# Rational Terms at Two Loops

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# Motivation: towards two-loop numerical calculation

- Aim to $\mathcal{O}(1\%)$ precision for LHC processes $\Rightarrow$ **Automation of two-loop calculation**
- Higher-order calculations are usually performed in $D$ dimension to regularise divergences in Feynman integrals, but $D$-dim vector cannot be implemented in a numerical program.
- Automated numerical calculation requires the numerator of loop integrand constructed in $4$-dim, e.g. **OpenLoops 2** [Buccioni et al., 19'] at one-loop level.
- **Rational terms** originates from discrepancy between $4$- and $D$-dim numerator in loop integrands
  $\Rightarrow$ one loop: rational terms of type $R_2$ [Ossola, Papadopoulos, Pittau, Garzelli et al., 08', 09' ]
  $\Rightarrow$ in this talk: **general method for two-loop UV rational terms**

# Outline

I. Introduction to one-loop rational terms and tadpole decomposition

II. One-loop subdiagram with $D$-dim external loop momenta

III. Structure of two-loop rational terms

IV. Proof and recipe to compute two-loop rational terms

V. Two-loop rational terms in QED

VI. A brief introduction to OpenLoops 2 (Backup)

# Introduction to one-loop rational terms

**Amplitude of one-loop diagram** $\gamma$ in $D = 4 - 2\varepsilon$ dimension in HV scheme

$$\bar{\mathcal{A}}_{1,\gamma} = \mu^{2\varepsilon} \int \mathrm{d}\bar{q}_1 \, \frac{\bar{\mathcal{N}}(\bar{q}_1)}{D_0(\bar{q}_1) \cdots D_{N-1}(\bar{q}_1)}, \quad \text{with} \quad D_k(\bar{q}_1) = (\bar{q}_1 + p_k)^2 - m_k^2$$

**Rational term** emerges by splitting numerator into $4$-dim and $\varepsilon$-dim parts

$$\bar{\mathcal{N}}(\bar{q}_1) = \mathcal{N}(q_1) + \tilde{\mathcal{N}}(\bar{q}_1), \quad \text{with} \quad \begin{cases} \bar{q} & = q + \tilde{q} \\ \bar{\gamma}^{\bar{\mu}} & = \gamma^{\mu} + \tilde{\gamma}^{\tilde{\mu}} \\ \bar{g}^{\bar{\mu}\bar{\nu}} & = g^{\mu\nu} + \tilde{g}^{\tilde{\mu}\tilde{\nu}} \end{cases}$$

leads to

$$\bar{\mathcal{A}}_{1,\gamma} = \underbrace{\mathcal{A}_{1,\gamma}}_{\substack{\text{compute} \\ \text{numerically}}} + \underbrace{\delta\mathcal{R}_{1,\gamma}}_{\substack{\text{compute} \\ \text{analytically}}}$$

- $\delta\mathcal{R}_{1,\gamma}$ from interplay between $\varepsilon$-dim $\tilde{\mathcal{N}}$ and $\frac{1}{\varepsilon}$ UV pole. $\Rightarrow$ requires technique to extract UV pole

## Tadpole decomposition [Chetyrkin, Misiak, Münz, 98', Zoller, 14']

The UV divergence can be captured by **massive tadpole decomposition** of denominators

$$\frac{1}{D_k(\bar{q}_1)} = \underbrace{\frac{1}{\bar{q}_1^2 - M^2}}_{\substack{\text{leading UV term} \\ \mathcal{O}(1/\bar{q}_1^2)}} + \underbrace{\frac{\Delta_k(\bar{q}_1, p_k)}{\bar{q}_1^2 - M^2}\frac{1}{D_k(\bar{q}_1)}}_{\substack{\text{subleading UV term} \\ \mathcal{O}(1/\bar{q}_1^3)}}$$

with

$$\Delta_k(\bar{q}_1, p_k) = -p_k^2 - 2\,\bar{q}_1 \cdot p_k + m_k^2 - M^2$$

**Apply recursively** to obtain **tadpole expansion** up to order $(1/\bar{q}_1)^{X+2}$

$$\frac{1}{D_k(\bar{q}_1)} = \underbrace{\sum_{\sigma=0}^{X} \text{tadpoles}}_{\text{expansion}} + \text{UV-finite remainder}$$

# Tadpole decomposition

**Decomposition of tensor integral** with degree of divergence $X$

$$T_N^{\bar{\mu}_1\cdots\bar{\mu}_r} = \int \mathrm{d}\bar{q}_1 \, \frac{\bar{q}_1^{\bar{\mu}_1}\cdots\bar{q}_1^{\bar{\mu}_r}}{D^0(\bar{q}_1)\cdots D^{N-1}(\bar{q}_1)} = \underbrace{\mathbf{S}_X \, T_N^{\bar{\mu}_1\cdots\bar{\mu}_r}}_{\text{expansion}} + \underbrace{\mathbf{F}_X \, T_N^{\bar{\mu}_1\cdots\bar{\mu}_r}}_{\text{remainder}}$$

**UV divergent part** is fully isolated in tadpoles

$$\mathbf{S}_X \, T_N^{\bar{\mu}_1\cdots\bar{\mu}_r} = \underbrace{\sum_{\sigma=0}^{X} \int \mathrm{d}\bar{q}_1 \, \frac{\bar{q}_1^{\bar{\mu}_1}\cdots\bar{q}_1^{\bar{\mu}_r}\,\Delta^{(\sigma)}(\bar{q}_1, p_k)}{\left(\bar{q}_1^2 - M^2\right)^{N+\sigma}}}_{\text{simple IR finite tadpole integrals}}$$

Define $\mathbf{K}$ operator that **extracts the pole part**

$$\underbrace{\mathbf{K} \, T_N^{\bar{\mu}_1\cdots\bar{\mu}_r}}_{\text{UV pole of } T_N} = \mathbf{K}\,\mathbf{S}_X \, T_N^{\bar{\mu}_1\cdots\bar{\mu}_r} \propto \frac{1}{\varepsilon}$$

## Rational terms from UV singularities

Define the $\bar{\mathbf{K}}$ operator that extracts **full pole contribution**

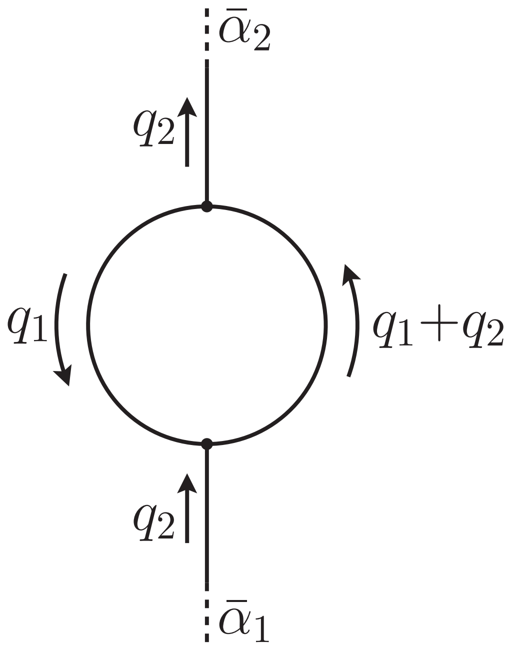
$$\bar{\mathbf{K}}\,\bar{\mathcal{A}}_{1,\gamma} \;:=\; \underbrace{\sum_r \bar{\mathcal{N}}_{\bar{\mu}_1\cdots\bar{\mu}_r}\,\mathbf{K}\,T_N^{\bar{\mu}_1\cdots\bar{\mu}_r}}_{\text{tensorial decomposition}} \;=\; \sum_r (\mathcal{N}_{\mu_1\cdots\mu_r} + \tilde{\mathcal{N}}_{\bar{\mu}_1\cdots\bar{\mu}_r})\;\underbrace{\mathbf{K}\,\mathbf{S}_X\,T_N^{\bar{\mu}_1\cdots\bar{\mu}_r}}_{\text{UV pole}}$$

splitting into

$$\bar{\mathbf{K}}\,\bar{\mathcal{A}}_{1,\gamma} \;=\; \underbrace{-\delta Z_{1,\gamma}}_{\overline{\mathrm{MS}}\text{ pole}} + \underbrace{\delta\mathcal{R}_{1,\gamma}}_{\text{rational}}$$

- $\delta\mathcal{R}_{1,\gamma}$ and $\delta Z_{1,\gamma}$ from same UV singularity of $T_N \Rightarrow \delta\mathcal{R}_{1,\gamma}$ local counterterm like $\delta Z_{1,\gamma}$
- $\delta\mathcal{R}_{1,\gamma}$ is NOT a finite renormalisation of fields and couplings in bare Lagrangian, e.g. there is a rational term of 4-photon vertice.

# One-loop subdiagram with $D$-dim external loop momenta




One-loop diagram with $4$-dim $q_2$:

$$D_k(\bar{q}_1, q_2) = (\bar{q}_1 + q_2)^2 = \bar{q}_1^2 + \underbrace{2\,\bar{q}_1 \cdot q_2 + q_2^2}_{4\text{-dim}}$$

One-loop subdiagram with $D$-dim $\bar{q}_2 = q_2 + \tilde{q}_2$:

$$D_k(\bar{q}_1, \bar{q}_2) = D_k(\bar{q}_1, q_2) + \underbrace{(2\,\bar{q}_1 \cdot \tilde{q}_2 + \tilde{q}_2^2)}_{\varepsilon\text{-dim}}$$

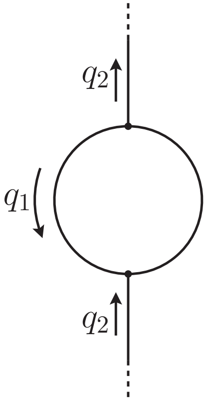
$\Rightarrow$ **extra term** of order $\tilde{q}_2^2/\varepsilon$ contributes to **two loops** (see later) $\Rightarrow$ **rational structure changes**

## Subdiagram with $D$-dim external momentum $\tilde{q}_2$ and $4$-dim numerator

Tadpole expansion

$$\mathbf{S}_X \frac{1}{(\bar{q}_1 + q_2 + \tilde{q}_2)^2} = \frac{1}{\bar{q}_1^2 - M^2} + \frac{-(q_2 + \tilde{q}_2)^2 - 2\,\bar{q}_1 \cdot (q_2 + \tilde{q}_2) - M^2}{(\bar{q}_1^2 - M^2)^2} + \ldots$$

Contribution to UV pole




$$\begin{aligned}\mathbf{K}\,\mathcal{A}^{\alpha}_{1,\gamma}(q_2) &= \mathbf{K} \sum_r \mathcal{N}^{\alpha}_{\mu_1 \cdots \mu_r}(q_2) \int \mathrm{d}\bar{q}_1 \frac{\bar{q}_1^{\bar{\mu}_1} \cdots \bar{q}_1^{\bar{\mu}_r}\, \Delta^{(\sigma)}(\bar{q}_1,\, q_2 + \tilde{q}_2)}{\left(\bar{q}_1^2 - M^2\right)^{N+\sigma}} \\ &= \underbrace{-\,\delta Z^{\alpha}_{1,\gamma}(q_2)}_{\overline{\mathrm{MS}}\text{ pole}} \quad \underbrace{-\,\delta \tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2)}_{\substack{\textbf{extra pole} \\ \textbf{new rational part}}}\end{aligned}$$

- $\delta \tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2)$ is **non-vanishing only in quadratic divergent subdiagrams**, and has the form

$$\delta \tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2) \;\propto\; \frac{\tilde{q}_2^2}{\varepsilon} = \mathcal{O}(1)$$

# Renormalised one-loop subdiagrams

**Subtract poles and rational terms**, we can **identify amplitudes** with $D$-dim and $4$-dim numerator

$$\underbrace{\bar{\mathcal{A}}^{\bar{\alpha}}_{1,\gamma}(\bar{q}_2) - \bar{\mathbf{K}}\,\bar{\mathcal{A}}^{\bar{\alpha}}_{1,\gamma}(\bar{q}_2)}_{D\text{-dim full subtraction}} \;=\; \underbrace{\mathcal{A}^{\alpha}_{1,\gamma}(q_2) - \mathbf{K}\,\mathcal{A}^{\alpha}_{1,\gamma}(q_2)}_{4\text{-dim full subtraction}} + \mathcal{O}(\varepsilon, \tilde{q})$$

Recall

$$\begin{aligned}
\bar{\mathbf{K}}\,\bar{\mathcal{A}}^{\bar{\alpha}}_{1,\gamma}(\bar{q}_2) &= -\delta Z^{\bar{\alpha}}_{1,\gamma}(\bar{q}_2) + \delta\mathcal{R}^{\alpha}_{1,\gamma}(q_2) + \mathcal{O}(\varepsilon) \\
\mathbf{K}\,\mathcal{A}^{\alpha}_{1,\gamma}(q_2) &= -\delta Z^{\alpha}_{1,\gamma}(q_2) - \delta\tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2)
\end{aligned}$$

⇒ **Renormalised one-loop amplitude**

$$\underbrace{\bar{\mathcal{A}}^{\bar{\alpha}}_{1,\gamma}(\bar{q}_2) + \delta Z^{\bar{\alpha}}_{1,\gamma}(\bar{q}_2)}_{D\text{-dim renormalisation}} \;=\; \underbrace{\mathcal{A}^{\alpha}_{1,\gamma}(q_2) + \delta Z^{\alpha}_{1,\gamma}(q_2)}_{\substack{4\text{-dim renormalisation} \\ \text{compute numerically}}} + \underbrace{\delta\tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2) + \delta\mathcal{R}^{\alpha}_{1,\gamma}(q_2)}_{\text{rational parts}} + \mathcal{O}(\varepsilon, \tilde{q})$$

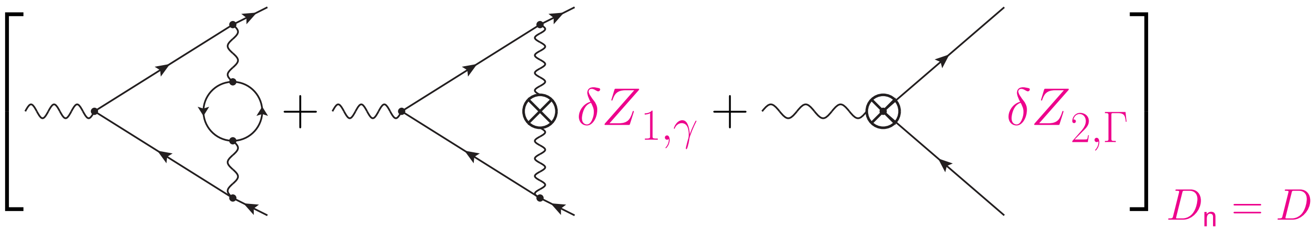
## Renormalisation of irreducible two-loop diagrams

**Renormalisation** of $D$-dim amplitude of diagram $\Gamma$ with $\mathbf{R}$ operation [Caswell and Kennedy, 82']

$$\mathbf{R}\,\bar{\mathcal{A}}_{2,\Gamma} \;=\; \bar{\mathcal{A}}_{2,\Gamma} + \sum_{\gamma_i} \underbrace{\delta Z_{1,\gamma_i} \cdot \bar{\mathcal{A}}_{1,\Gamma/\gamma_i}}_{\text{subdivergences}} + \underbrace{\delta Z_{2,\Gamma}}_{\text{local two-loop divergence}}$$

Example: QED vertex ($D_{\mathsf{n}} \in \{D, 4\}$ be the numerator dimension)

$$\mathbf{R}\,\bar{\mathcal{A}}_{2,\Gamma} =$$

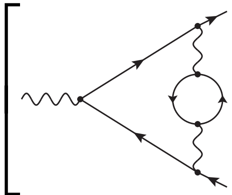
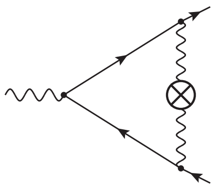
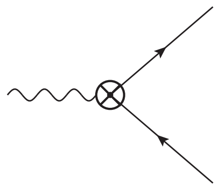


# Structure of two-loop UV rational terms (Ansatz)

**Relation between renormalised amplitude** in $D_\mathrm{n} = D$ and $D_\mathrm{n} = 4$:

$$\mathbf{R}\,\bar{\mathcal{A}}_{2,\Gamma} \;=\; \mathcal{A}_{2,\Gamma} + \sum_{\gamma_i} \underbrace{\left(\delta Z_{1,\gamma_i} + \delta\tilde{Z}_{1,\gamma_i} + \delta\mathcal{R}_{1,\gamma}\right)\cdot\mathcal{A}_{1,\Gamma/\gamma_i}}_{\text{subdivergences}} + \underbrace{\left(\delta Z_{2,\Gamma} + \delta\mathcal{R}_{2,\Gamma}\right)}_{\text{local two-loop divergence}} + \mathcal{O}(\varepsilon)$$

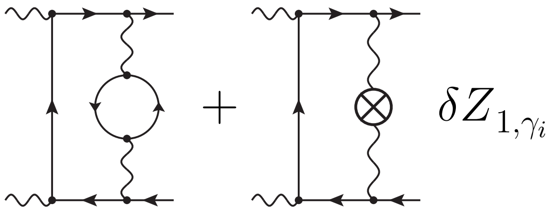
Example: QED vertex

$\mathbf{R}\,\bar{\mathcal{A}}_{2,\Gamma} = \Big[$  $+$  $\Big(\delta Z_{1,\gamma_i} + \underbrace{\delta\tilde{Z}_{1,\gamma_i} + \delta\mathcal{R}_{1,\gamma}}_{\text{extended rational insertion}}\Big) +$  $\left(\delta Z_{2,\Gamma} + \delta\mathcal{R}_{2,\Gamma}\right)\Big]_{D_\mathrm{n}=4} + \mathcal{O}(\varepsilon)$

# Two-loop diagrams without global divergence (Proof)

**No global divergence $\Rightarrow$ at most one subdivergence to be subtracted**

$$\mathbf{R}\bar{\mathcal{A}}_{2,\Gamma} = \underbrace{\left(\bar{\mathcal{A}}_{1,\gamma_i} + \delta Z_{1,\gamma_i}\right)}_{\text{(a) UV pole subtracted}} \cdot \underbrace{\bar{\mathcal{A}}_{1,\Gamma/\gamma_i}}_{\text{(b) no divergence}} \quad \Leftarrow \quad \text{e.g.}$$



$$= \underbrace{\left(\mathcal{A}_{1,\gamma_i} + \delta Z_{1,\gamma_i} + \delta\tilde{Z}_{1,\gamma_i} + \delta\mathcal{R}_{1,\gamma}\right) \cdot \mathcal{A}_{1,\Gamma/\gamma_i}}_{\text{with 4-dim numerator}} + \mathcal{O}(\varepsilon)$$

$$= \mathcal{A}_{2,\Gamma} + \left(\delta Z_{1,\gamma_i} + \delta\tilde{Z}_{1,\gamma_i} + \delta\mathcal{R}_{1,\gamma}\right) \cdot \mathcal{A}_{1,\Gamma/\gamma_i} + \mathcal{O}(\varepsilon)$$

Hence we prove

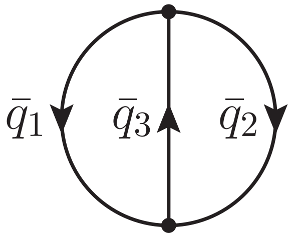
two-loop $\delta\mathcal{R}_{2,\Gamma} = 0$ and $\delta Z_{2,\Gamma} = 0$

$\Rightarrow$ only globally divergent two-loop diagrams contribute to $\delta\mathcal{R}_{2,\Gamma}$ and $\delta Z_{2,\Gamma}$

$\Rightarrow$ **finite set of $\delta\mathcal{R}_{2,\Gamma}$ and $\delta Z_{2,\Gamma}$ counterterms** in renormalisable theories

# Two-loop diagrams with global divergence (Proof)

**Isolates all divergences** from three chains of loop momenta $\bar{q}_i$ into tadpoles




$$\bar{\mathcal{A}}_{2,\Gamma} = \underbrace{\left(\mathbf{S}^{(1)}_{X_1} + \mathbf{F}^{(1)}_{X_1}\right)}_{\bar{q}_1\text{-denominators}} \underbrace{\left(\mathbf{S}^{(2)}_{X_2} + \mathbf{F}^{(2)}_{X_2}\right)}_{\bar{q}_2\text{-denominators}} \underbrace{\left(\mathbf{S}^{(3)}_{X_3} + \mathbf{F}^{(3)}_{X_3}\right)}_{\substack{\bar{q}_3 = \bar{q}_1 + \bar{q}_2 \\ \text{-denominators}}} \bar{\mathcal{A}}_{2,\Gamma}$$

$$= \underbrace{\mathbf{S}^{(1)}_{X_1}\mathbf{S}^{(2)}_{X_2}\mathbf{S}^{(3)}_{X_3}\,\bar{\mathcal{A}}_{2,\Gamma}}_{\substack{\text{tadpoles } \bar{\mathcal{A}}_{2,\Gamma_{\text{tad}}} \\ \text{one master integral } F(1,1,1)}} + \text{non-global divergent terms}$$

where $\mathbf{S}^{(i)}_{X_i}$ := tadpole expansion of $\bar{q}_i$-dependent denominators that captures related sub- and global-divergences.

- **Only "simple" tadpoles** $\mathcal{A}_{2,\Gamma_{\text{tad}}}$ **contributes to two-loop** $\delta\mathcal{R}_{2,\Gamma}$ & $\delta Z_{2,\Gamma}$

  $\Rightarrow$ **polynomial** in external momenta and masses (upon subdivergence subtraction)

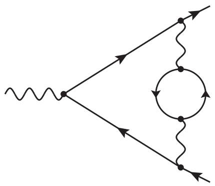
## Calculations of two-loop rational terms

**For practical calculation in terms of tadpoles** (recasting Ansatz)

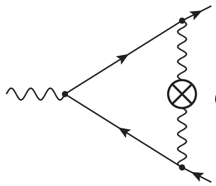
$$\delta\mathcal{R}_{2,\Gamma} = \left[\prod_{i=1}^{3}\mathbf{S}_{X_i}^{(i)}\,\bar{\mathcal{A}}_{2,\Gamma} + \sum_{i=1}^{3}\delta Z_{1,\gamma_i}\cdot\mathbf{S}_{X_i}^{(i)}\bar{\mathcal{A}}_{1,\Gamma/\gamma_i}\right]_{D_\mathsf{n}=D} - \left[\prod_{i=1}^{3}\mathbf{S}_{X_i}^{(i)}\,\mathcal{A}_{2,\Gamma} + \sum_{i=1}^{3}\left(\delta Z_{1,\gamma_i} + \delta\tilde{Z}_{1,\gamma_i} + \delta\mathcal{R}_{1,\gamma_i}\right)\cdot\left(\mathbf{S}_{X_i}^{(i)}\mathcal{A}_{1,\Gamma/\gamma_i}\right)\right]_{D_\mathsf{n}=4}$$

Example: QED vertex

$$\delta\mathcal{R}_{2,\Gamma} = \Bigg[\prod_{i=1}^{3}\mathbf{S}_{X_i}^{(i)}$$

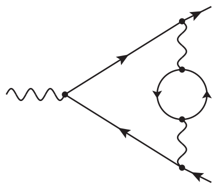


$$+\ \mathbf{S}_{X_1}^{(1)}$$

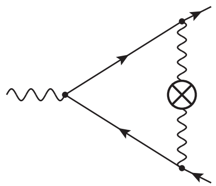


$$\delta Z_{1,\gamma}\Bigg]_{D_\mathsf{n}=D}$$

$$-\ \Bigg[\prod_{i=1}^{3}\mathbf{S}_{X_i}^{(i)}$$



$$+\ \mathbf{S}_{X_1}^{(1)}$$



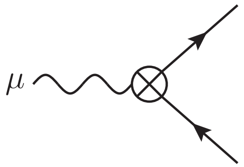
$$\left(\delta Z_{1,\gamma_i} + \delta\tilde{Z}_{1,\gamma_i} + \delta\mathcal{R}_{1,\gamma}\right)\Bigg]_{D_\mathsf{n}=4}$$

# QED two-loop rational terms in $\overline{\mathrm{MS}}$ scheme ($\xi = 0, m = 0$)

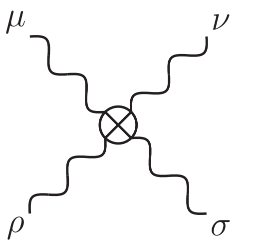
Calculating in GEXCOM [Chetyrkin,Zoller] framework: QGRAF [Noguira] →Q2E+EXP [Seidesticker, Harlander, Steinhauser] →FORM [Vermaseren] code →MATAD [Steinhauser]

$$\delta\mathcal{R}_{2,e} = i\frac{\alpha^2}{16\pi^2}\left(\frac{19}{18}\frac{1}{\varepsilon} + \frac{247}{108}\right)\not{p}$$


$$\delta\mathcal{R}^{\mu\nu}_{2,\gamma} = i\frac{\alpha^2}{16\pi^2}\left[(p^\mu p^\nu - g^{\mu\nu}p^2)\left(\frac{2}{3}\frac{1}{\varepsilon} - \frac{71}{18}\right) + g^{\mu\nu}\left(-\frac{11}{12}p^2\right)\right]$$




$$\delta\mathcal{R}^{\mu}_{2,ee\gamma} = ie\frac{\alpha^2}{16\pi^2}\gamma^\mu\left(\frac{13}{9}\frac{1}{\varepsilon} + \frac{191}{27}\right)$$




$$\delta\mathcal{R}^{\mu\nu\rho\sigma}_{2,4\gamma} = i\frac{\alpha^3}{4\pi}(-3)\left(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right)$$

- $\delta\mathcal{R}_{2,\Gamma}$ are **polynomial** in $p$ and **independent** of auxiliary tadpole mass $M$ ⇒ local counterterm
  (Full results with $R_\xi$ gauge and electron mass dependence in upcoming paper)

## Summary

- **Renormalised** $D$-dim two-loop amplitude can be **reconstructed** by amplitude with $4$-dim numerator

$$\mathbf{R}\,\bar{\mathcal{A}}_{2,\Gamma} \;=\; \mathcal{A}_{2,\Gamma} + \sum_{i=1}^{3}(\delta Z_{1,\gamma_i} + \delta\tilde{Z}_{1,\gamma_i} + \delta\mathcal{R}_{1,\gamma}) \cdot \mathcal{A}_{1,\Gamma/\gamma_i} + (\delta Z_{2,\Gamma} + \delta\mathcal{R}_{2,\Gamma}) + \mathcal{O}(\varepsilon)\,.$$

$\Rightarrow$ numerical implementation in automated tools becomes possible.

- We provide a **generic method to compute** $\delta\mathcal{R}_{2,\Gamma}$ from one-scale tadpoles, and show that $\delta\mathcal{R}_{2,\Gamma}$ is **local counterterm**.

- Full set of QED rational terms at two loops.

# Backup

## A brief introduction to OpenLoops 2 [ArXiv:1907.13071]

**OpenLoops** is a fully automated numerical tool for tree and one-loop amplitudes computation.

- Download at https://openloops.hepforge.org
- Full NLO QCD and EW corrections available
  More than 200 processes libraries available for all relevant SM processes (+HEFT)
  Additional libraries provided upon user requests
- Fast CPU performance and excellent numerical stability

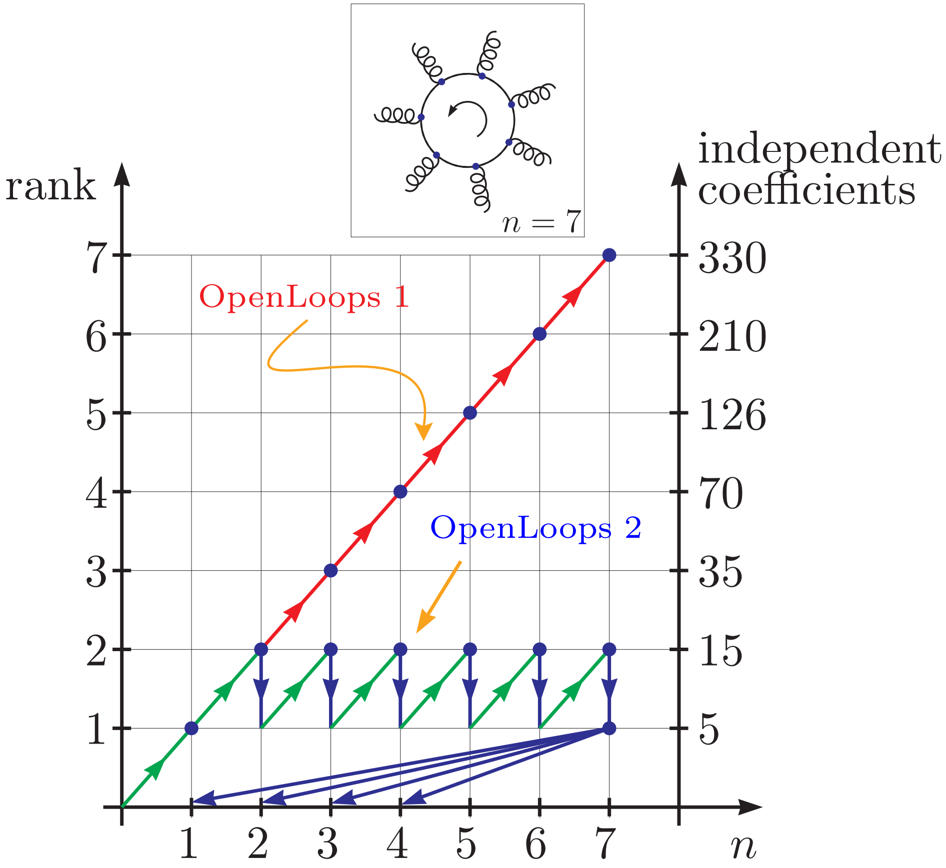
# Applications of OpenLoops 2

- **Interfaces to many Monte Carlo programs**
  Sherpa [Höche, Krauss, Schönherr, Siegert et al.]
  Munich/Matrix [Grazzini, Kallweit, Rathlev, Wiesemann]
  NNLOJET [Currie, Chen, Gehrmann, Glover, Huss et al.]
  Powheg [Nason, Oleari et al.], Herwig [Gieseke, Plätzer et al.]
  Geneva [Alioli, Bauer, Tackmann et al.], Whizard [Kilian, Ohl, Reuter et al.]

- **OpenLoops 2 applications (2019)**
  NNLO QCD + NLO EW Vector-boson pair production [Grazzini, Kallweit, Lindert, Pozzorini, Wiesemann]
  NNLO QCD Three-photon production [Chawdhry, Czakon, Mitov, Poncelet]
  NNLO QCD Spin correlations in $t\bar{t}$ production [Behring, Czakon, Mitov, Papanastasiou, Poncelet]
  NLO QCD $t\bar{t}b\bar{b}$+jet production [Buccioni, Kallweit, Pozzorini, Zoller]
  and more NNLO QCD results to appear with Matrix and NNLOJET collaborations

# Why OpenLoops 2 is used at NNLO? ⇒ fast and accurate

OpenLoops 2 implements **on-the-fly reduction** [Buccioni, Pozzorini, Zoller, 18'] that unifies amplitude constructions and reductions in a single recursion




| processes | # diagrams | $t$ [s] per PSP |
|---|---|---|
| $gg \to t\bar{t}gg$ | $\sim 9 \times 10^3$ | 0.60 |
| $gg \to t\bar{t}ggg$ | $\sim 160 \times 10^3$ | 21.15 |
| $u\bar{u} \to t\bar{t}gg$ | $\sim 1.6 \times 10^3$ | 0.07 |
| $u\bar{u} \to t\bar{t}ggg$ | $\sim 25 \times 10^3$ | 2.08 |
| $u\bar{u} \to W^+W^-gg$ | $\sim 1 \times 10^3$ | 0.15 |
| $u\bar{u} \to W^+W^-ggg$ | $\sim 13 \times 10^3$ | 3.66 |

## Why OpenLoops 2 is used at NNLO? $\Rightarrow$ fast and accurate

**Numerical instabilities** arise form reduction methods for evaluation of coefficients of master integrals, which can have large cancellations due to spurious singularities.

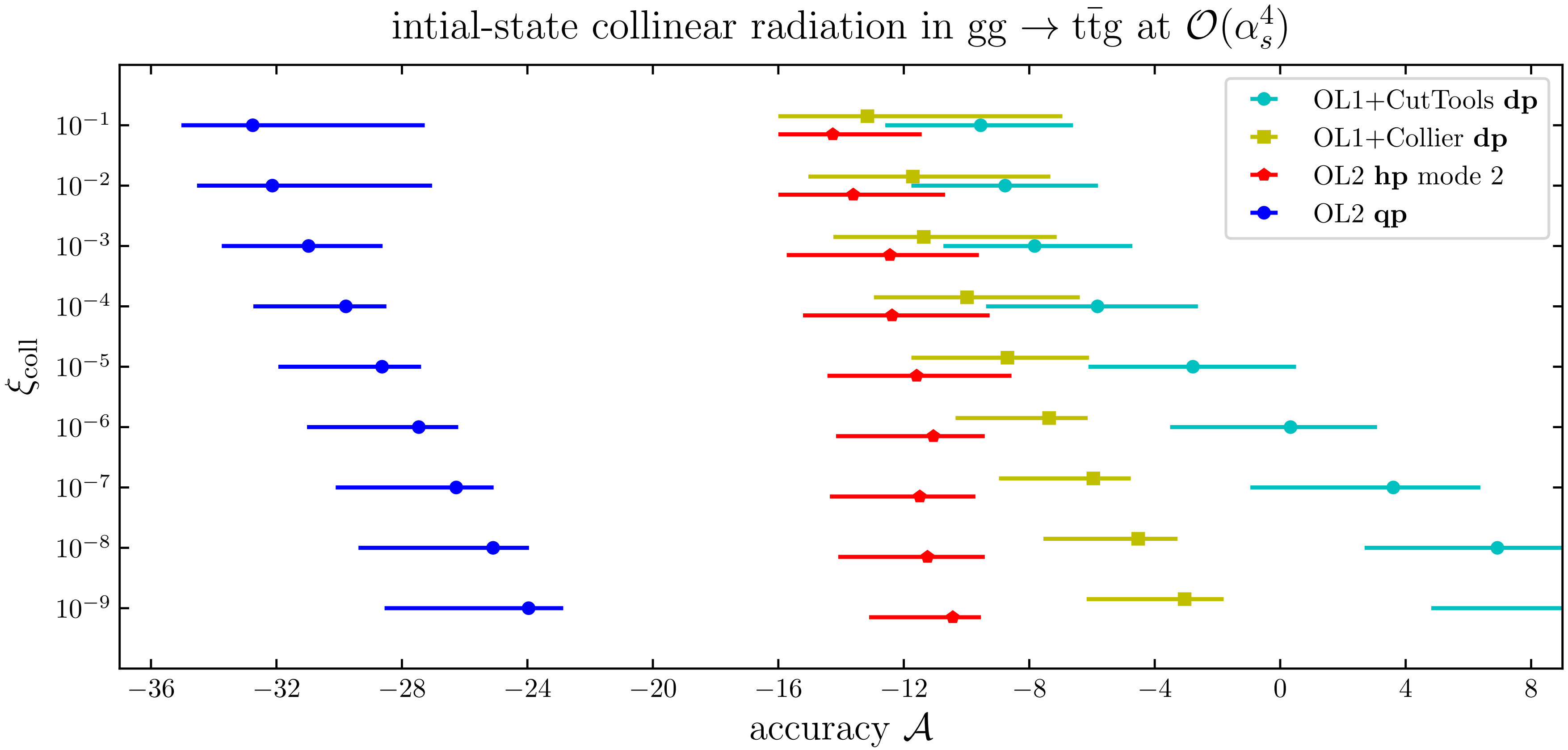
**Source of instabilities** are the inverse of small rank-2 & rank-3 Gram determinants in critical kinematical regions (both hard and infrared regions)

**Solutions:** on-the-fly stability system

- **Analytical:** Any-order triangle diagram expansion (reduction + master integral expansion)
- **Numerical:** Hybrid precision framework (double + quadruple precision)

# OpenLoops 2 stability

$2 \to 3$ process: Real-Virtual contribution (NNLO) with collinear gluon pair ($\xi_{\text{coll}} = \theta_{ij}^2$)



## One-loop subdiagram example: photon self-energy

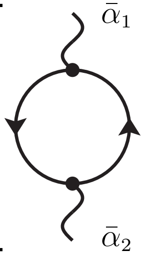
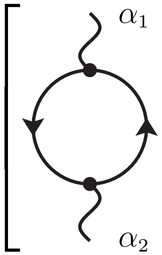
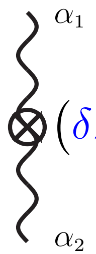
Let $D_{\mathsf{n}} \in \{D, 4\}$ be the dimension of numerator, we have

$$D_{\mathsf{n}} = D \quad \Rightarrow \quad \bar{\mathbf{K}} \int \mathrm{d}\bar{q}_1 \frac{-\mathrm{Tr}\left[\bar{\gamma}^{\bar{\alpha}_1} \bar{\not{q}}_1 \bar{\gamma}^{\bar{\alpha}_2} (\bar{\not{q}}_1 + \bar{\not{q}}_2)\right]}{\bar{q}_1^2\, (\bar{q}_1 + \bar{q}_2)^2} \;=\; \frac{1}{\varepsilon}\Bigg( \underbrace{-\frac{4}{3}\left(\bar{q}_2^2\, g^{\bar{\alpha}_1\bar{\alpha}_2} - \bar{q}_2^{\bar{\alpha}_1}\bar{q}_2^{\bar{\alpha}_2}\right)}_{-\delta Z_{1,\gamma}(\bar{q}_2)} \; \underbrace{+\frac{2\,\varepsilon}{3}\,\tilde{q}_2^2\, g^{\bar{\alpha}_1\bar{\alpha}_2}}_{\delta\mathcal{R}_{1,\gamma}(q_2) + \mathcal{O}(\varepsilon)} \Bigg)$$

and

$$D_{\mathsf{n}} = 4 \quad \Rightarrow \quad \mathbf{K} \int \mathrm{d}\bar{q}_1 \frac{-\mathrm{Tr}\left[\gamma^{\alpha_1} \not{q}_1 \gamma^{\alpha_2} (\not{q}_1 + \not{q}_2)\right]}{\bar{q}_1^2\, (\bar{q}_1 + q_2 + \tilde{q}_2)^2} \;=\; \frac{1}{\varepsilon}\Bigg( \underbrace{-\frac{4}{3}\left(q_2^2\, g^{\alpha_1\alpha_2} - q_2^{\alpha_1} q_2^{\alpha_2}\right)}_{-\delta Z_{1,\gamma}(q_2)} \; \underbrace{-\frac{2}{3}\,\tilde{q}_2^2\, g^{\alpha_1\alpha_2}}_{-\delta\tilde{Z}_{1,\gamma}(\tilde{q}_2)} \Bigg)$$

$\Rightarrow$ Renormalised photon self-energy insertion:

$\Bigg[$  $+$  $\delta Z_{1,\gamma}(\bar{q}_2) \Bigg]_{D_{\mathsf{n}}=D} = \Bigg[$  $+$  $\Big(\delta Z_{1,\gamma}(q_2) + \delta\tilde{Z}_{1,\gamma}(\tilde{q}_2) + \delta\mathcal{R}_{1,\gamma}(q_2)\Big) \Bigg]_{D_{\mathsf{n}}=4} + \mathcal{O}(\varepsilon)$