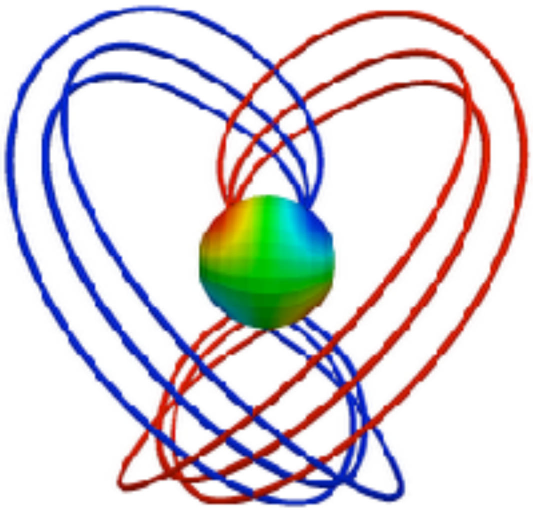


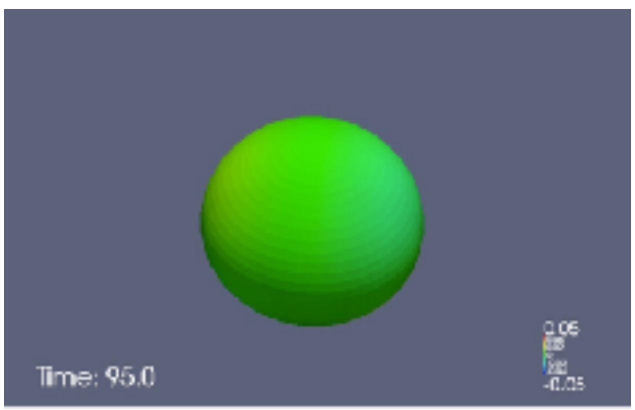




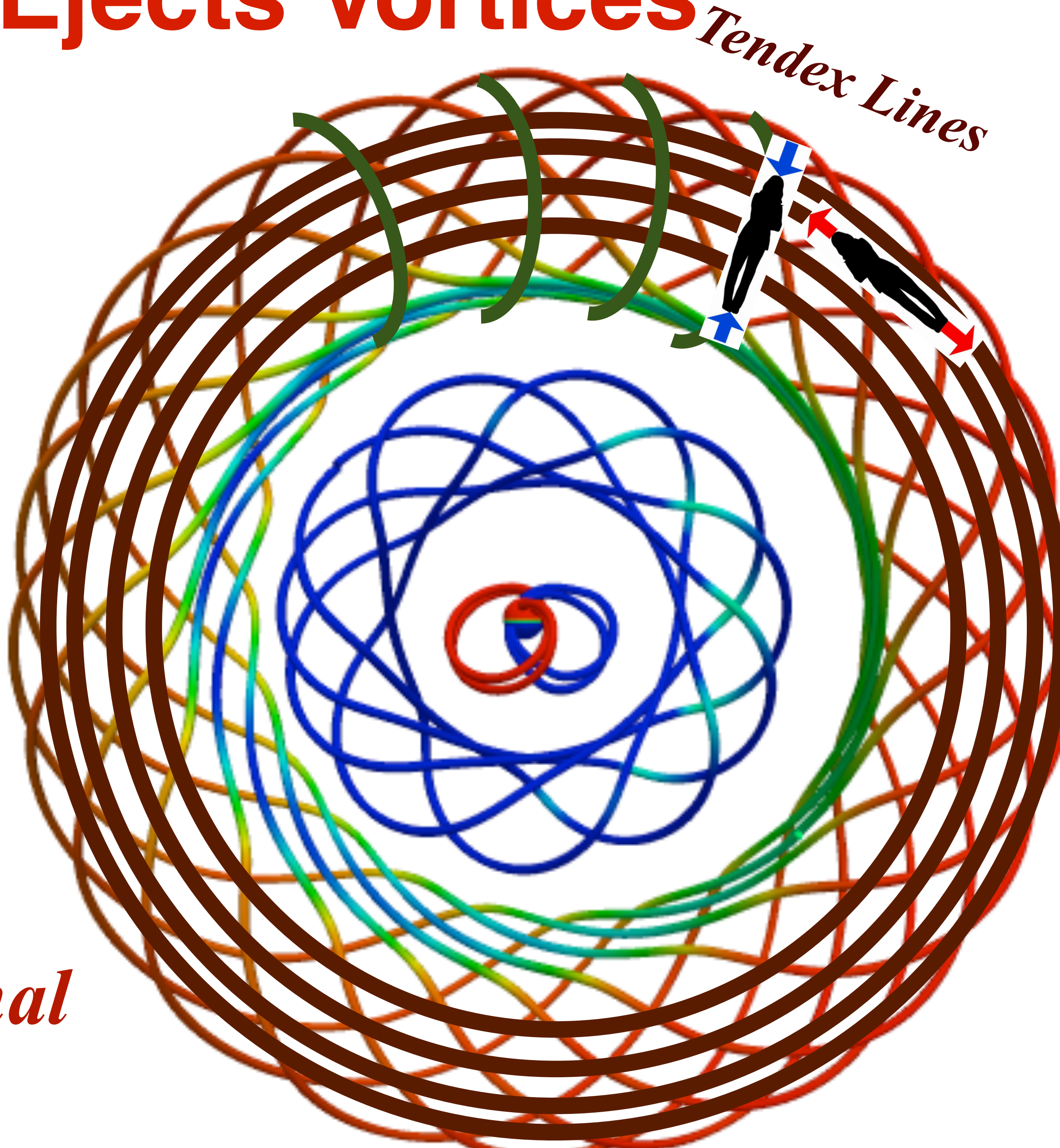
gravitational waves

Sloshing Ejects Vortices

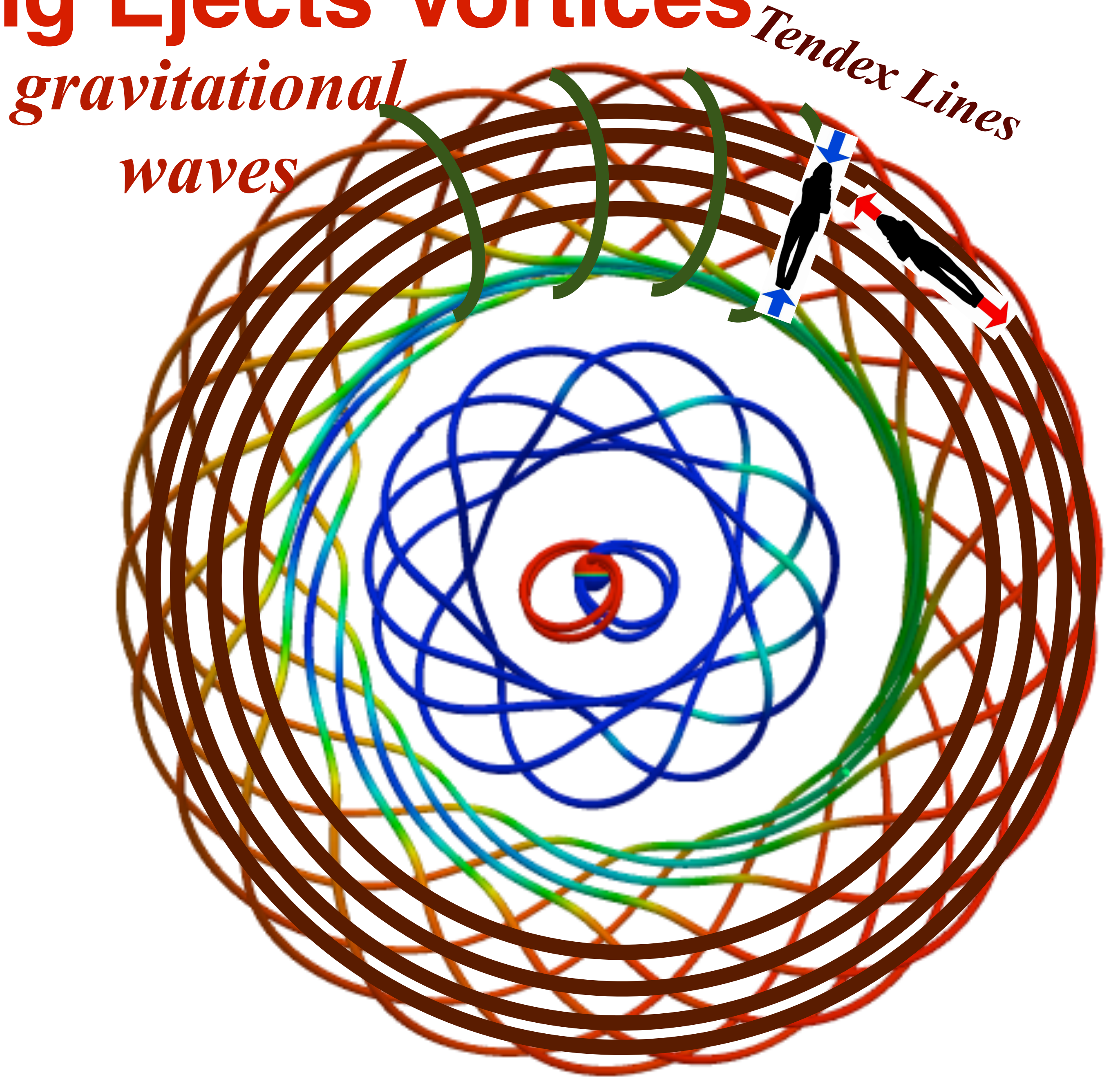




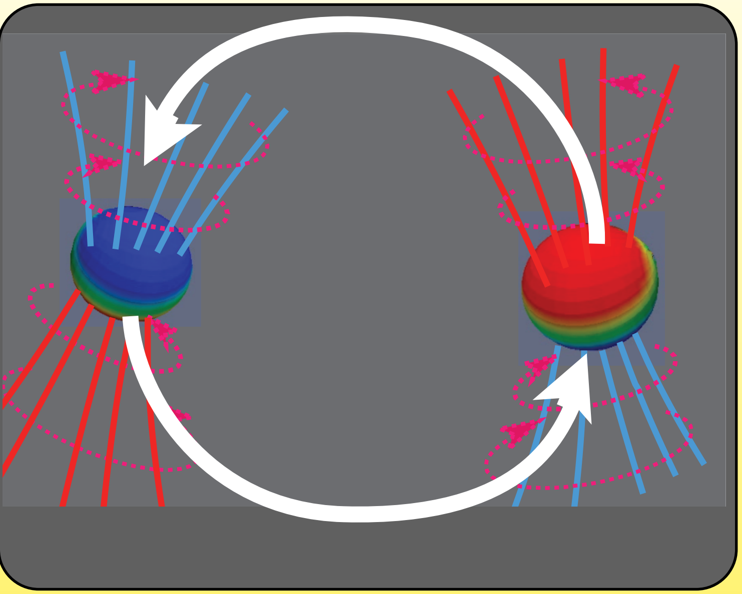
gravitational waves

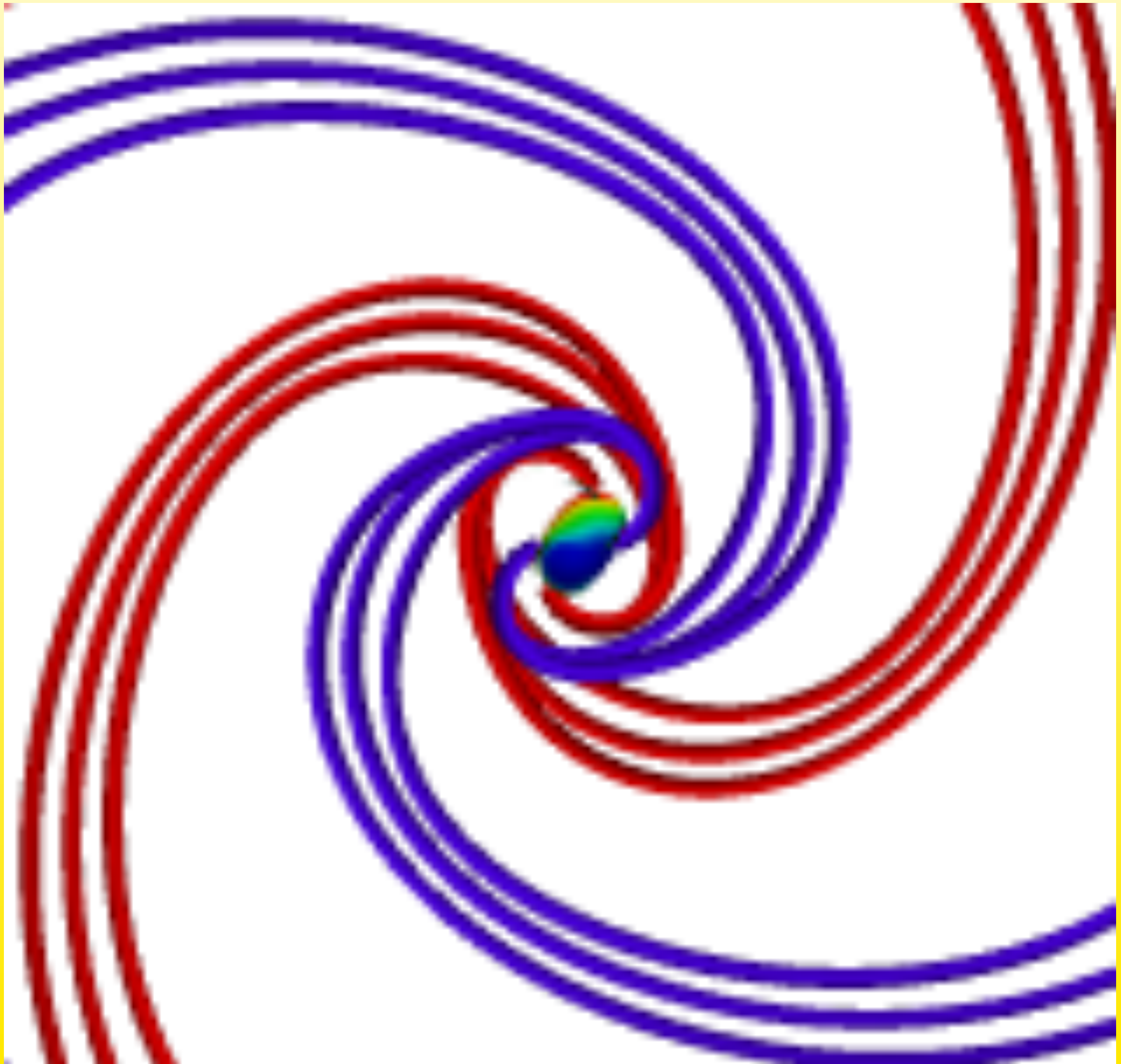


Sloshing Ejects Vortices



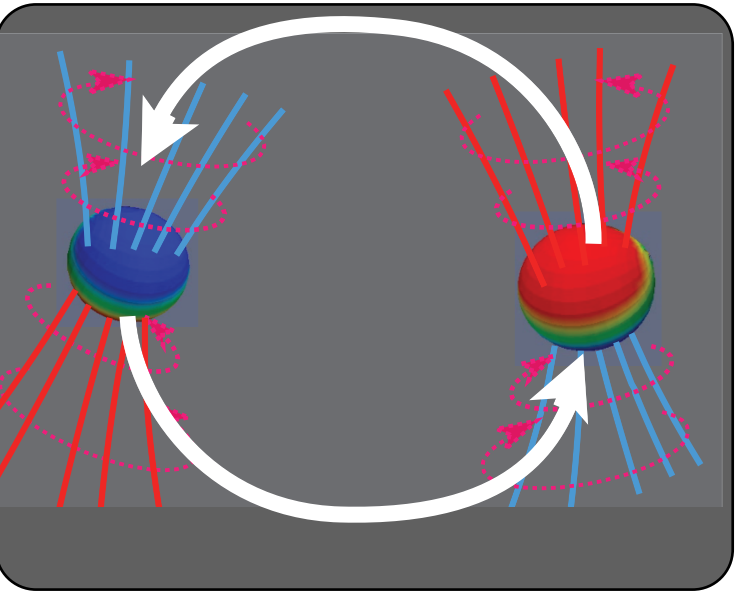
Orbiting Collision

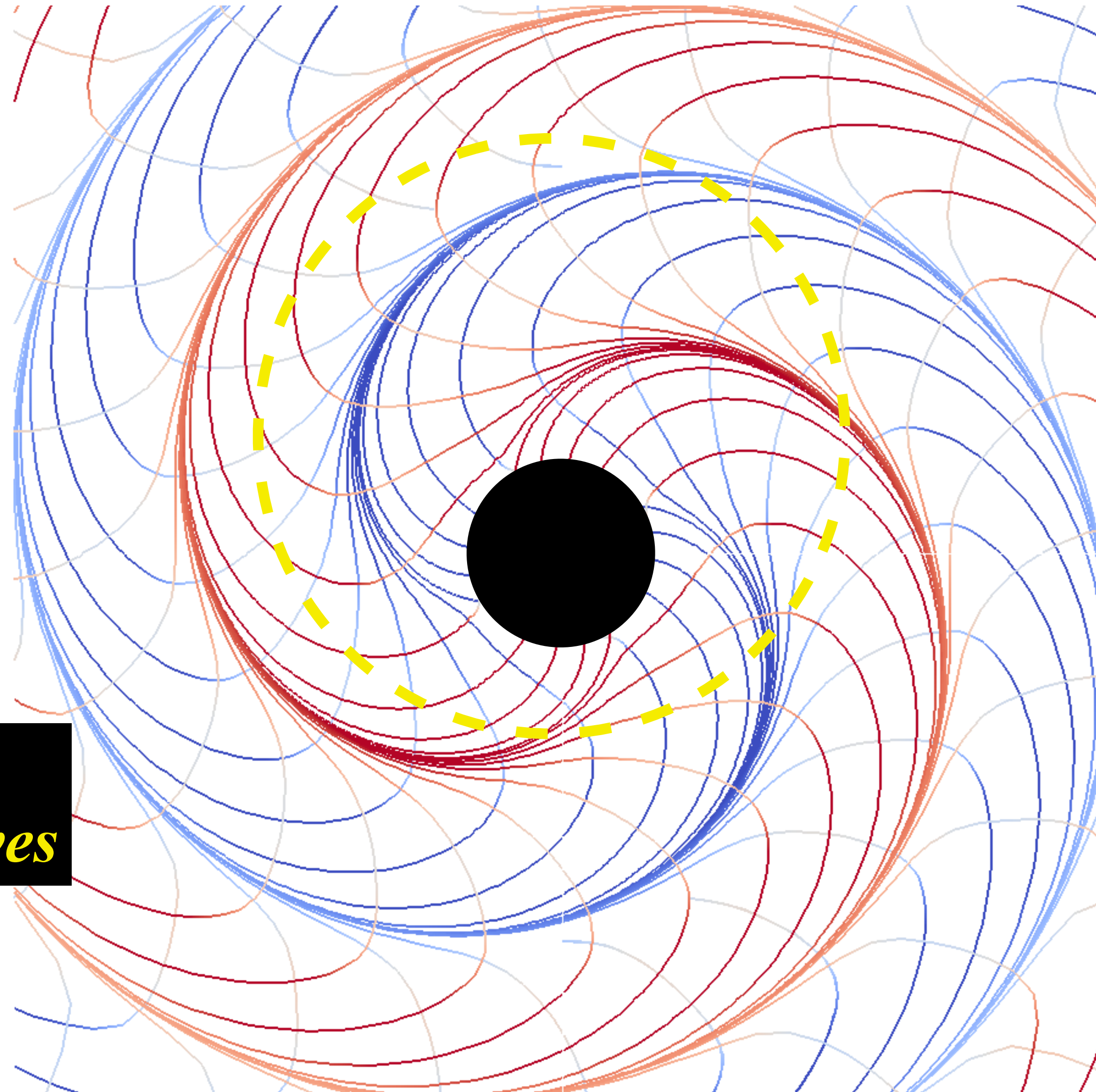




gravitational waves

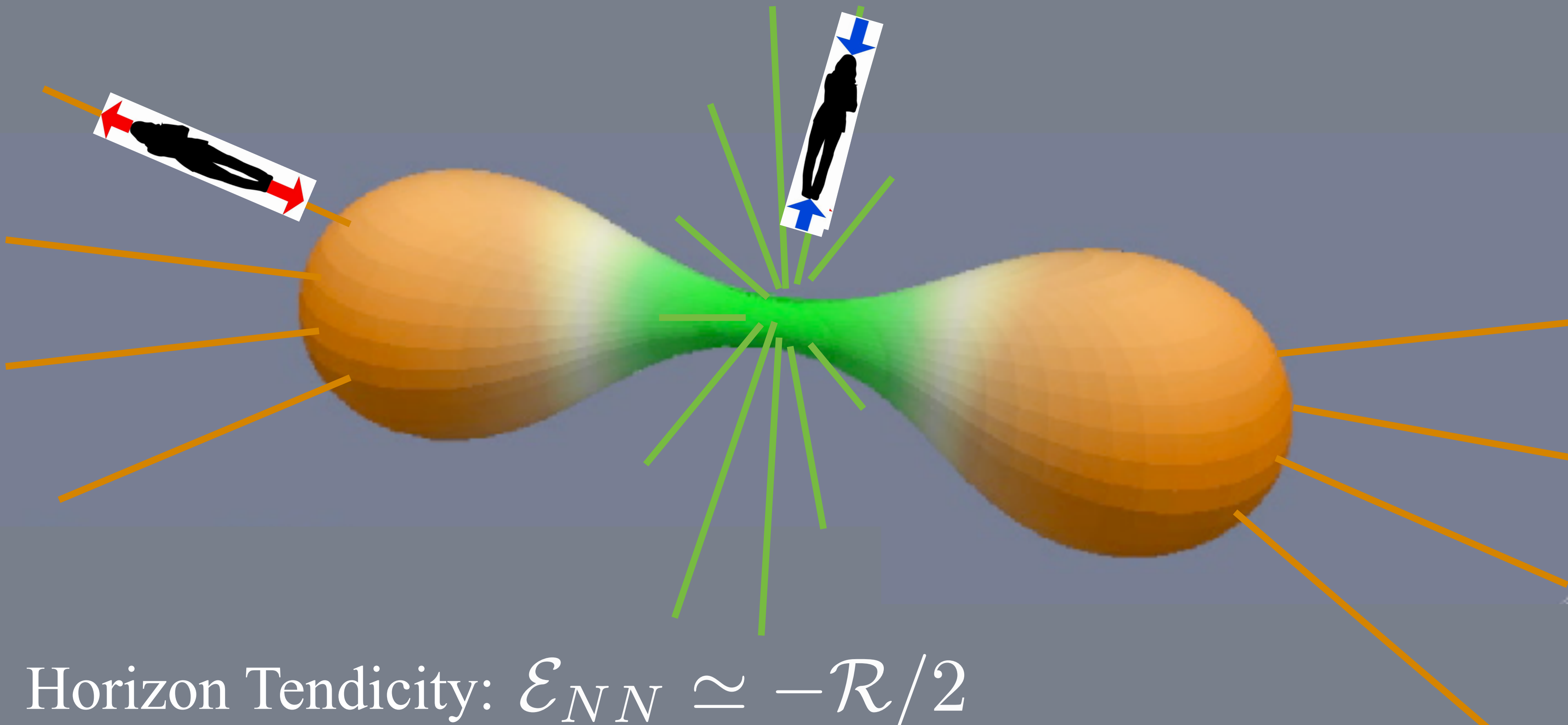
Vortex Lines in Orbital Plane at Late Times





Near-hole vortices generate gravitational waves

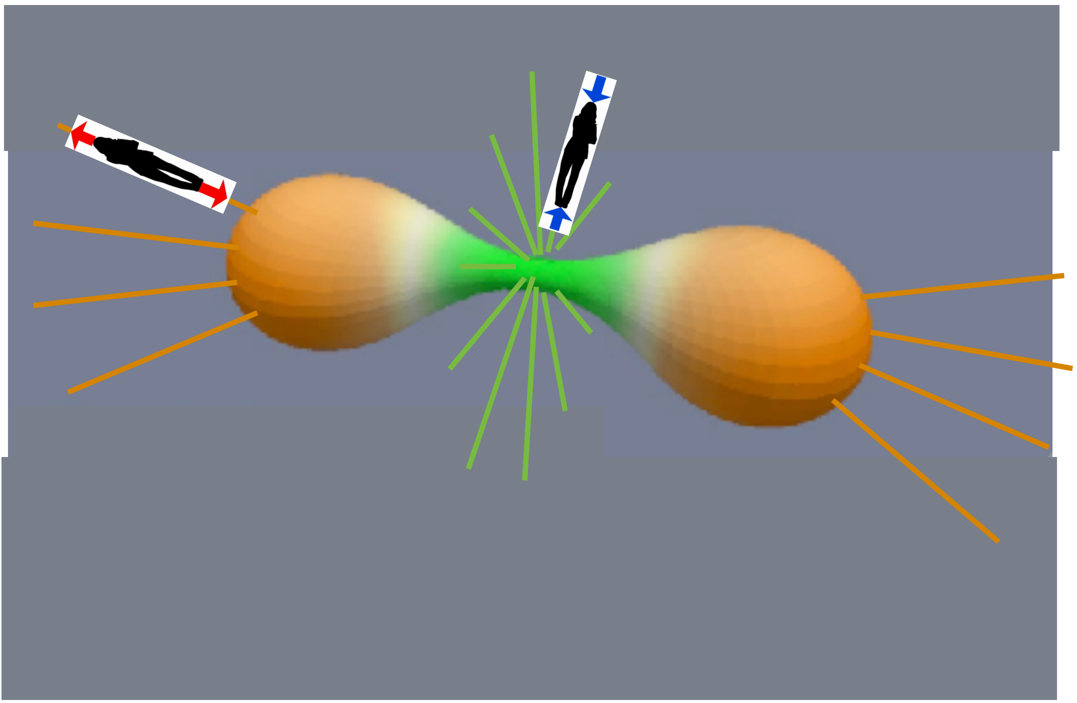
Horizon Tendicity and Tendex Lines



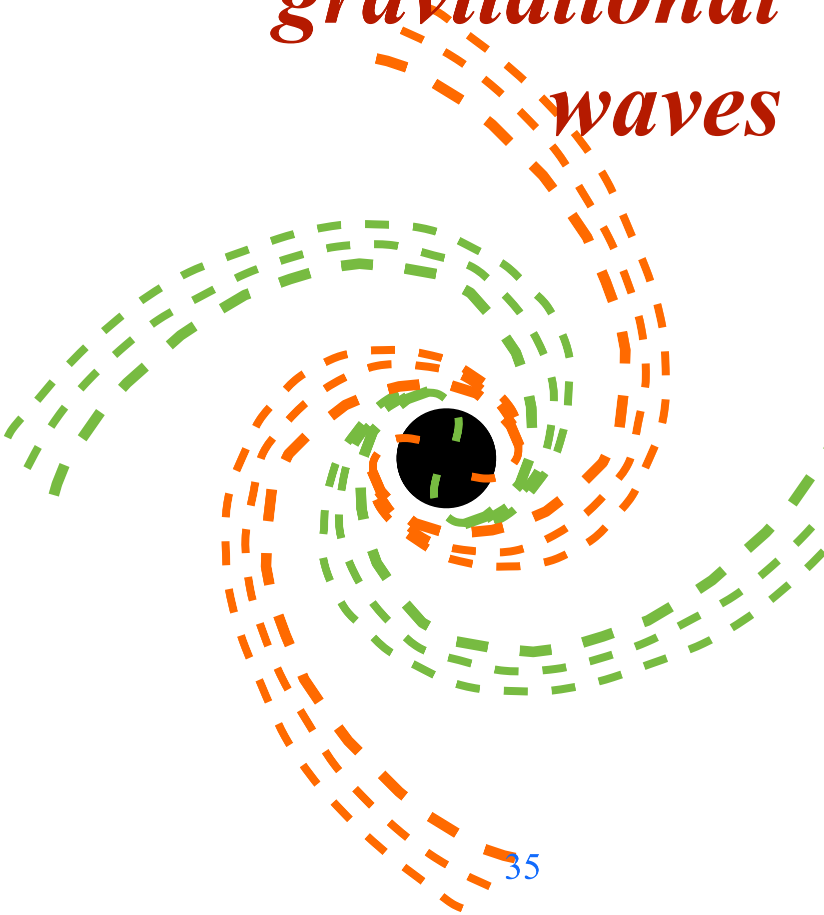
Horizon Tendicity: $\mathcal{E}_{NN} \simeq -\mathcal{R}/2$

Tendex Lines: Integral Curves of Eigenvectors of $\mathcal{E}_{ij}$

Orbiting Collision



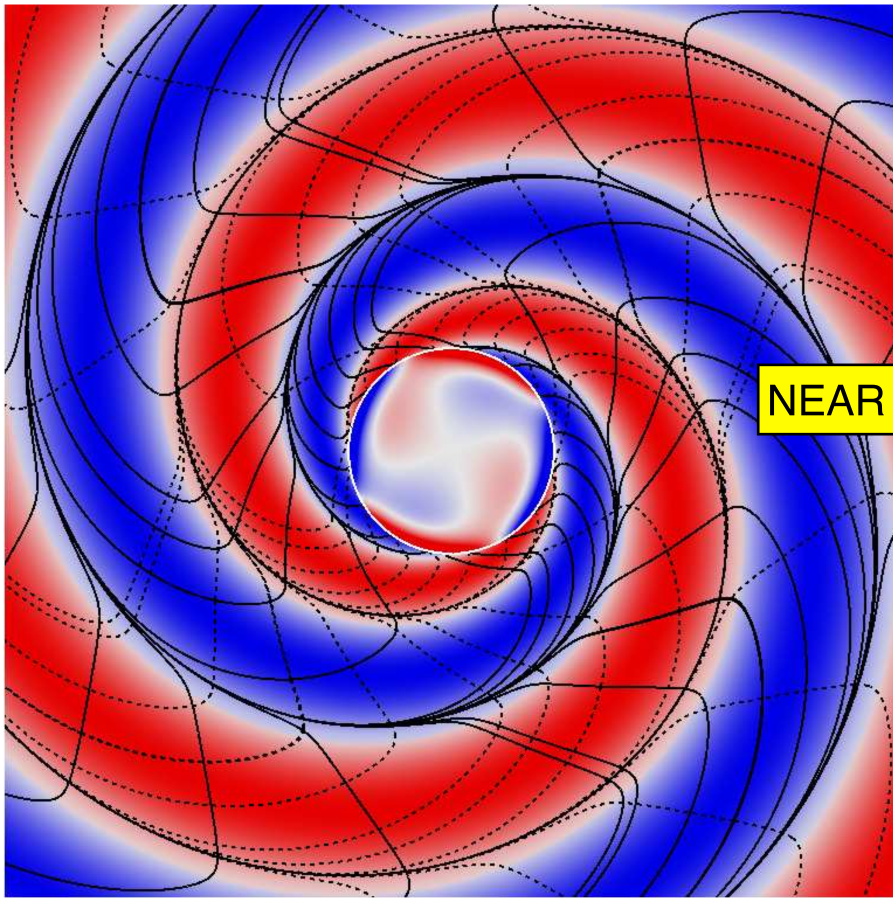
Tendex-generated gravitational waves



At late times, a/M=0.945

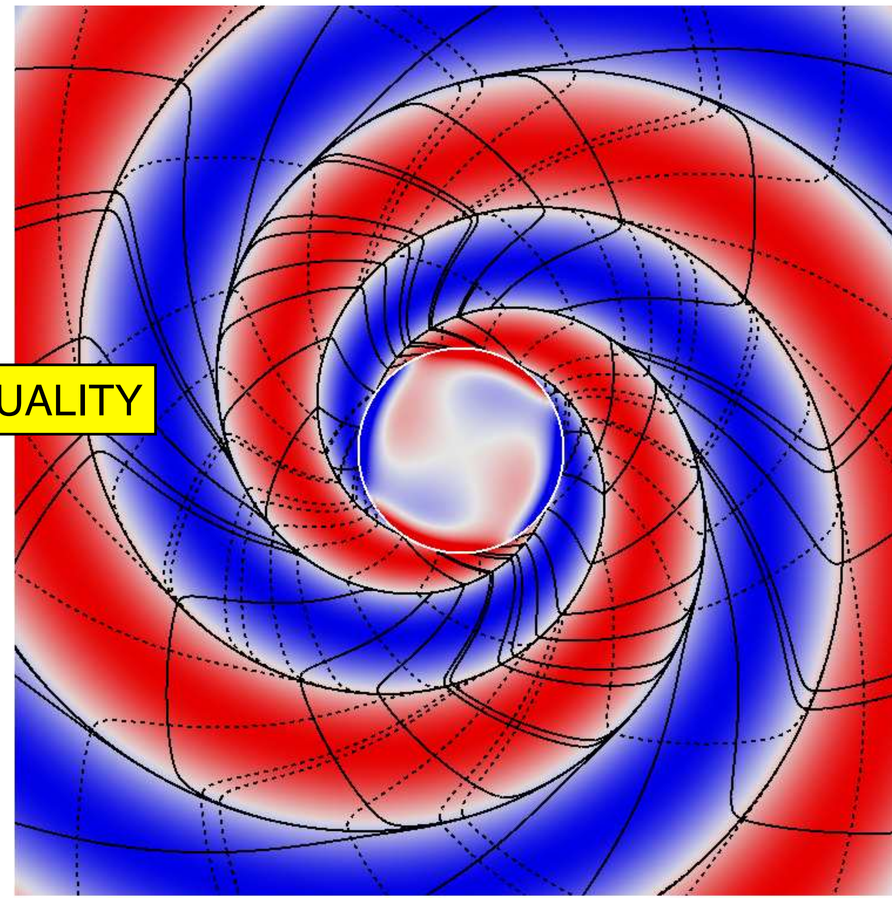
Vortex-generated waves

Equatorial vortex lines and vortices



Tendex-generated waves

Equatorial tendex lines and tendices



NEAR DUALITY

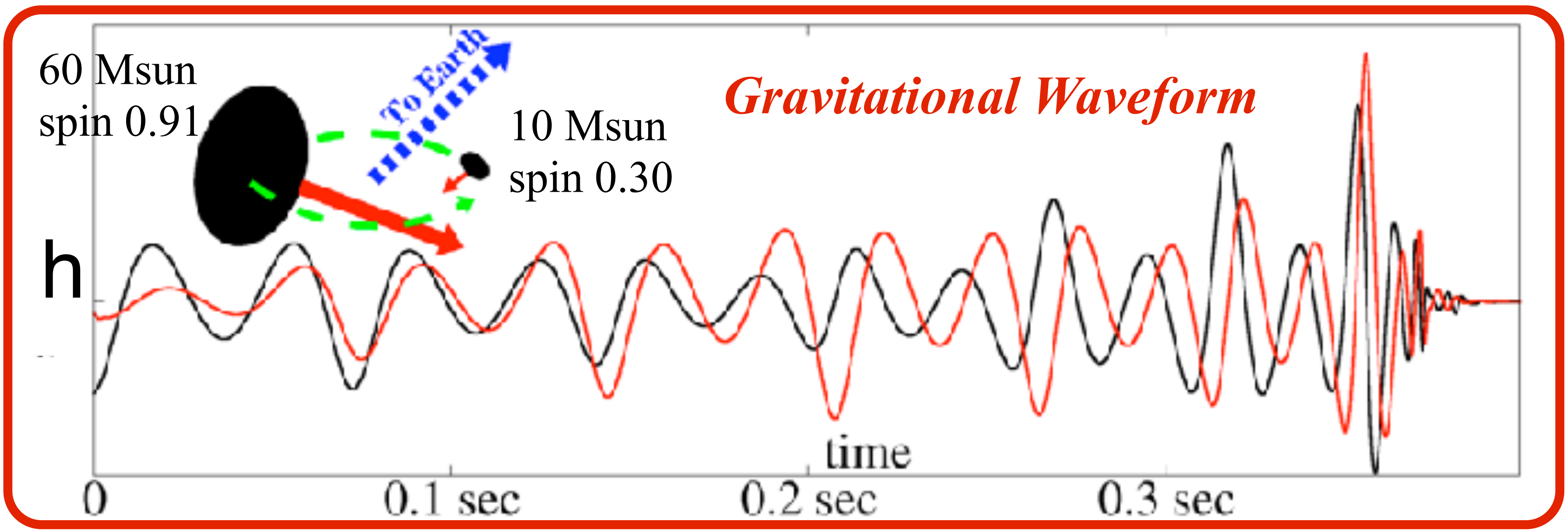
Super Kicks

Gravitational-Wave Observations

- ***Challenge:*** From observed BBH gravitational waveforms, how can we read off the geometrodynamics?
  - » i.e., the dynamics of near-zone vortices and tendices?

Gravitational-Wave Observations

- ***Challenge:*** From observed BBH gravitational waveforms, how can we read off the geometrodynamics?
  - » i.e., the dynamics of near-zone vortices and tendices?
- ***Answer:*** Identify BBH parameters by comparing observed waveforms with waveforms from simulations; then look at the simulations' dynamics



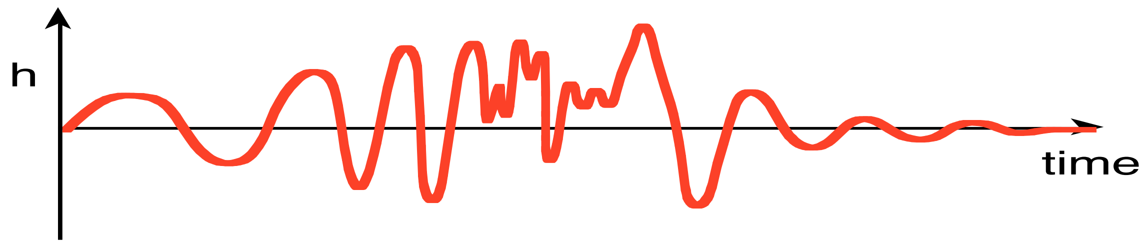
Six BBH's So Far

All observed waveforms agree beautifully with simulations

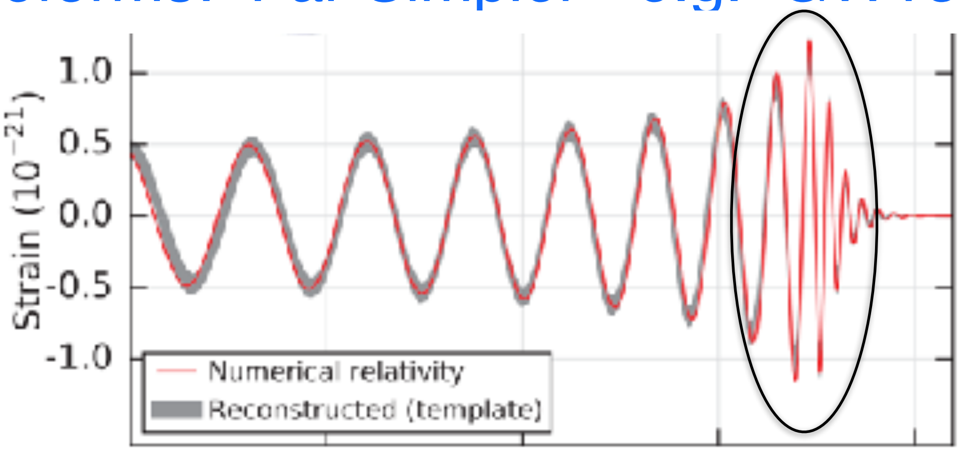
- GW150914: 36 & 29 Msun @ 1.3 billion lt yrs
- LVT151012: 23 & 13 Msun @ 3 billion lt yrs
- GW151226: 14 & 7.5 Msun @ 1.4 billion lt yrs
- GW170104: 31 & 19 Msun @ 2.9 billion lt yrs
- GW170608: 12 & 7 Msun @1.1 billion lt yrs
- GW170814: 31 & 25 Msun @ 1.4 billion lt yrs (LIGO/VIRGO)

Simplicity of BBH Collision Waveforms

- My speculation in 1984



- Actual Waveforms: Far Simpler - e.g. GW150914:

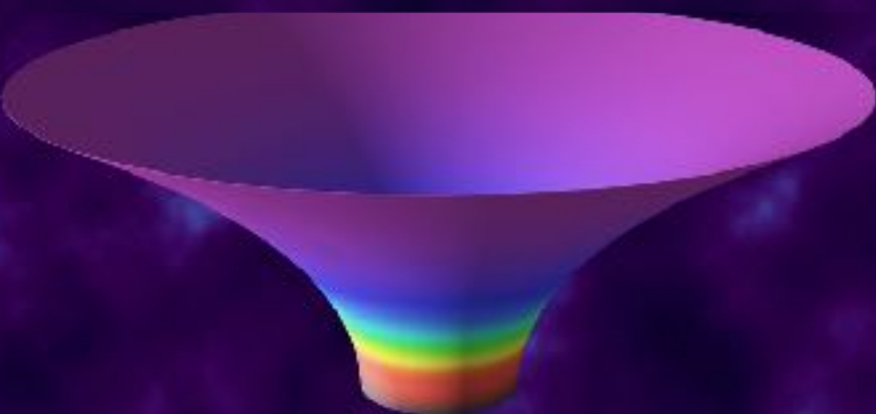


- Disturbances depart very quickly!

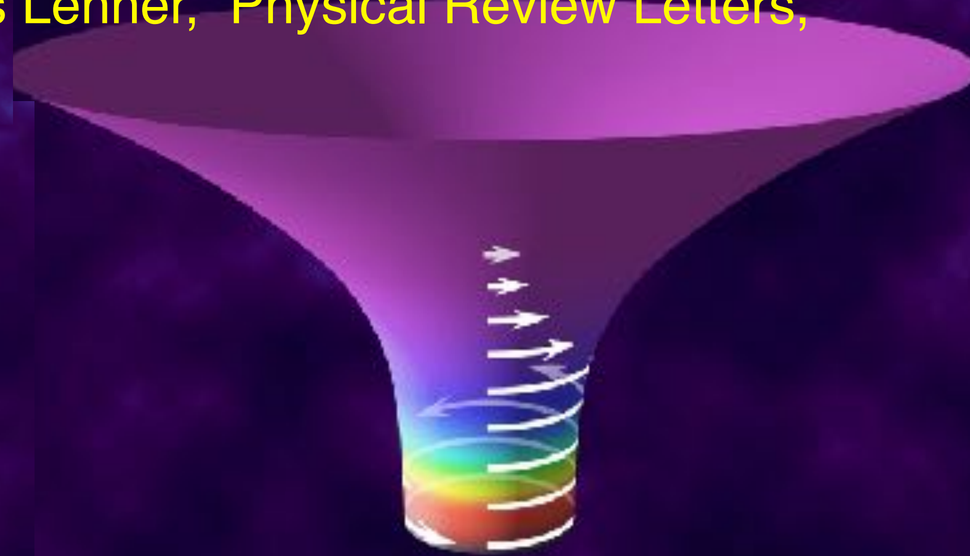
Increase a (black hole spin).

Trap Dynamical disturbances long enough for mode-mode coupling - 2D Turbulence!

Huan Yang, Aaron Zimmerman, Luis Lehner, Physical Review Letters, 114, 081101 (2015)



$a/M = 0$



$a/M = 0.998$

family of modes with zero damping in limit $a \rightarrow M$ (ZDMs)

Summary

- Five current arenas for geometrodynamics
  - » critical collapse
  - » singularities
  - » black-string instability
  - » binary black holes
  - » gravitational-wave observations
- In all, I suspect we have barely scratched the surface.